All-sky search for isolated pulsars using the Hough transform

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Organization of the talk

- The Hough transform
- Expected sensitivity for S2
- Search parameters and the search pipeline
- Statistical properties
- Setting the thresholds
- Hardware injections
- The search results
- The Monte-Carlo strategy
- Future work

The Hough transform

- The Hough transform is an incoherent method based on looking for patterns in the time-frequency plane. Less sensitive but computationally inexpensive compared to full coherent search
- Start with coherent stretches of data and combine them incoherently – present search combines 30 Min long SFTs
- Similar to power summing (Stack-slide) algorithm but instead of adding power, after sliding we add zeros and ones depending on whether power in SFT bin meets a certain criteria – input to Hough algorithm is set of zeros and ones
- Advantage of Hough is computational speed – can consider large region in parameter space at once without stepping through templates one-by-one.
- Final result of Hough search is a histogram in parameter space

The Hough transform

- Time-frequency pattern follows Doppler shift equation

$$f(t) - f_0(t) = f_0(t)\frac{\mathbf{v}(t)\cdot\mathbf{n}}{c}$$

- $f(t) \rightarrow$ observed frequency at time t
- $f_0(t) \rightarrow$ intrinsic signal frequency at time t
- $\mathbf{v}(t) \rightarrow$ detector velocity at time t
- $\mathbf{n} \rightarrow$ sky-position

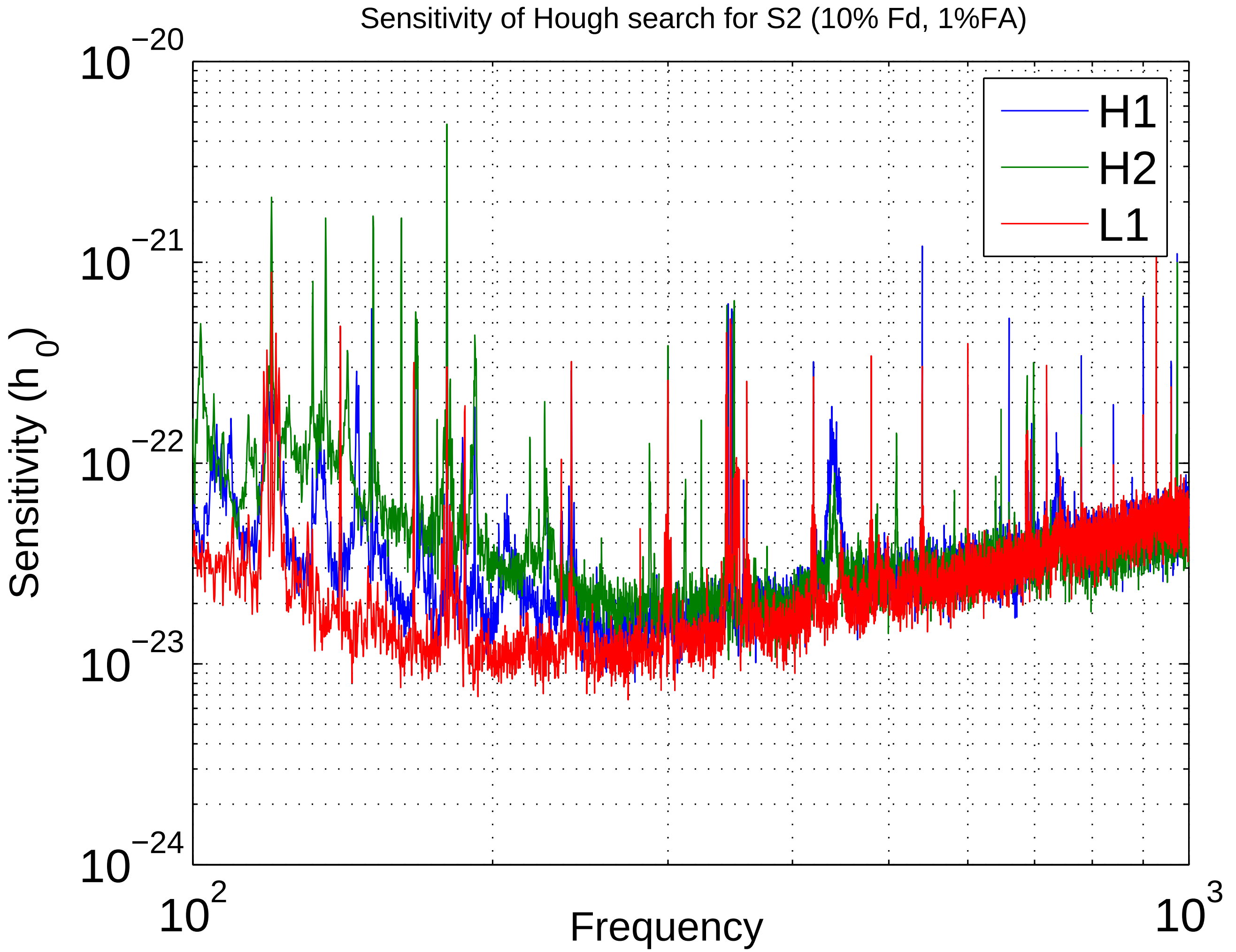
The Hough transform

- Smallest signal that can be detected with 1% false alarm and 10% false dismissal in ideal stationary Gaussian noise:

