Estimating spinning binary parameters and testing alternative theories of gravity with LISA

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Content:

• Binary-parameter–accuracy reduction when spin couplings are included

• How results depend on sensitivity of LISA at low frequencies

9th Gravitational-Wave Data Analysis Workshop
Testing alternative theories of gravity with LISA

[Will 94; Krolak et al. 95; Damour et al. 97; Will 98; Scharre & Will 02; Will & Yunes 04]

- **Scalar-tensor theories**: phasing modified by GW dipole radiation
- **Massive graviton theories**: GW–propagation-speed depends on wavelength \( \Rightarrow \) distortion in time of arrival with respect to GR

\[
\frac{df}{dt} = \frac{96}{5 \pi \mathcal{M}^2} \left( \pi \mathcal{M} f \right)^{11/3} \left\{ 1 + \frac{5 \hat{\alpha}^2 \eta^{2/5}}{192 \omega_{BD}} \left( \pi \mathcal{M} f \right)^{-2/3} + \frac{96 \pi^2 \mathcal{M} D}{5 (1+z) \chi_g^2} \left( \pi \mathcal{M} f \right)^{2/3} + \text{PN corr.} \right\}
\]

\[\mathcal{M} = M \eta^{3/5} \rightarrow \text{observed chirp mass}; \quad M = m_1 + m_2 \text{ and } \eta = m_1 m_2 / M^2\]

- \( \hat{\alpha} = \hat{\alpha}_1 - \hat{\alpha}_2, \quad \hat{\alpha}_{\text{BH}} = 0, \quad \hat{\alpha}_{\text{NS}} \sim 0.6 - 0.8, \quad \hat{\alpha}_{\text{WD}} > 0.998\)
  \[\omega_{\text{BD}} > 4 \times 10^4 \text{ (Cassini)}\]
- \( \lambda_g \rightarrow \text{graviton Compton length}; \quad \lambda_g > 10^{12} \text{ km (solar-system), } \lambda_g > 6 \times 10^{19} \text{ km (cluster dynamics)}\)
Effective LISA noise curve and binary investigated

- BH-BH binaries with $M = 10^4 - 10^7 M_\odot$
- NS-IMBH binaries with $M_{\text{IMBH}} = 10^2 - 10^4 M_\odot$

- At low frequency the effective noise curve coincides with non-sky-averaged LISA curve
- Galactic and extragalactic WD confusion noise included [Barack & Cutler 03]
GW templates for binaries moving along circular orbits

\[ \tilde{h}_\alpha(f) = A f^{-7/6} e^{i\psi(f)} A_\alpha(t(f)) e^{-i\phi(t(f))} \]

\[ \psi(f) = 2\pi ft_c - \phi_c + \frac{3}{128} (\pi M f)^{-5/3} \left\{ 1 - \frac{5\alpha^2}{336\omega_{BD}} \eta^{2/5} (\pi M f)^{-2/3} - \frac{128}{3} \frac{\pi^2 D M}{\lambda_g^2 (1 + z)} (\pi M f)^{2/3} \right\} \]

\[ + \left( \frac{3715}{756} + \frac{55}{9} \eta \right) \eta^{-2/5} (\pi M f)^{2/3} - 16\pi \eta^{-3/5} (\pi M f) + 4\beta \eta^{-3/5} (\pi M f) \]

\[ + \left( \frac{15293365}{508032} + \frac{27145}{504} \eta + \frac{3085}{72} \eta^2 \right) \eta^{-4/5} (\pi M f)^{4/3} - 10\sigma \eta^{-4/5} (\pi M f)^{4/3} \left\} \right. \]

\[ \beta = \frac{1}{12} \sum_{i=1}^{2} \chi_i \left[ 113 \frac{m_i^2}{M^2} + 75\eta \right] \mathbf{\hat{L}} \cdot \mathbf{\hat{S}}_i, \quad \sigma = \frac{\eta}{48} \chi_1 \chi_2 \left( -27 \mathbf{\hat{S}}_1 \cdot \mathbf{\hat{S}}_2 + 721 \mathbf{\hat{L}} \cdot \mathbf{\hat{S}}_1 \mathbf{\hat{L}} \cdot \mathbf{\hat{S}}_2 \right) \]

Effects of spins in GR for ground-based detectors [Poisson et al. 95; Krolak et al. 95]
LISA model and Monte Carlo simulation

• **We assume two independent Michelson outputs** [Cutler 97; Hughes 01; Vecchio 03]

• **Parameter estimation using Fisher matrix formalism with and without averaging over the relative orientation of the binary with respect to LISA**

• **Monte Carlo simulation using** \(10^4\) **sources distributed over sky positions and orientation**

\[
\begin{align*}
(\overline{\phi}_S, \cos \overline{\theta}_S) & \Rightarrow \text{binary position with respect to solar-system baricenter} \\
(\overline{\phi}_L, \cos \overline{\theta}_L) & \Rightarrow \text{binary orientation with respect to solar system baricenter}
\end{align*}
\]

