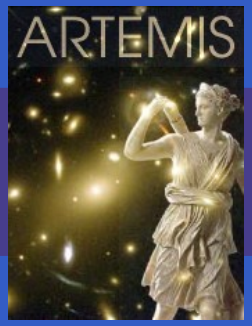


Rational Approximation applied to Spectral Estimation and Whitening

Orsay - 2|XI|2004

Spectral Analysis

Power Spectrum Density $\mathscr{E}_N(\nu) = \sum_{k\in\mathbb{Z}} C_k \; e^{ik\nu}$

correlation

frequency ν

for the whitening of the data we need an estimator of the PSD

in the complex plane of the variable $z \equiv e^{i\nu}$

the real axis of frequencies is mapped on the unit circle

Linear Model

AR(p) system

gaussian white noise

$$X(n) = \sum_{k=1}^{p} a_k \, X(n-k) \; + \; \phi(n)$$

PSD $\quad \mathscr{E}_N(z) \simeq \dfrac{1}{L_p(z) L_p(1/z)}$

characteristic polynomial

$L_p(z) \equiv 1 - a_1 z - \cdots a_p \; z^p$

Rational Estimator with poles only

Padé Approximants

Taylor serie $f(z)$ $[q/p]_f$ rational fraction

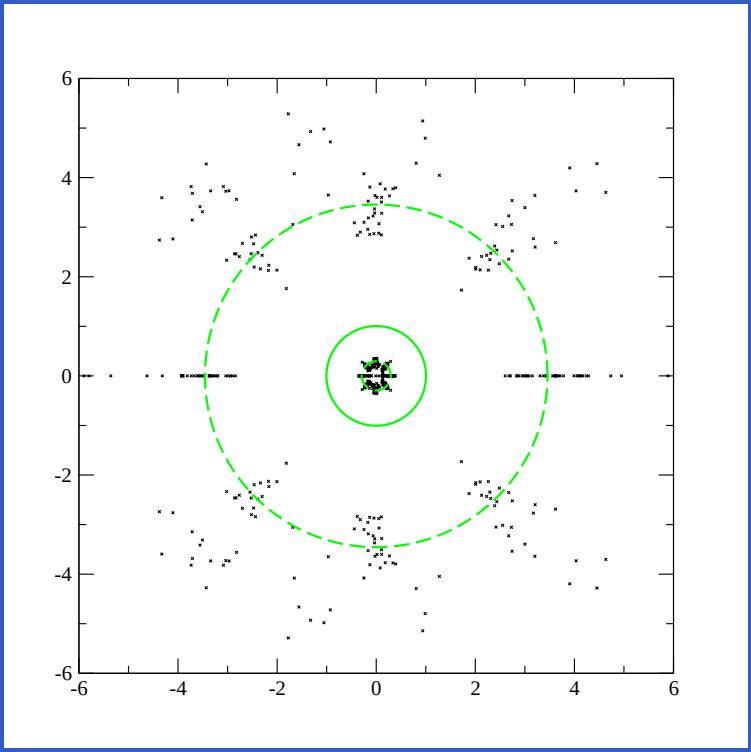
$$[q/p]_f(z) - f(z) = \mathscr{O}(z^{p+q+1})$$