$$\langle h_0 \rangle = \frac{8.54}{N^{1/4}} \sqrt{\frac{S_n}{T_{\rm coh}}}$$

  $T_{\rm coh} = 30$Min and N is number of SFTs
- Will be eventually combined with $\mathcal{F}$-statistic in a hierarchical scheme
- Reference with details of method, statistics etc. : PRD **70** 082001 (2004)
- We analyze data from S2 run (14 Feb – 14 Apr 2003) of the LIGO detectors using this method
- S2 data has 1761 SFTs from H1, 687 from L1 and 1384 from H2
- Less sensitive than full directed 2-month coherent search by factor of ~ 5

Expected sensitivity for S2



Expected sensitivity for S2

- This implies following astrophysical reach of the search

$$d = \frac{16\pi^2 G N^{1/4} I_{zz} \epsilon f^2}{8.54 c^4} \sqrt{\frac{T_{\text{coh}}}{S_n(f)}}$$

$$d^{L1} = 30.4\text{pc} \left(\frac{I_{zz}}{10^{38}\text{kg-m}^2}\right) \left(\frac{f}{300\text{Hz}}\right)^2 \left(\frac{\epsilon}{10^{-6}}\right) \sqrt{\frac{10^{-43}\text{Hz}^{-1}}{S_n}},$$

$$d^{H2} = 18.3\text{pc} \left(\frac{I_{zz}}{10^{38}\text{kg-m}^2}\right) \left(\frac{f}{300\text{Hz}}\right)^2 \left(\frac{\epsilon}{10^{-6}}\right) \sqrt{\frac{4 \times 10^{-43}\text{Hz}^{-1}}{S_n}}$$

$$d^{H1} = 22.2\text{pc} \left(\frac{I_{zz}}{10^{38}\text{kg-m}^2}\right) \left(\frac{f}{300\text{Hz}}\right)^2 \left(\frac{\epsilon}{10^{-6}}\right) \sqrt{\frac{3 \times 10^{-43}\text{Hz}^{-1}}{S_n}}.$$

Search parameters

- All sky search with $\sim 3 \times 10^5$ sky locations (frequency dependent)

$$\delta\theta = \frac{c}{2 v f T_{\text{coh}}}$$

  In practice, cannot analyze whole sky all at once – break up sky into 23 patches of roughly equal area (required because we set grid in stereographic plane)

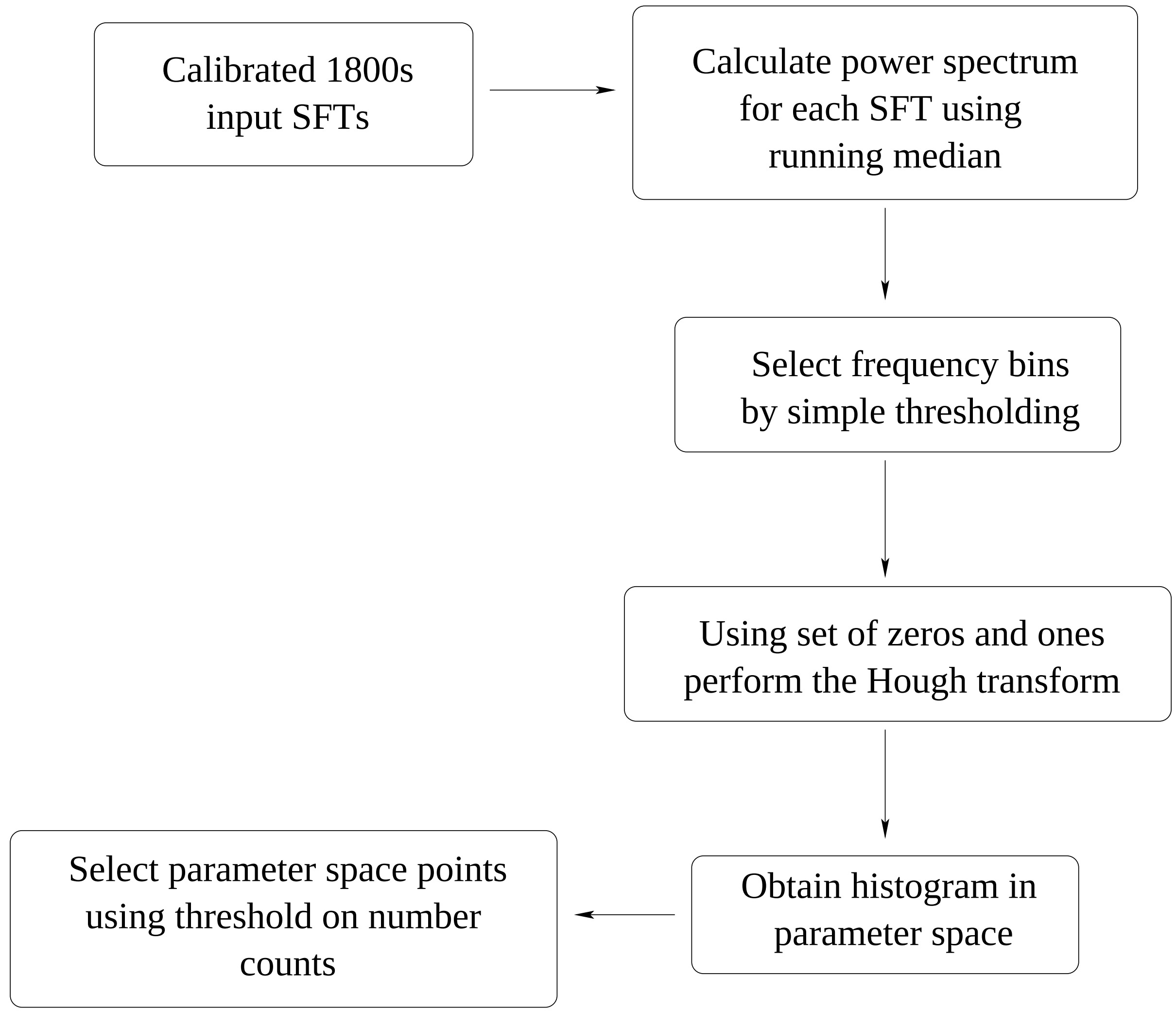
- Frequency band: 200-400 Hz
- One spindown parameter

