Directional Stochastic Search: a Gravitational Wave Radiometer

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Spatially Resolved Stochastic Searches

- Formalisms:
  - Cornish, Class. Quant. Grav. 18 (2001) 4277-4292

- Implementation of directed searches with LIGO interferometers for S4, S5
  - S. Ballmer (MIT) - targeted search for few specific objects for Ph.D. thesis

- Beyond S4, S5 for LIGO
  - Network application using multiple baselines among instruments with comparable sensitivity to produce a sky map
Gravitadiometer: beyond S2, S3

The Idea

- Long baseline sky-averaged stochastic search is limited to lower frequency
  - Higher frequencies (>\approx 100\text{Hz}) average out in source position integration

- But the nearby universe is not isotropic:
  - Galactic center
  - Nearest galaxy: M31 Andromeda
  - Virgo galaxy cluster
  - LMXBs, (Bildsten, astro-ph0404234)
  - Voids
    - Compare different sky patches -- "on-source" vs. "off-source"
      - Exploit signal modulation (dithering theme, ala' LIGO-ALLEGRO) to produce subtle changes in signals against detector noise
      - IF noise were stationary, MC injections show we could detect differential changes $\ll$ sky-averaged point estimate
        - Equivalent of a GW Dicke radiometer

- We can get source position information from:
  - Signal time delay between different sites (sidereal time dependent)
  - Sidereal variation of the antenna pattern
    - Recipe: Track targets by time-shifting and cross-correlate
Optimal filter will depend on sidereal time & sky position:

\[ S(\Omega) = \int dt \int dt' \, s_1(t) \cdot s_2(t') \cdot Q_{\text{sidereal}}(t - t') ; \quad s_i = n_i + h_0 F_{i,\text{sidereal}}(\Omega) \]

\[ \mu = \langle S \rangle = \int dt_{\text{sidereal}} \int df \left| \tilde{h}_0(f) \right|^2 \gamma_{\text{sidereal}}(f) \tilde{Q}_{\text{sidereal}}^*(f) \tilde{Q}_{\text{sidereal}}(f) \]

\[ \sigma_S^2 = \frac{1}{4} \int dt_{\text{sidereal}} \int df \, P_1(f) P_2(f) \left| \tilde{Q}_{\text{sidereal}}(f) \right|^2 \]

Where,

\[ \gamma_{\text{sidereal}}(f) = \sum_{A=+x} e^{\frac{2\pi f}{c} \Omega \cdot \Delta x^{(A,\text{sidereal})}} A^{\Omega} F_{1,\text{sidereal}}^A(\Omega) F_{2,\text{sidereal}}^A(\Omega) \]

\[ \tilde{Q}_{\text{sidereal}}(f) = \lambda \left( \left| \tilde{h}_0(f) \right|^2 \gamma_{\text{sidereal}}(f) P_1(f) P_2(f) \right) \]

Following notation of gr-qc/9710117
Comparison of (normalized) $\gamma(f)$ vs. $\gamma(f, T_{opt}, \Delta\Omega_{Virgo})$

- Reducing $\Delta\Omega$ increases frequency response

**Virgo Cluster of 2096 Galaxies**
(160 Brightest Highlighted)

- $\gamma_{sidereal}$, $\Omega_{Virgo}(f)$
- for H-L

Frequency-time map of targeting Virgo cluster

- $4\pi$ average (H-L)
- $\text{Imag}[^{\gamma}]$
- $\text{Real}[^{\gamma}]$
Antenna Pattern vs. Frequency