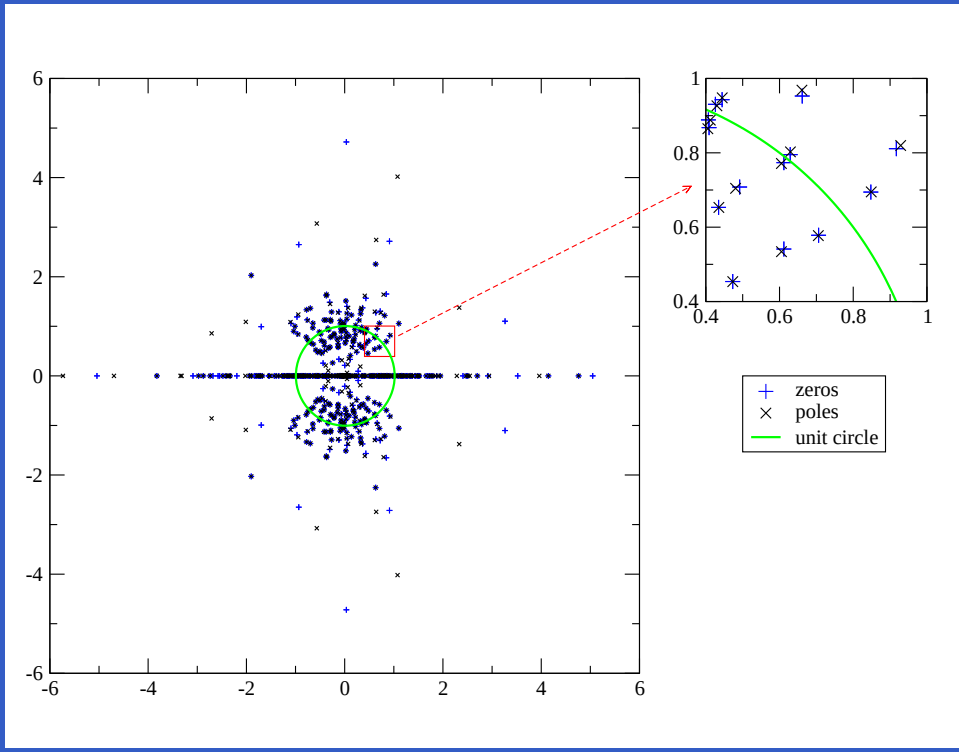
$$\mathscr{E}_+(z) \equiv \sum_{k\geq 0} C_k z^k \qquad \mathscr{E}_N(z) = \mathscr{E}_+(z) + \mathscr{E}_+(1/z) - C_0$$

New rational estimator (with poles *and* zeros)

$$[p\text{-}1/p]_{\mathscr{E}_+}(z) \; + \; [p\text{-}1/p]_{\mathscr{E}_+}(\tfrac{1}{z}) - C_0$$