$$\delta f_{(1)} = \frac{1}{T_{\text{obs}} T_{\text{coh}}} \approx -1.1 \times 10^{-10} \text{Hz/s}$$

  Largest spindown parameter: -1.1×10^{-9}Hz/s $\Longrightarrow$ 11 spindown values considered

- Search takes ~ 0.3 days for each detector

The search pipeline



Statistical properties

Peak selection statistics

- We set threshold ρ_{th} on normalized SFT power ρ to select frequency bins
- Noise floor estimated by running median for each sft
- In absence of signal ρ follows exponential distribution
- False alarm rate for peak selection is $\alpha = e^{-\rho_{\text{th}}}$
- In presence of signal, distribution of 2ρ is the non-central χ^2 distribution

$$p(\rho|\lambda) = e^{-\rho-\lambda/2} I_0(\sqrt{2\rho\lambda})$$

  where $\lambda = 4|\tilde{h}|^2/(T_{\text{coh}} S_n)$ is the coherent SNR and non-centrality parameter
- Presence of signal increases peak selection rate

$$\eta \approx \alpha + \frac{1}{2}\alpha\rho_{\text{th}}\lambda$$

Statistical properties

Hough map statistics

- In absence of signal distribution of number counts is binomial

$$p(n|N) = \begin{pmatrix} N \\ n \end{pmatrix} \alpha^n (1-\alpha)^{N-n}$$

- Select candidates using threshold n_{th} on number counts
- False alarm rate is

$$\alpha_H(n_{\text{th}}, \rho_{\text{th}}, N) = \sum_{n=n_{\text{th}}}^{N} p(n|\rho_{\text{th}})$$

- Mean and standard deviation are

$$\bar{n} = N\alpha \qquad \sigma^2 = N\alpha(1-\alpha)$$

Statistical properties

Hough map statistics

- In presence of signal distribution of number counts is binomial only if amplitude modulation and non-stationarity is neglected

$$p(n|\lambda) = \binom{N}{n} \eta^n (1-\eta)^{N-n}$$

- False dismissal rate is

$$\beta_H(n_{\mathrm{th}}, \rho_{\mathrm{th}}, \lambda, N) = \sum_{n=0}^{n_{\mathrm{th}}-1} p(n|\rho_{\mathrm{th}}, \lambda)$$

- One possible way to choose thresholds $(\rho_{\mathrm{th}}, n_{\mathrm{th}})$ is to minimize β_H for a given choice of α_H. For weak signals this leads to $\rho_{\mathrm{th}} = 1.6$ ($\alpha = 0.20$) and

$$n_{\mathrm{th}} = N\alpha + \sqrt{2N\alpha(1-\alpha)}\ \mathrm{erfc}^{-1}(2\alpha_H^{\star}) \,.$$

Hardware Injections

Two artificial pulsar signals were injected into all LIGO IFOs during S2 for a 12h period with $h_0 = 2 \times 10^{-21}$, $\psi = 0$, $\cos\iota = 0$, $\phi_0 = 0$

Pulsar P1 with constant intrinsic frequency:

- $f = 1279.123\text{Hz}$
- $\alpha = 5.147162\text{rad}$, $\delta = 0.376696\text{rad}$