**angular resolution:**

\[
\Delta \Omega_S = 2\pi \left\{ \langle \Delta \overline{\mu}_S^2 \rangle \langle \Delta \overline{\phi}_S^2 \rangle - \langle \Delta \overline{\mu}_S \Delta \overline{\phi}_S \rangle^2 \right\}^{1/2}
\]

\[
\overline{\mu}_S = \cos \overline{\theta}_S
\]
Results in Einstein’s theory when including spin couplings

Compact body inspiralling into IMBH: $M = (1.4 + 10^3)M_\odot$ with SNR = 10

$\Delta \beta \propto \text{spin-orbit}$
Results for Brans-Dicke parameter

Compact body inspiralling into IMBH: $M = (1.4 + 10^3)M_\odot$ with SNR = 10

\[ f_{\text{in}} = 3.7 \times 10^{-2} \text{ Hz}; \quad f_{\text{fin}} = 1 \text{ Hz} \]

- Number of cycles
  - Newtonian: $1.83 \times 10^6$
  - 1PN: 44712
  - 1.5PN: -29081
  - Spin-orbit: 2314 $\beta$
  - 2PN: 868
  - Spin-spin: -288 $\sigma$
  - Brans-Dicke: $-45 \omega_{\text{Cassini}}/\omega_{\text{BD}}$

Bound from Cassini measurements of Shapiro time delay: $\omega_{\text{BD}} = 40000$
Summary of results for Brans-Dicke parameter

NS-IMBH binaries

\[ \omega_{BD} \]

- BD
- BD+SO
- BD+SO+SS

\[ M_{IMBH}(M_{\text{sun}}) \]

\[ 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \]

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Results in Einstein’s theory when including spin couplings

\[ M = (10^6 + 10^6) M_\odot \text{ at 3 Gpc} \]

\[ \beta \propto \text{spin-orbit} \]
Results for graviton’s Compton length

\[ M = (10^6 + 10^6) M_\odot \text{ at } 3 \text{ Gpc} \]

\[ f_{\text{in}} = 4.5 \times 10^{-5} \text{ Hz}; \quad f_{\text{fin}} = 2.2 \times 10^{-3} \text{ Hz} \]

\[ \lambda_g = h/c/m_g \]

Number of cycles

- Newtonian: 2266
- 1PN: 134
- 1.5PN: -92
- Spin-orbit: 7 \beta
- 2PN: 6
- Spin-spin: -1 \sigma
- Massive graviton: \(-217(10^{14} \text{ km}/\lambda_g)^2\)
Summary of results for graviton’s Compton length

Equal-mass binaries at 3 Gpc

\begin{center}
\begin{tikzpicture}
\begin{loglogaxis}[
   title={\textit{\lambda_g} vs. $M (M_{\odot})$},
   xlabel={$M (M_{\odot})$},
   ylabel={$\lambda_g$},
   xtick={1e4,1e5,1e6,1e7,1e8},
   ytick={1e14,1e15,1e16,1e17},
   grid=major,
   width=\textwidth,]

\addplot [solid,mark=o,mark options={solid},mark size=2pt] coordinates {
(1e4,1e14)
(1e5,1e15)
(1e6,1e16)
(1e7,1e17)
(1e8,1e17)
};
\addlegendentry{MG}

\addplot [dashed,mark=square,mark options={solid},mark size=2pt] coordinates {
(1e4,1e14)
(1e5,1e15)
(1e6,1e16)
(1e7,1e17)
(1e8,1e17)
};
\addlegendentry{MG+SO}

\end{tikzpicture}
\end{center}
Effect of worsening LISA sensitivity at low frequencies

\[ D_L = 3 \text{ Gpc} \quad f_{\text{cut}} = 10^{-5} \text{ Hz} \text{ (continuous lines)} \quad f_{\text{cut}} = 10^{-4} \text{ Hz} \text{ (dashed lines)} \]
Effect of worsening LISA sensitivity at low frequencies and the bound on graviton Compton wavelength

\[ D_L = 3 \text{ Gpc} \quad f_{\text{cut}} = 10^{-5} \text{ Hz (dashed continuous)} \quad f_{\text{cut}} = 10^{-4} \text{ Hz (dashed lines)} \]
Conclusions/future work:

- Binary parameters in gravity-wave’s phase are highly correlated ⇒ adding parameters effectively dilutes the available information

- Accuracy’s degradation of binary parameters when black holes carry spins aligned or antialigned with angular momentum
  \[ \sim 10 \text{ - } 20 \text{ with SO term; } \sim 30 \text{ - } 80 \text{ with SO } + \text{ SS terms for Brans-Dicke bound} \]
  \[ \sim 4 \text{ - } 5 \text{ with SO term for massive graviton bound} \]

- Spin precession could help in decorrelating the parameters [Vecchio 03]

- Inclusion of eccentricity when studying binaries in scalar-tensor theories

- Systematic errors might not be negligible with respect to statistical errors for high SNR