Geodesic motion and kludge waveforms from a test-mass orbiting around 'quasi-Kerr' object.

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- provide evidence in favour of GR’s ‘no-hair’ theorem (uniqueness of Kerr metric)
- reveal the true identity of the ‘dark objects’ in galactic nuclei (Kerr BH vs Boson stars, ...)

Building ‘quasi-Kerr’ metric

- Start with the exterior Hartle-Thorne metric describing the space-time of any axisymmetric & stationary body up to $\mathcal{O}(J^2)$ accuracy:

$$g_{\alpha\beta}^{HT} = g_{\alpha\beta}^{HT,Kerr}(up\, to\, \mathcal{O}(J^2)) + \epsilon h^{HT} + \mathcal{O}(J^{3+}),$$

where $\epsilon = -Q/M^3 - \left(J/M^2\right)^2$. For Kerr BH $\epsilon = 0$, $Q/M^3 = -\left(J/M^2\right)^2$, for NS $Q/M^3 = -\alpha \left(J/M^2\right)^2$, where $\alpha \in [2, 12]$.

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- Transform to Boyer-Lindquist coordinates and

$$g_{\alpha\beta}^{QK} = g_{\alpha\beta}^{Kerr} + \epsilon h_{\alpha\beta}$$

Depends on M, $a = J/M^2$, ϵ.

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- Non-separable for generic orbits, lack of Carter constant.
- Employ canonical perturbation theory: $H = H_0 + \epsilon H_1$. Constants of motion $\{Q_0, P_0\}$ at zero-order, now should satisfy (at first order)

$$\dot{Q} = -\epsilon \left(\frac{\partial H_1}{\partial P}\right)_0 \qquad \dot{P} = \epsilon \left(\frac{\partial H_1}{\partial Q}\right)_0 .$$

Isolate secular changes by time averaging.