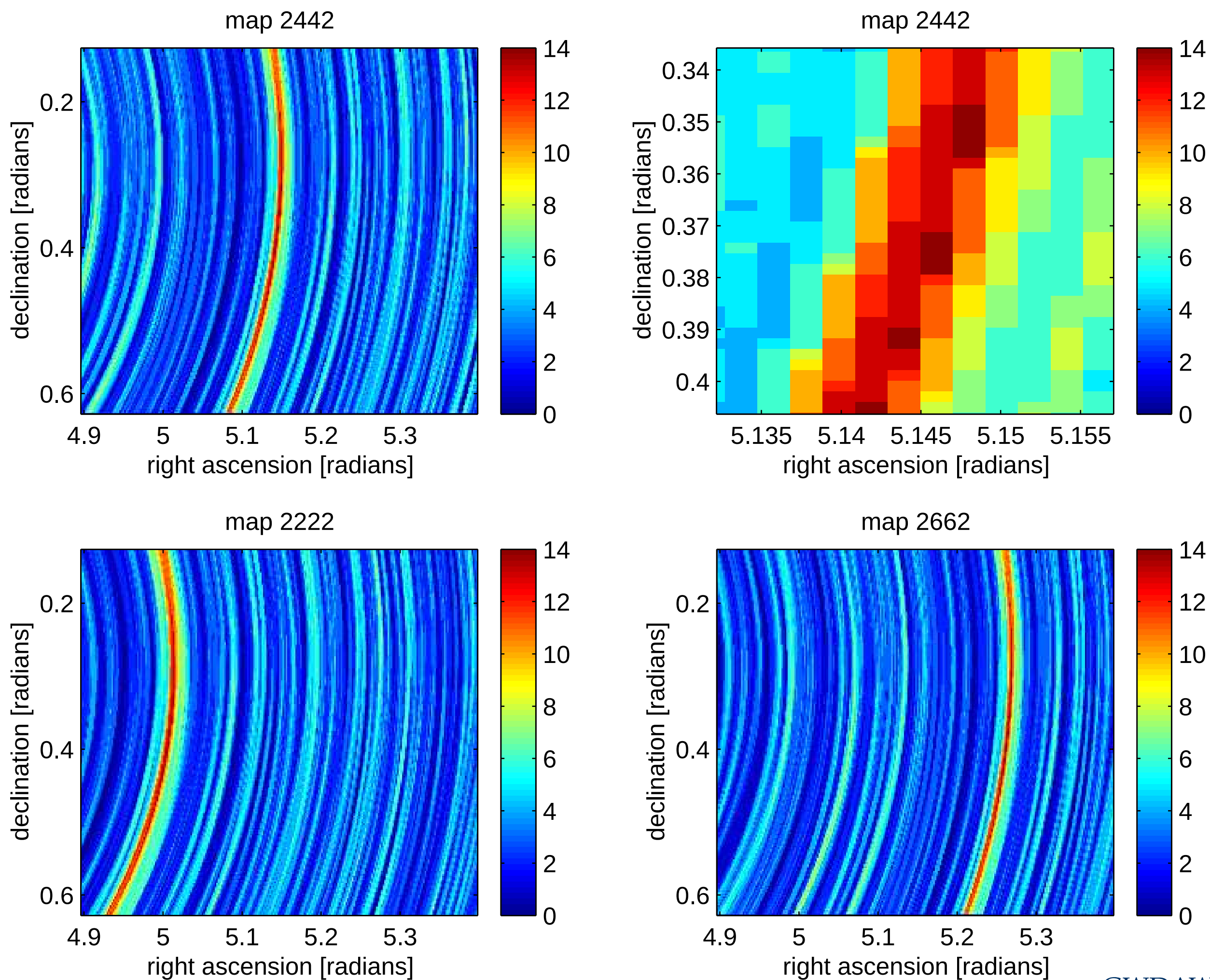
Pulsar P2 with spindown:

- $f = 1288.901\text{Hz}$
- $\alpha = 2.34567\text{rad}$, $\delta = 1.23456\text{rad}$
- $\dot{f} = -10^{-8}\text{rad/s}$

Both signals are clearly detected but observation time of 12h too small to pin-down pulsar parameters exactly

