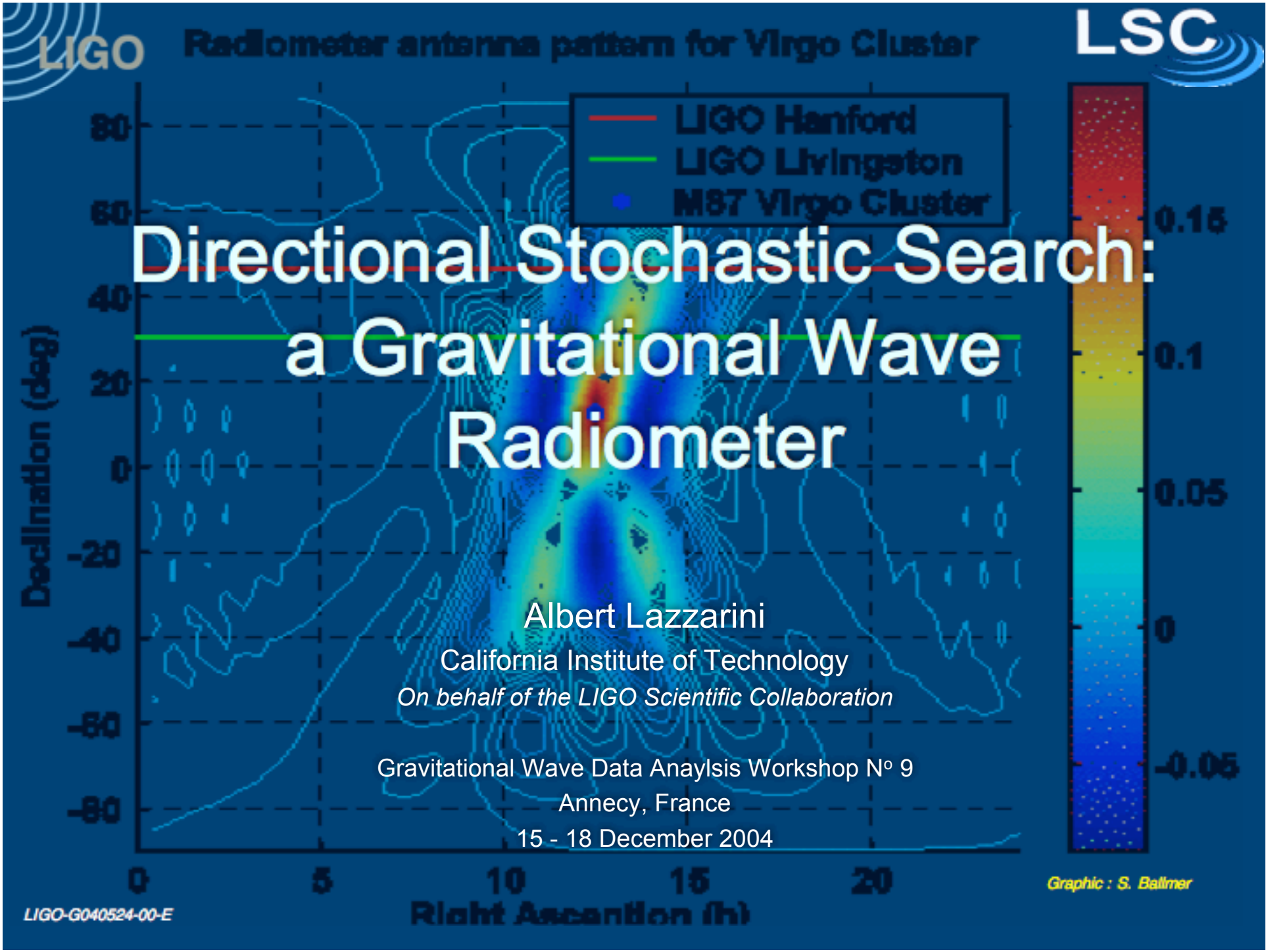

Directional Stochastic Search: a Gravitational Wave Radiometer

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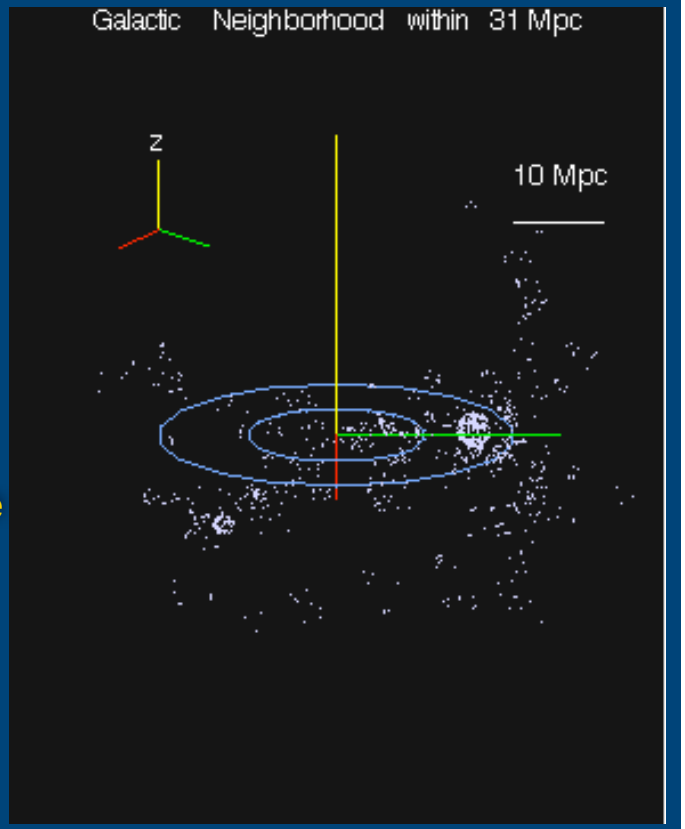
Spatially Resolved Stochastic Searches

- Formalisms:
  - Allen & Ottewill, Phys.Rev. D56 (1997) 545-563
  - Cornish, Class. Quant. Grav. 18 (2001) 4277-4292
- Implementation of directed searches with LIGO interferometers for S4, S5
  - S. Ballmer (MIT) - targeted search for few specific objects for Ph.D. thesis
- Beyond S4, S5 for LIGO
  - Network application using multiple baselines among instruments with comparable sensitivity to produce a sky map

Graviradiometer: beyond S2, S3

The Idea

- Long baseline sky-averaged stochastic search is limited to lower frequency
  - Higher frequencies (>~100Hz) average out in source position integration
- But the nearby universe is not isotropic:
  - Galactic center
  - Nearest galaxy: M31 Andromeda
  - Virgo galaxy cluster
  - LMXBs, (Bildsten, astro-ph0404234)
  - Voids
  - ➔ Compare different sky patches -- “on-source” vs. ”off-source”
    - Exploit signal modulation (dithering theme, ala’ LIGO-ALLEGRO) to produce subtle changes in signals against detector noise