$$P = < \dot{P} > t + P_0, \qquad Q = < \dot{Q} > t + Q_0$$

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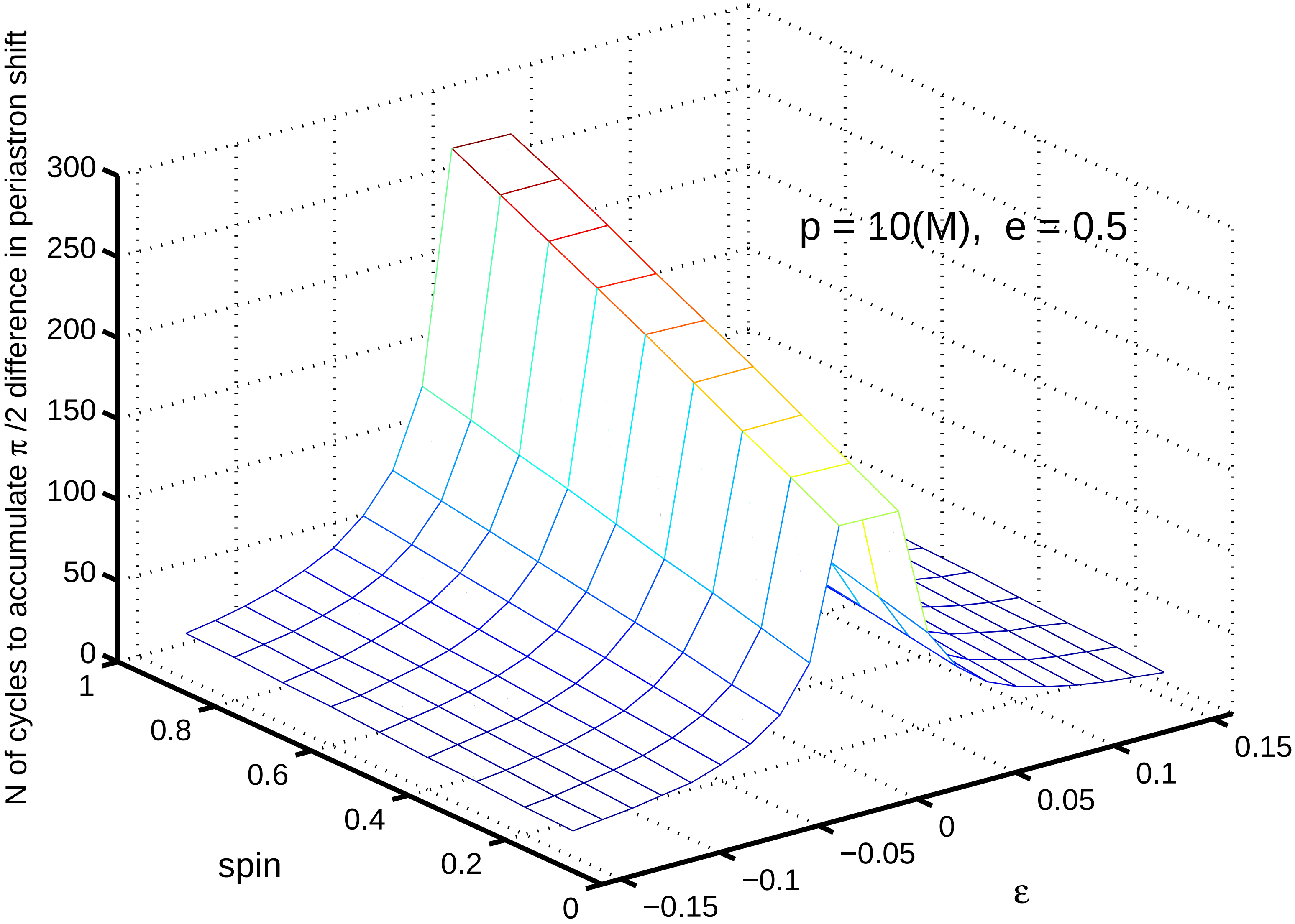
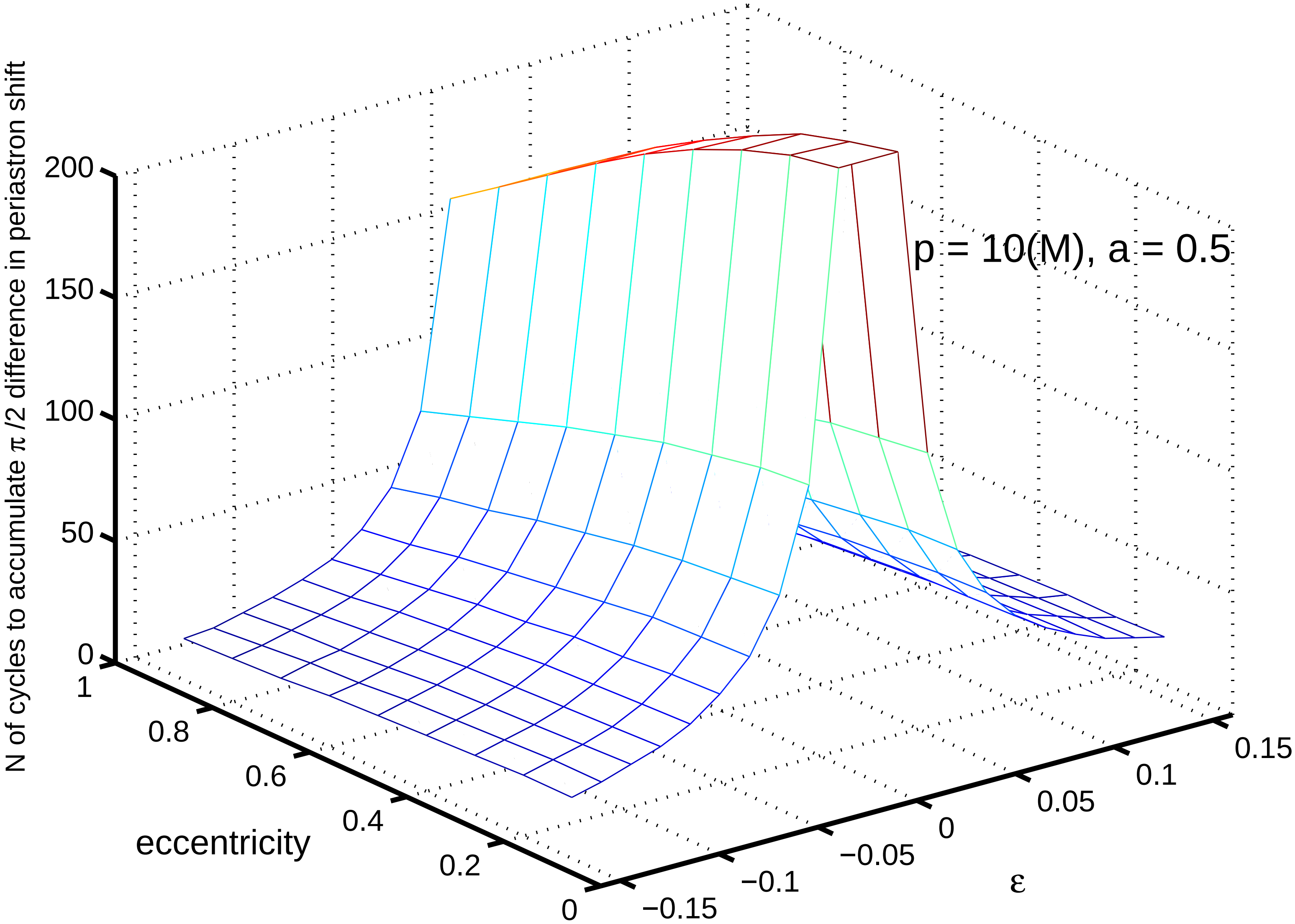
- Study periastron shift for Kerr and ‘quasi-Kerr’ orbits: $\Delta\phi_{Kerr} - \Delta\phi_{qK}$, where