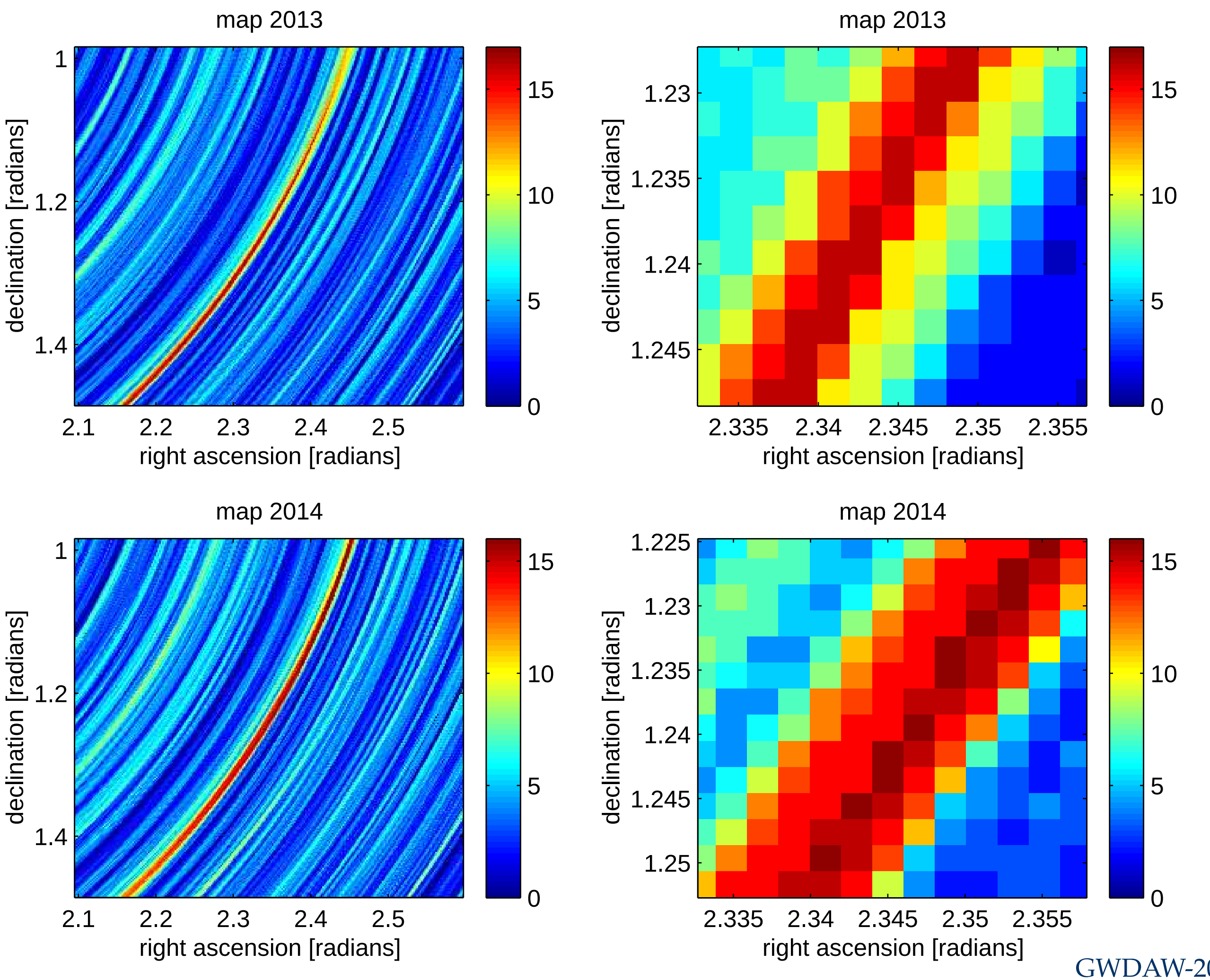
Hardware Injections

Pulsar P1: L1 data



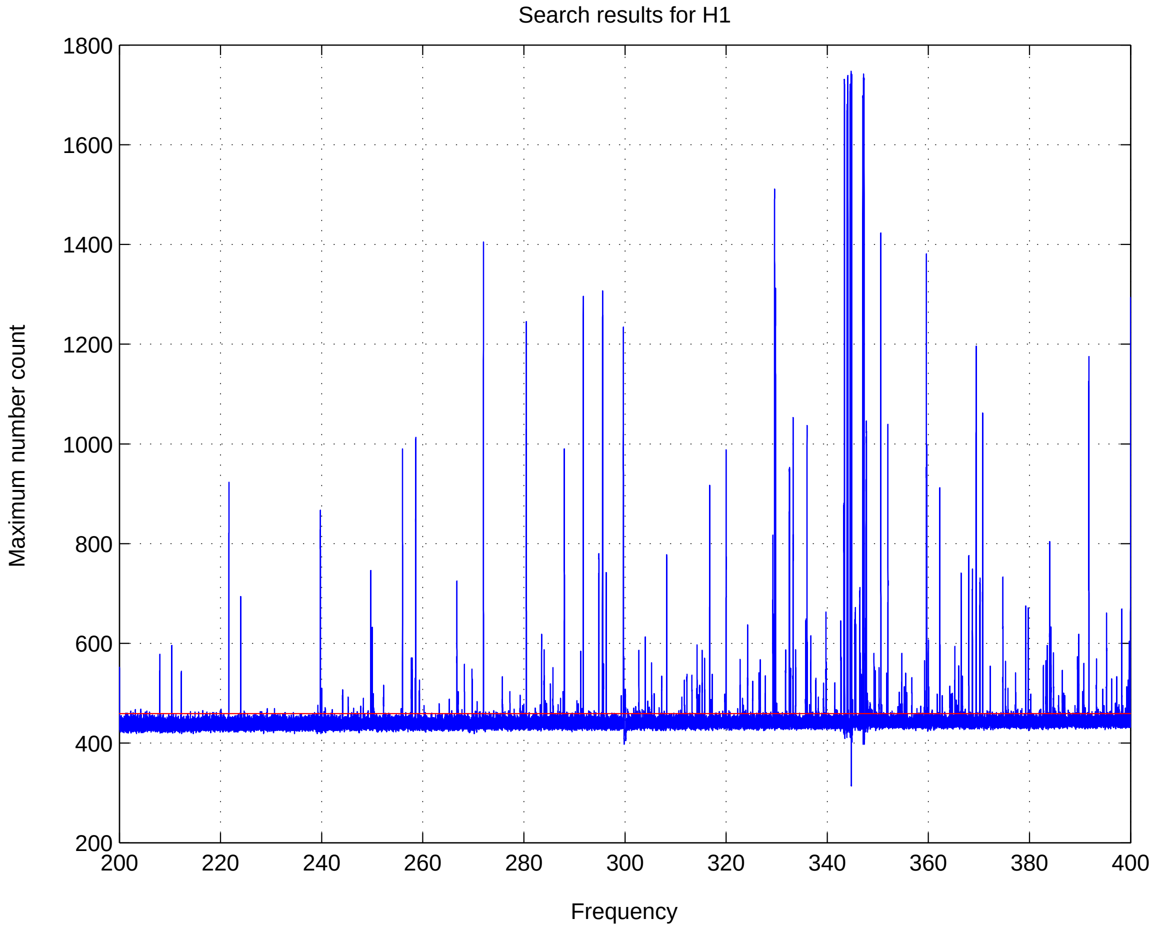
Hardware Injections

Pulsar P2: H1 data



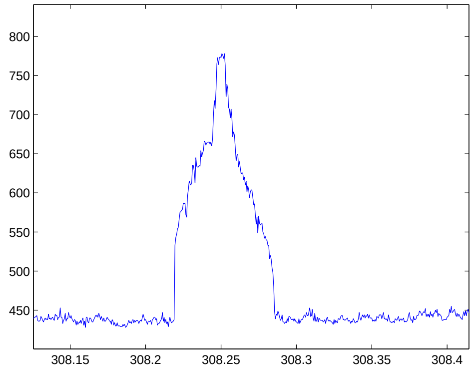
Search results

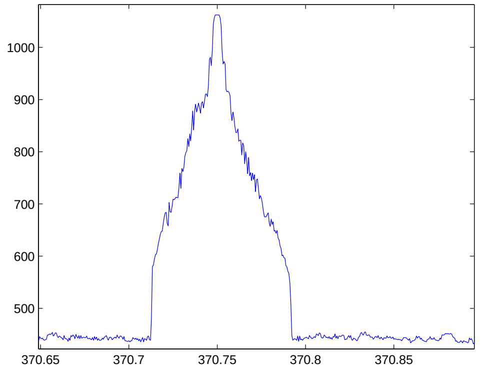
Hough number count maximized over whole sky and spindowns – H1 data

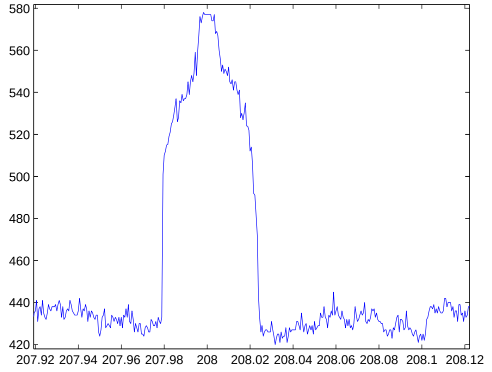


Search Results

- Expected threshold on number counts for $\alpha_H = 10^{-10}$ should ideally be 459 – should lead to $\sim$ a few candidates every 1Hz band
- Agrees with results in "good" frequency bands
- There are many sharp lines which have been smeared out due to Doppler effect
- Apart from violin modes, 60 Hz lines, 16Hz data acquisition lines, there are many 0.25 Hz harmonics