Test on a gaussian white noise



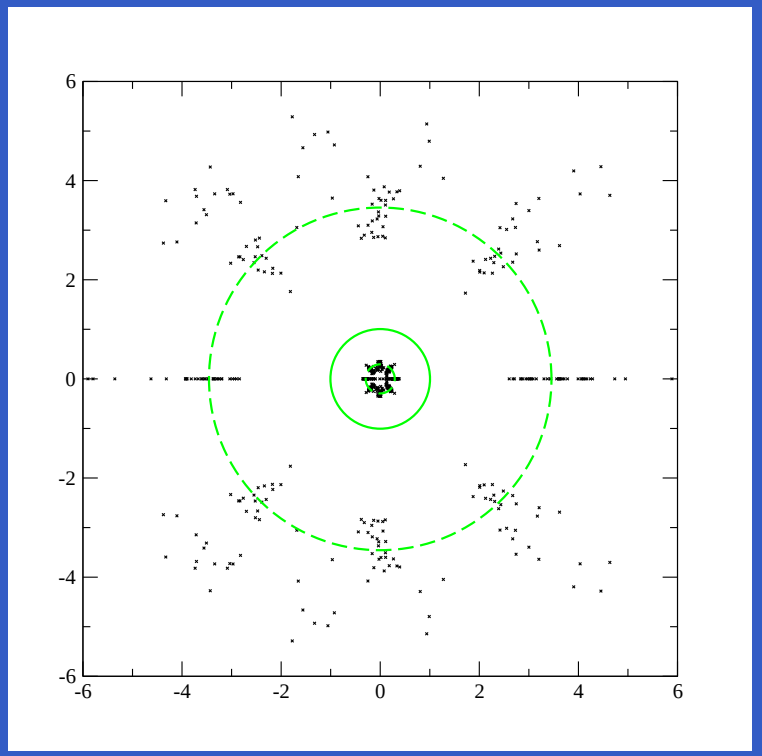


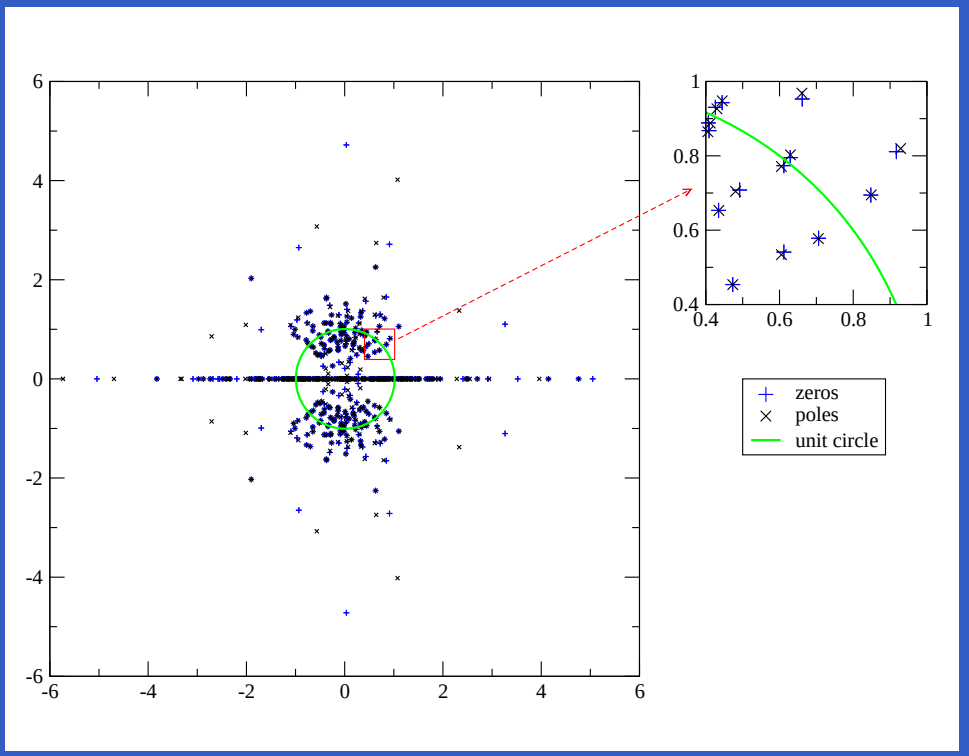
poles at the origin and infinity

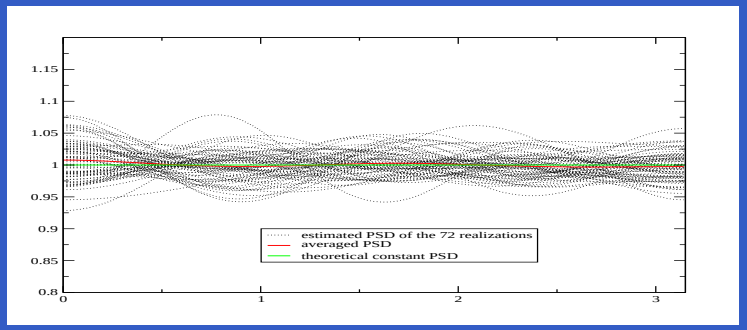
zero-pole pairs "Froissart doublets"

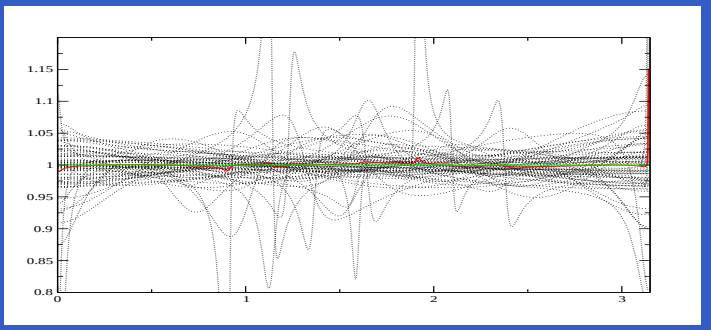
(the theoretical PSD is constant)