$$\Delta\phi = \int_0^{2\pi} \phi(\chi)\, d\chi.$$

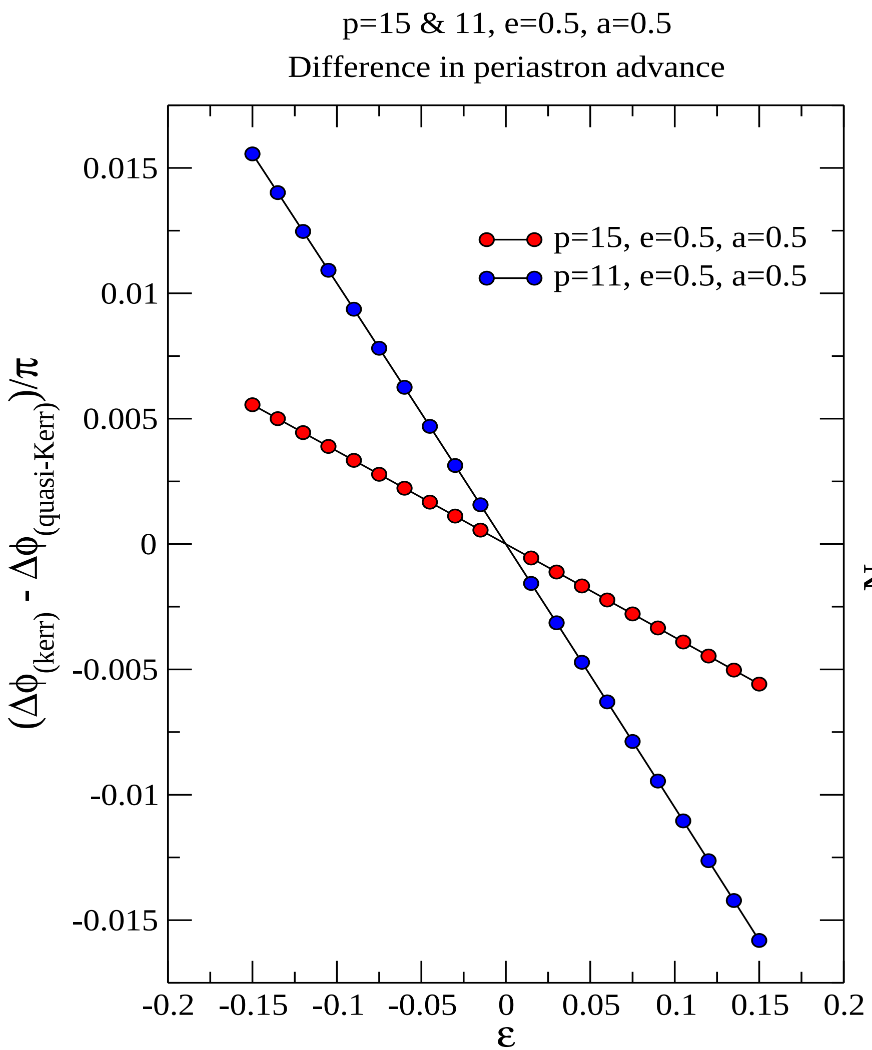
Periastron shift 1

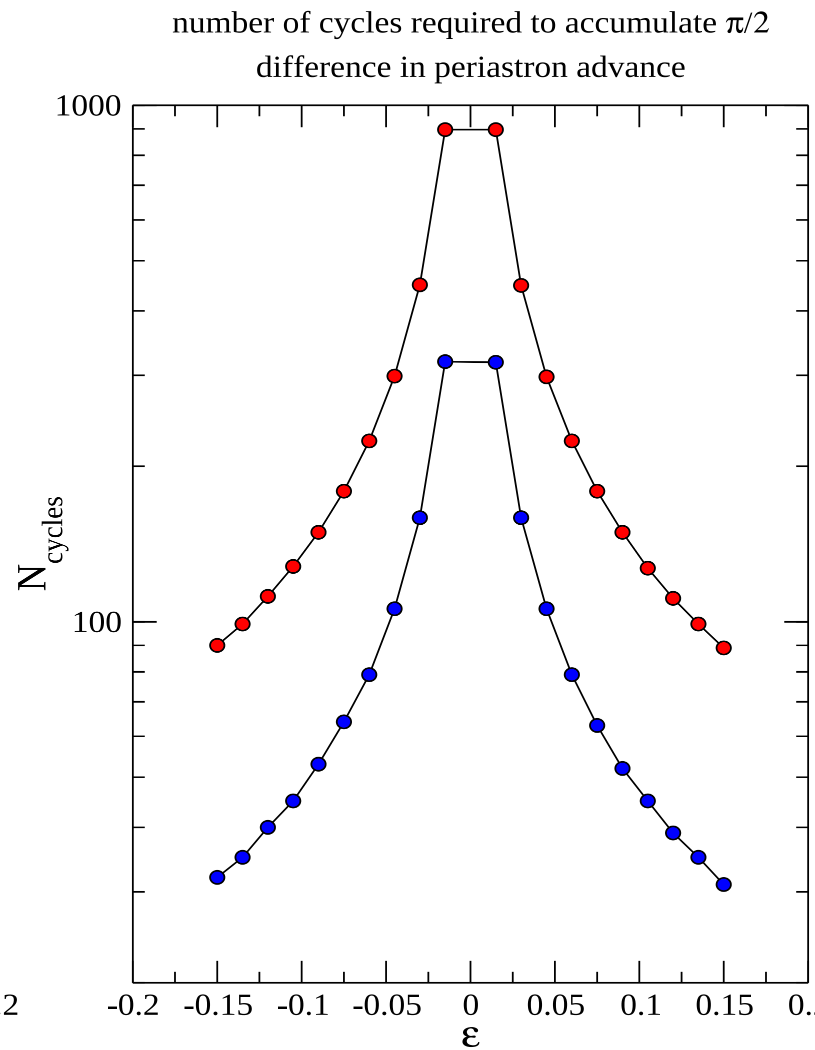