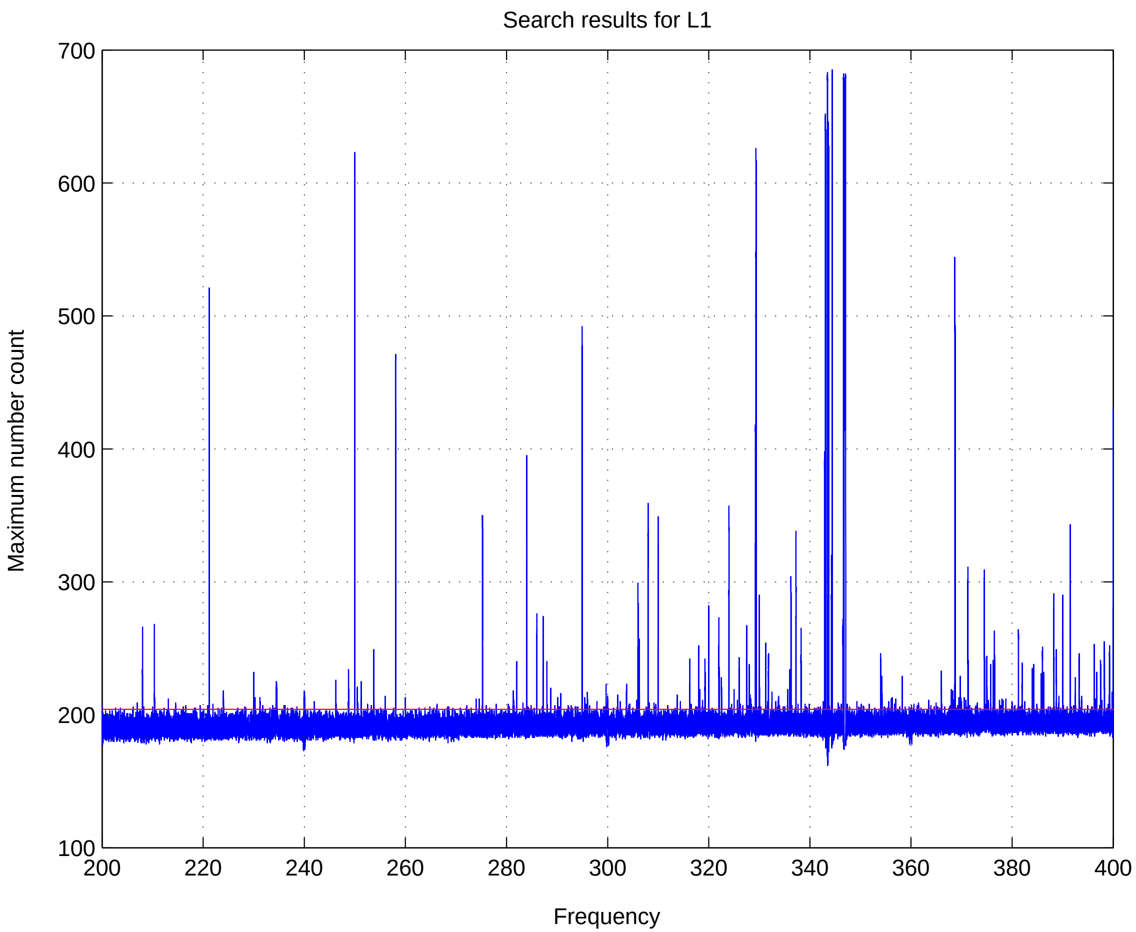






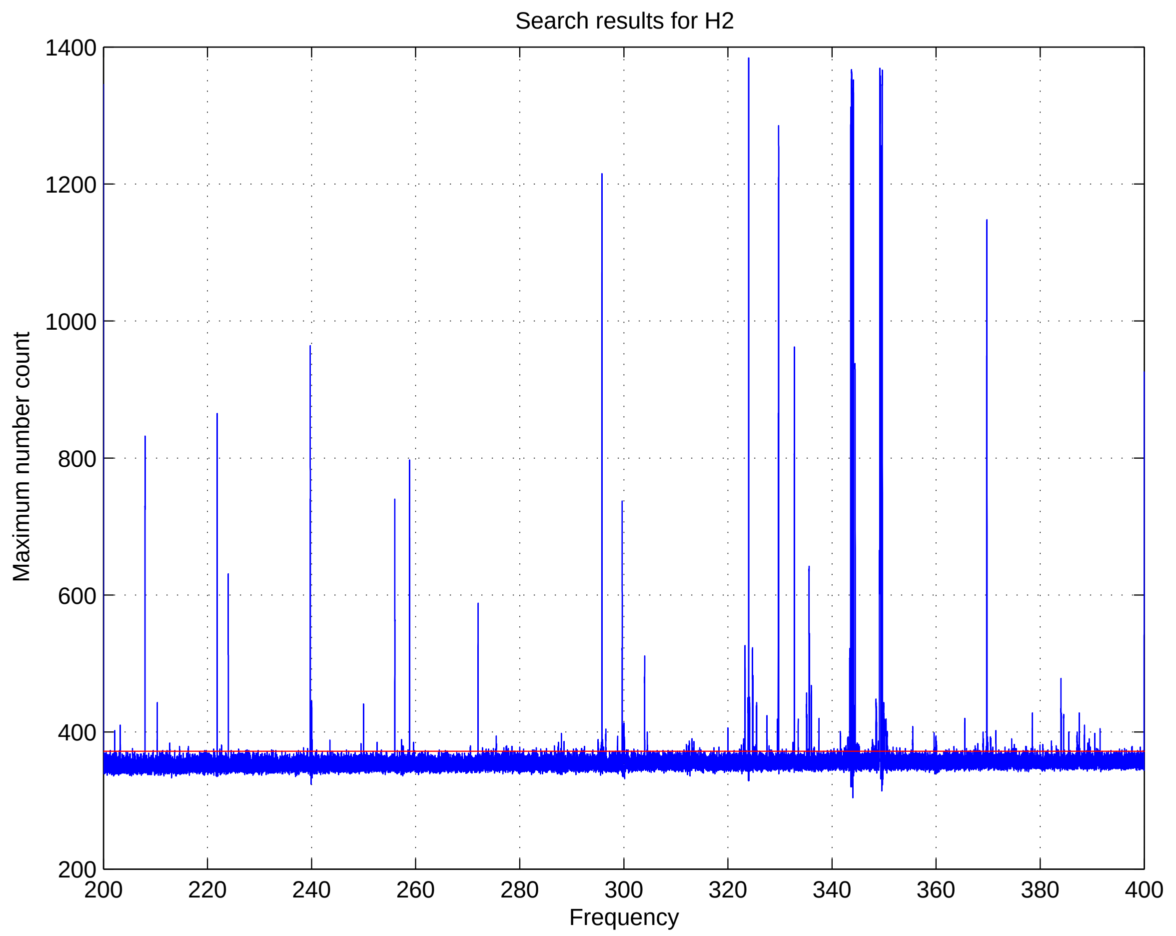
Search results

Hough number count maximized over whole sky and spindowns – L1 data



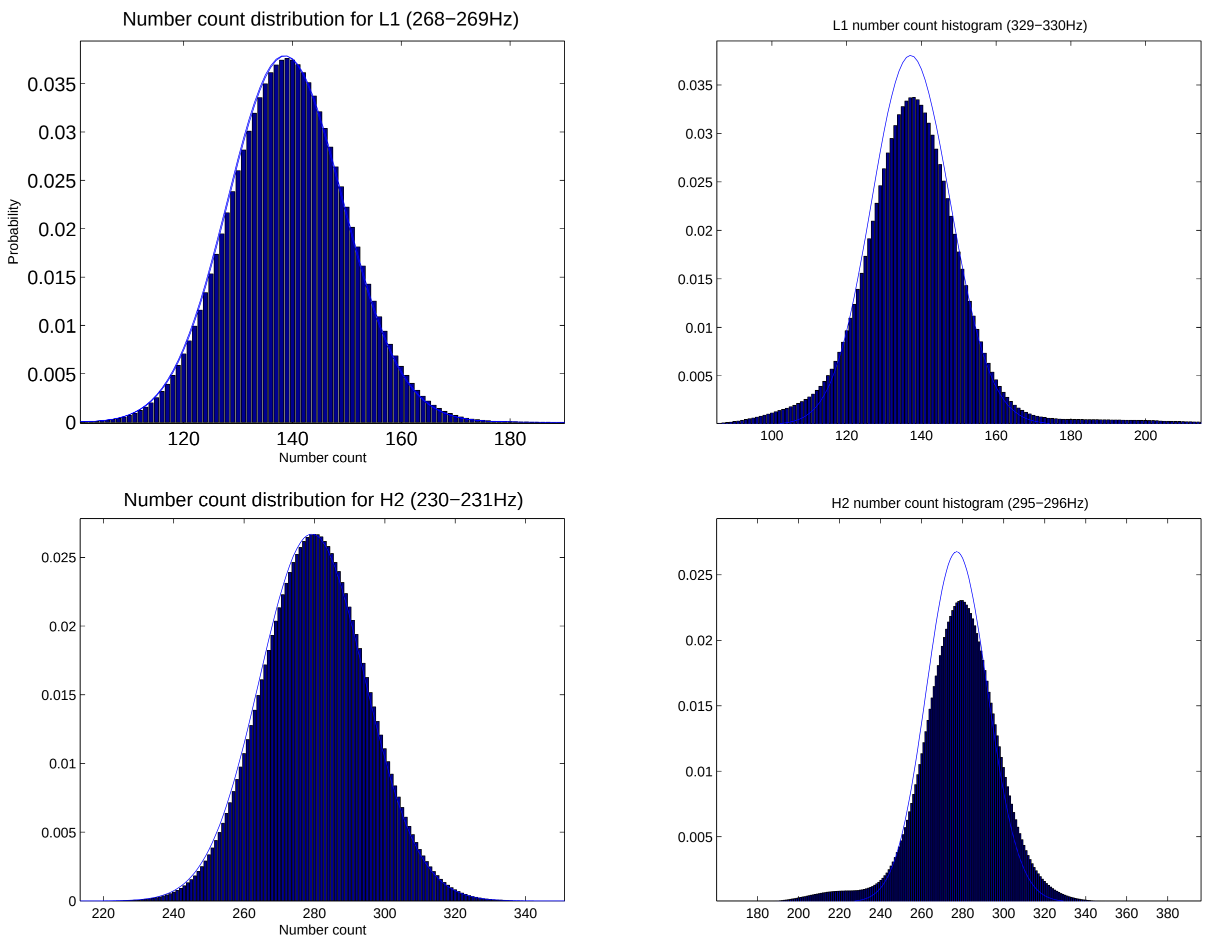
Search results

Hough number count maximized over whole sky and spindowns – H2 data



Search Results

- Distributions in clean bands agree very well with expected binomial distribution but not in bands with strong disturbances



Setting Upper limits

- We set all-sky upper limits over frequency bands of $\sim$ 1Hz – in progress
- To avoid setting unnecessarily bad limits, we should exclude a priori the known broad freatures – violin modes, 60Hz lines etc.
- Upper limits will be set by Monte-Carlo injections
- Aim is to set 95% upper limits based on the loudest event in each band, i.e. find the smallest h_0^{95} such that

$$\sum_{n=n_{max}}^{N} p(n|h_0^{95}) = 0.95$$

  where n_{max} is largest number count in that band
- We will inject signals with random parameters and random mismatch from search templates and find h_0^{95} satisfying above equation.
- Monte-Carlo results being validated within LSC

To-do list

- Validating the S2 upper limits
- Analyze S3 data
- Compare results with other incoherent methods
- Medium term plan – Combine $\mathcal{F}$-stat with Hough algorithm to get longer coherent integration times and thus better sensitivity
- Longer term plan – Use Hough as part of a multi-stage hierarchical search