DC pattern vs. sidereal time

Pattern evolution with frequency

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\[ \Phi = 2\pi f \tau = \frac{2 \pi c \tau}{\lambda} = \frac{2 \pi D \cos \theta}{\lambda}; \]

\[ \Delta \Phi = \frac{2 \pi c \delta \tau}{\lambda} = \frac{2 \pi D \sin \theta \delta \theta}{\lambda} \]

\[ = 2\pi \delta f \tau = \delta f \frac{2 \pi D \cos \theta}{c} \]

\[
\text{for } \Delta \Phi \sim \frac{\pi}{2} \\
\delta \theta \sim \frac{\lambda}{4D \sin \theta} \\
\delta f = f \tan \theta \delta \theta
\]

\[ \delta f = f \tan \theta \sqrt{\frac{\Delta \Omega}{\pi}} = \frac{c}{4D \cos \theta} \]

\[ N_{\text{pixel}} = \frac{4\pi}{\Delta \Omega} \]

\[ = 4 \left(4D \sin \theta \frac{f}{c}\right)^2 \]

\[ = 256 \text{ @ 200 Hz} \]

\[ = 6400 \text{ @ 1 kHz} \]
The antenna pattern for a flat $\Omega_{GW}(f)$ spectrum

- Optimal filter (and therefore spatial resolution in the sky) depends on:
  - Detector power spectra (known)
  - Frequency content of expected signal (unknown)

$$\tilde{Q}_{\text{sidereal}}(f) = \lambda \cdot \frac{|\tilde{h}_0(f)|^2 \gamma_{\text{sidereal}}(f)}{P_1(f)P_2(f)}$$

- Source modeling possible -- simplest assumption is a flat signal spectrum
  - For this case and assuming the LIGO design $h[f]$ curve (shape only) we can calculate the full antenna pattern.
The antenna pattern for a flat $\Omega_{GW}(f)$ spectrum from a spatially finite source.
The sensitivity

- For a flat $\Omega_{GW}(f)$ spectrum:

$$h_0 = \left( \frac{\text{SNR}}{2\sqrt{T} \gamma^D_{\Omega} \left( \int df \frac{1}{P_1(f)P_2(f)} \right)^{1/2}} \right)^{1/2}$$

- For 2 interferometers at the current H1 sensitivity this is

$$h_0 = 3 \times 10^{-24} \text{Hz}^{-1/2} \left( \text{SNR} \right)^{1/2} \left( \frac{T}{1\text{yr}} \right)^{-1/4}$$
To be done

- Full optimal filter for all sky
  - Declination generated by time shifting same filter

- Inversion of data into sky map
  - Bayesian maximum energy methods
  - Direct inversion (notoriously not robust)
  - Deconvolution
    - e.g., *CLEAN algorithm used in VLBI radio measurements*

- Simulations
FINIS
The antenna pattern
DC part

Look at frequency independent (geometric) part first:

\[
\frac{\mu}{\sigma} \propto \gamma^{\Omega}_{DC} := \left( \frac{1}{T_{\text{sidereal}}} \int dt \left( F_{1,t}^{+}(\Omega) F_{2,t}^{+}(\Omega) + F_{1,t}^{x}(\Omega) F_{2,t}^{x}(\Omega) \right) \right)^{1/2}
\]

Interpretation:

- Power coupling loss relative to 2 permanently optimally aligned detectors

How bad is it for existing interferometers?
Coupling from other sky directions
Antenna pattern, DC part
### $\gamma_{DC}$ for various targets

<table>
<thead>
<tr>
<th>Object</th>
<th>LHO-LLO</th>
<th>LHO-VIRGO</th>
<th>LLO-VIRGO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Galactic center</td>
<td>0.189</td>
<td>0.256</td>
<td>0.251</td>
</tr>
<tr>
<td>M31 Andromeda</td>
<td>0.158</td>
<td>0.296</td>
<td>0.231</td>
</tr>
<tr>
<td>M81</td>
<td>0.064</td>
<td>0.465</td>
<td>0.114</td>
</tr>
<tr>
<td>M87 Virgo cluster</td>
<td>0.202</td>
<td>0.252</td>
<td>0.262</td>
</tr>
</tbody>
</table>
Reducing angular size of patch on sky increases f-response

Comparison of (normalized) $\gamma(f)$ vs. $\gamma(f, T_{opt}, \Delta \Omega_{Virgo})$
The work to do

Implementation

- Almost identical to current Stochastic pipeline
  - Only modification: time-dependent overlap reduction function