Periastron shift 2



Periastron shift 3





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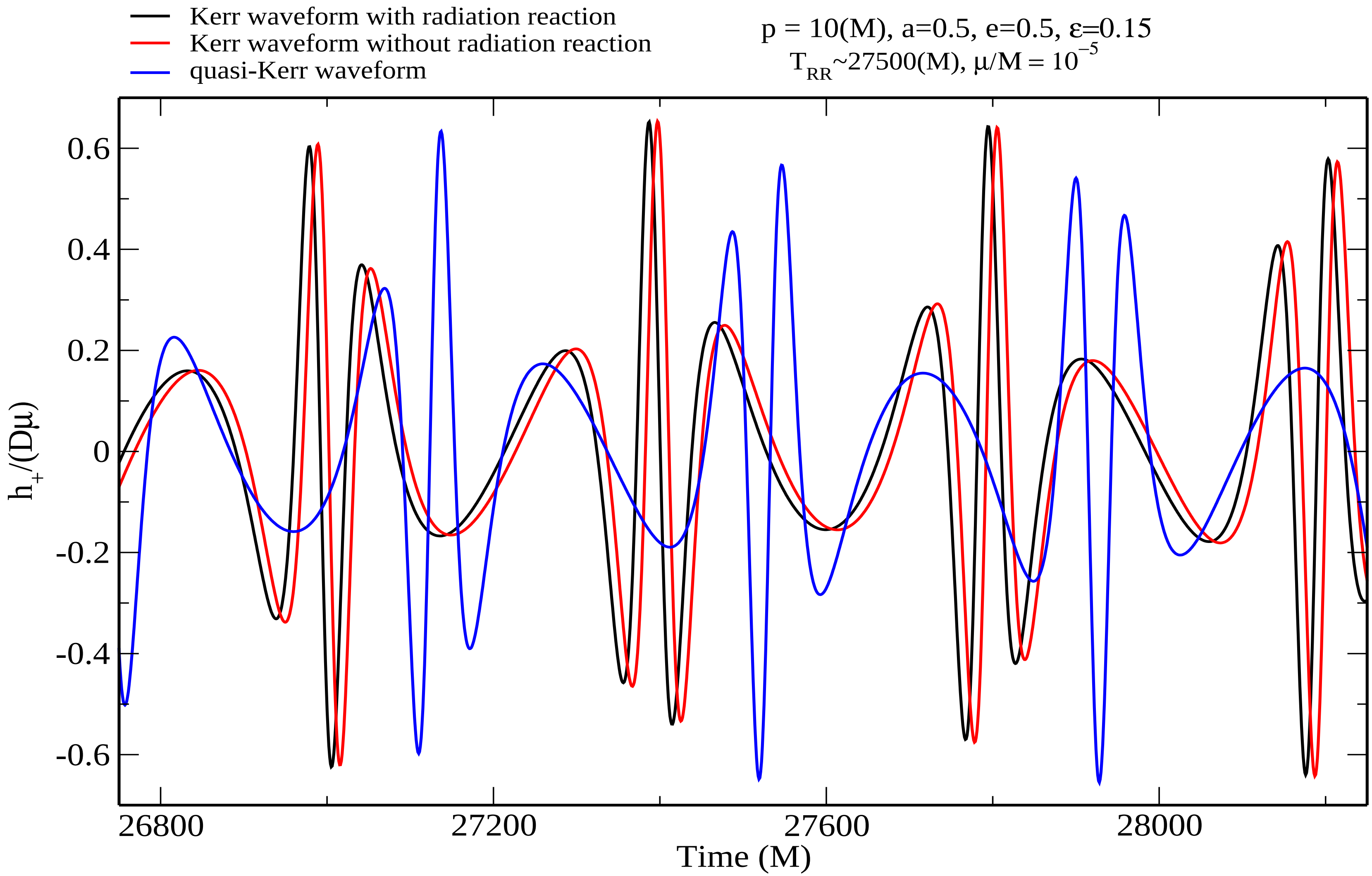