Test on a gaussian white noise



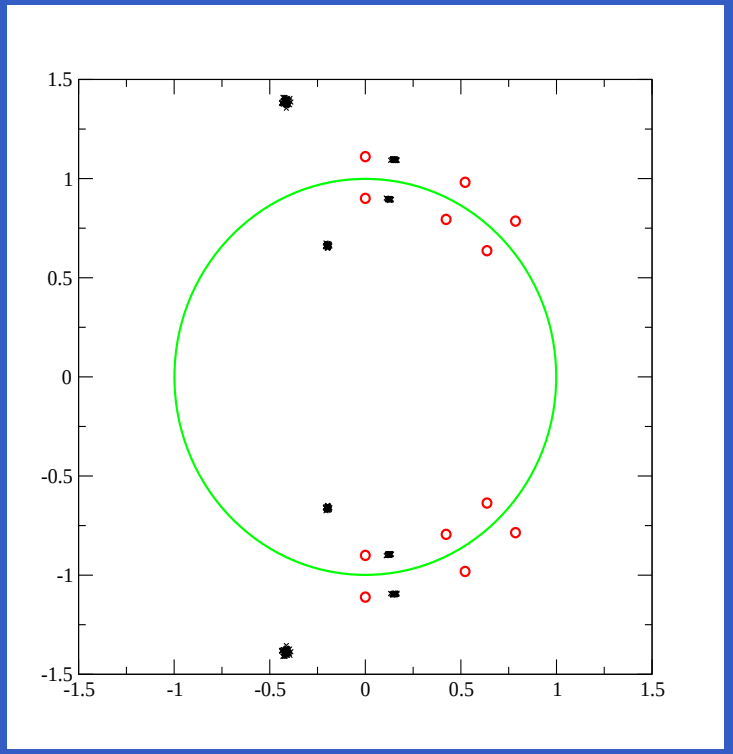


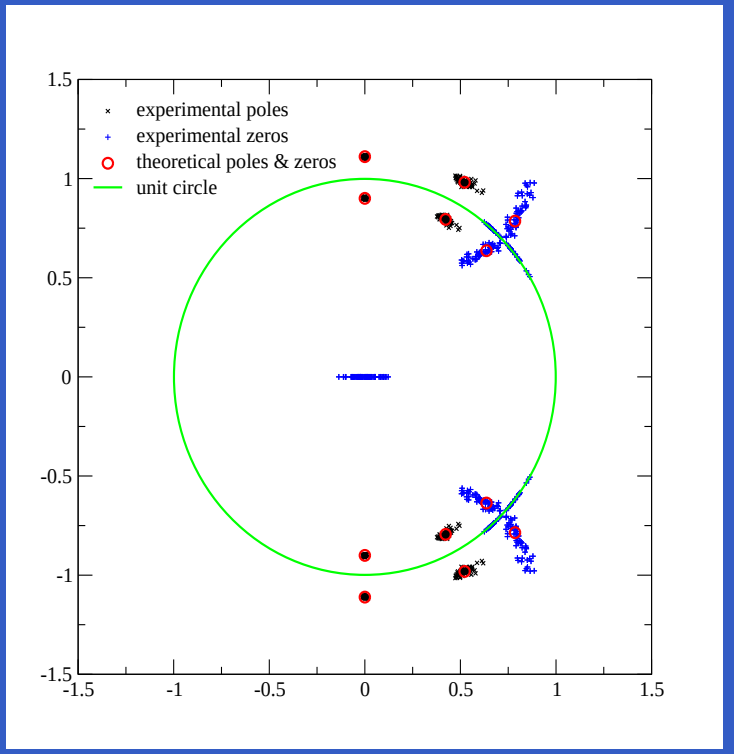


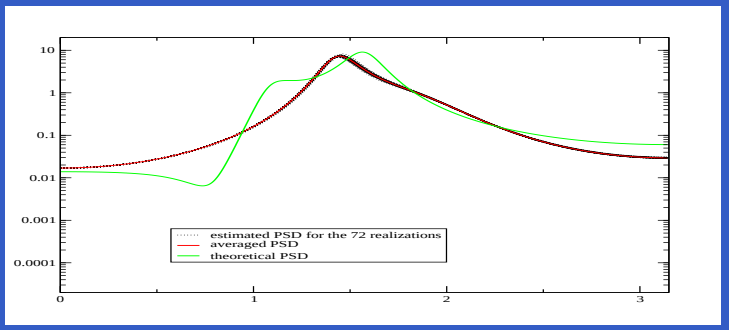


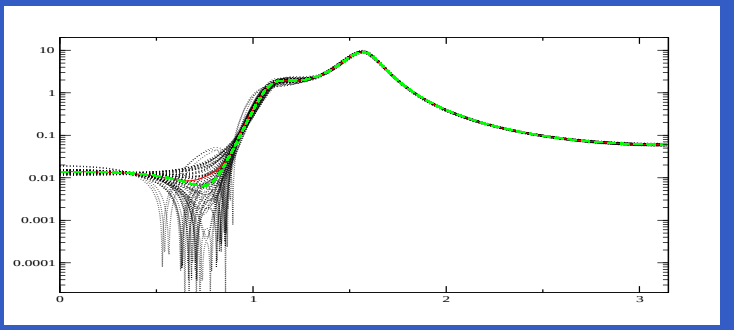
Test on a linear system AR4MA2

rational
PSD
with
6 zeros
and
8 poles

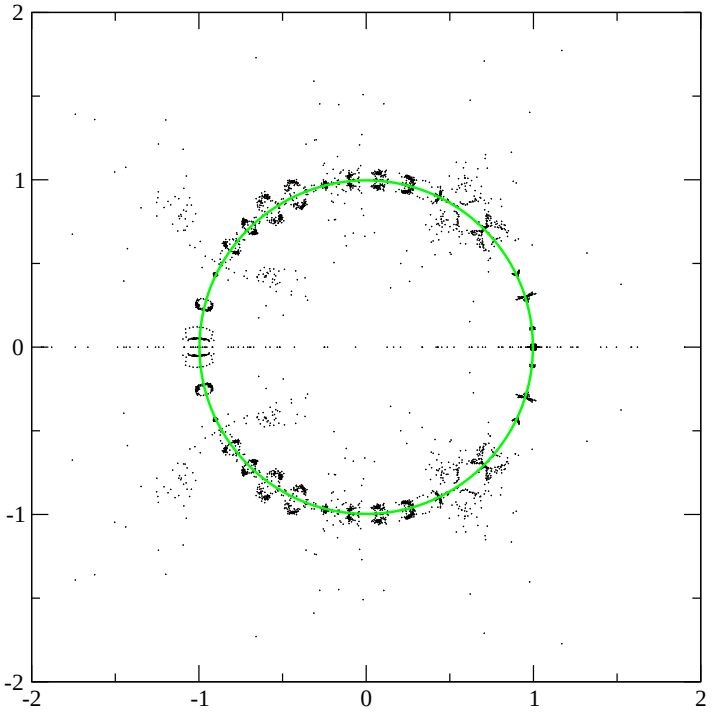


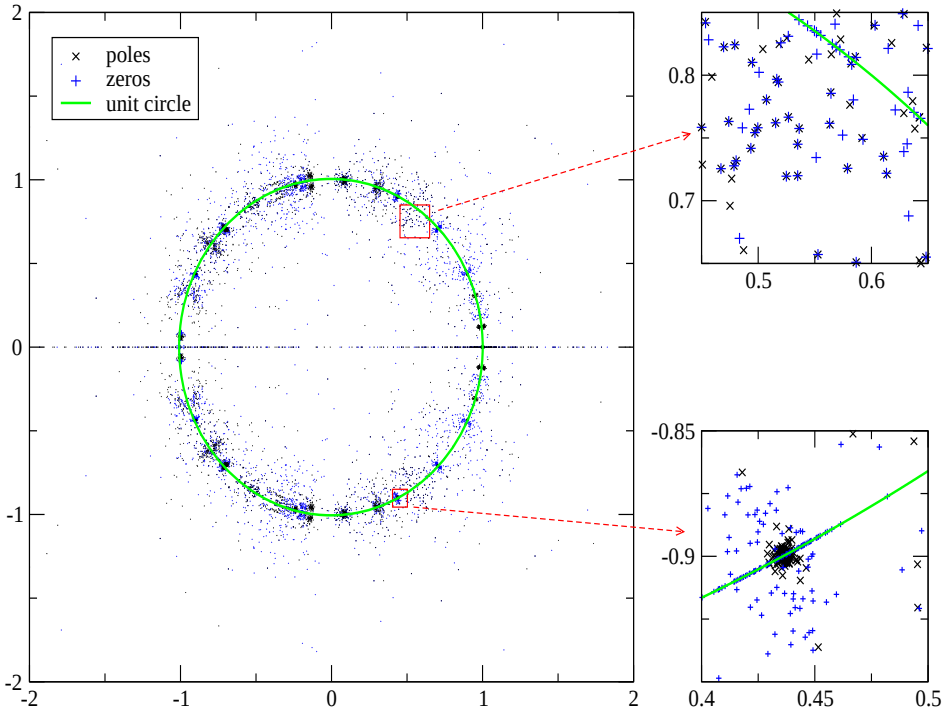