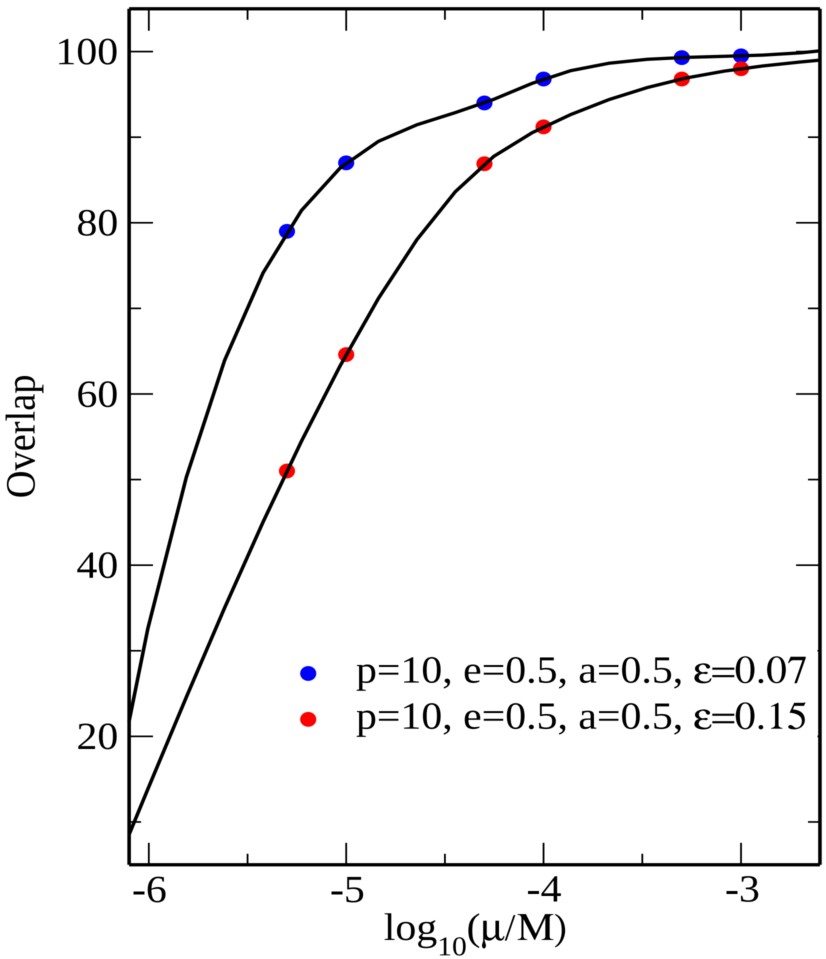
- Compute overlaps between Kerr and 'quasi-Kerr' waveforms without radiation reaction, but truncated at T_{RR} defined above.

Comparison of waveforms

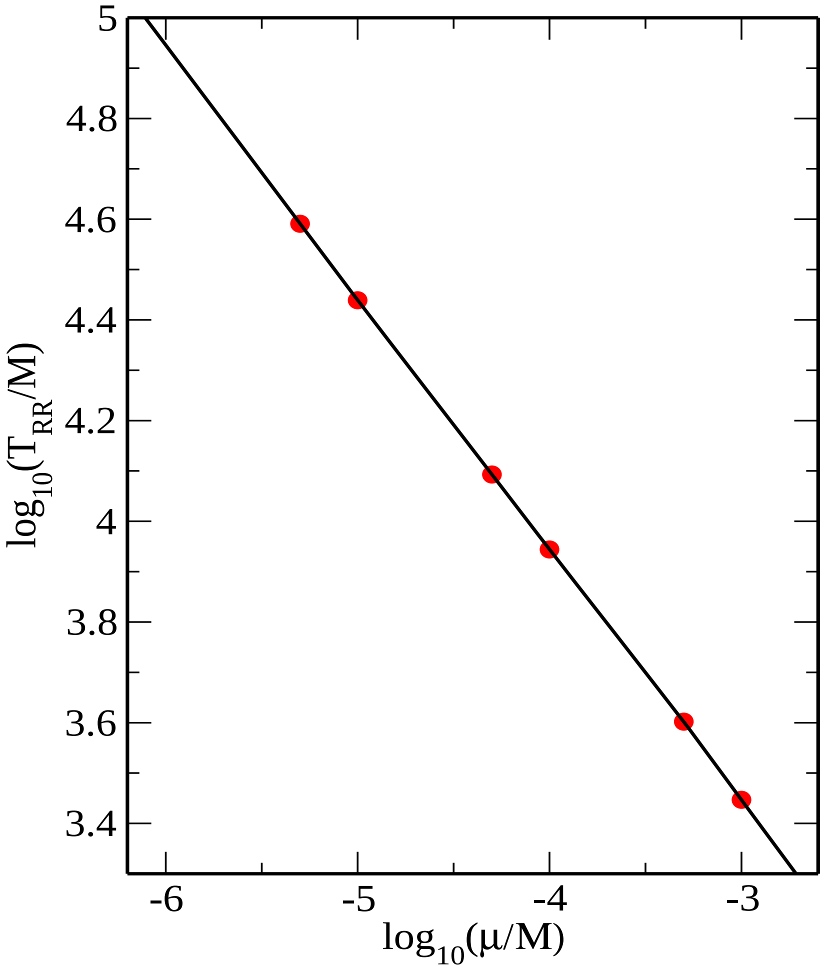


Overlaps

Overlap between waveforms (of T_{RR} duration) from test-body orbiting Kerr and quasi-Kerr objects as function of mass ratio



Radiation reaction (T_{RR}) time scale as a function of mass ratio



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- Compute geodesic motion
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- Construct kludge waveforms without RR.
- Quantify difference in waveforms by computing overlaps between Kerr and ‘quasi-Kerr’ waveforms truncated at T_{RR}.