Test on Virgo data -1





Virgo - E4 - July, 2002 - channel Pr_B1_ACq

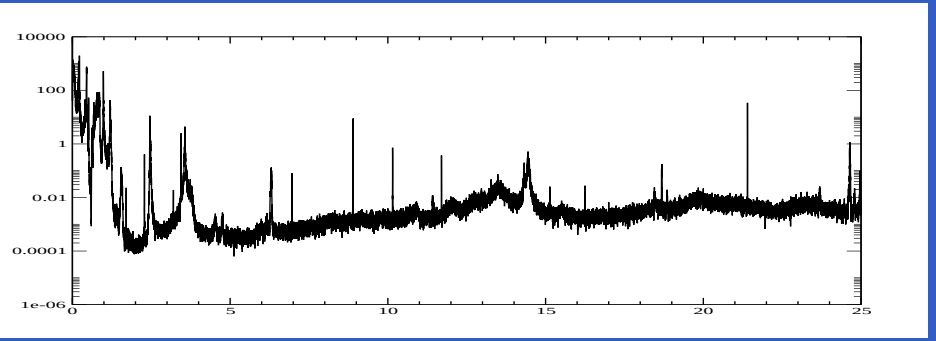
8h of 50Hz sampled data, divided into 72 files, N=20000.

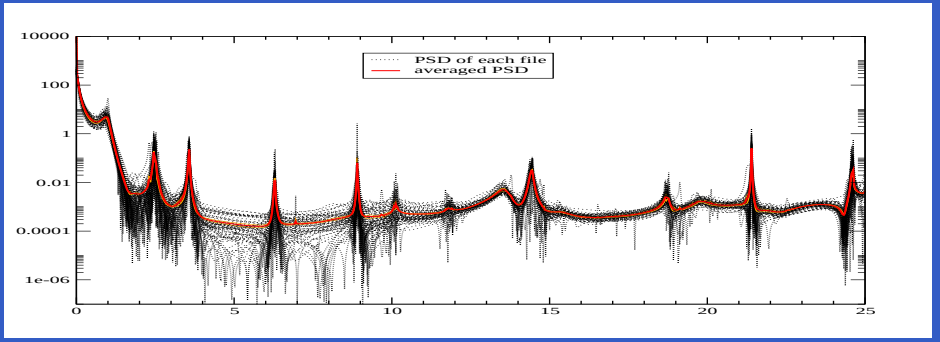
Test on Virgo data - 2

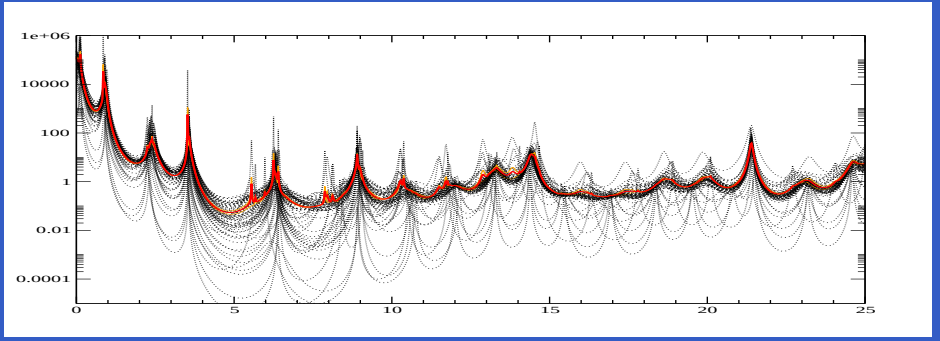
averaged periodogram

$[39/40]_{\mathscr{E}_+}$ estimator

AR(40) estimator







Conclusion

Arbitrary choice of [p-1/p]

$\mapsto$ Estimator with poles and zeros

Role of the random aspect of the signal

\+ Finite length effects

$\rightarrow$ Related mathematical topics :

Roots of random polynomials (position of the poles and zeros of the fraction)

Padé approximants of random series (Froissart doublets)