    - IF noise were stationary, MC injections show we could detect differential changes << sky-averaged point estimate
  - *Equivalent of a GW Dicke radiometer*
- We can get source position information from:
  - Signal time delay between different sites (sidereal time dependent)
  - Sidereal variation of the antenna pattern
  - ➔ Recipe: Track targets by time-shifting and cross-correlate



Signal and Noise: Matched Filtering

- Optimal filter will depend on sidereal time & sky position:

$$S(\Omega) = \int dt \int dt' \; s_1(t) \cdot s_2(t') \cdot Q_{t_{sidereal}\Omega}(t - t') \; ; \; s_i = n_i + h_0 F_{i,t_{sidereal}}(\Omega)$$

$$\mu = \langle S \rangle = \int dt_{sidereal} \int df \left| \tilde{h}_0(f) \right|^2 \gamma_{t_{sidereal}\Omega}(f) \tilde{Q}^*_{t_{sidereal}\Omega}(f)$$

$$\sigma_S^2 = \frac{1}{4} \int dt_{sidereal} \int df \;\; P_1(f) P_2(f) \left| \tilde{Q}_{t_{sidereal}\Omega}(f) \right|^2$$

Where,

$$\gamma_{t_{sidereal}\Omega}(f) = \sum_{A=+,\times} e^{i\frac{2\pi f}{c}\hat{\Omega}\cdot\Delta\vec{x}(t_{sidereal})} F^A_{1,t_{sidereal}}(\Omega) F^A_{2,t_{sidereal}}(\Omega)$$

$$\tilde{Q}_{t_{sidereal}\Omega}(f) = \lambda \cdot \left| \tilde{h}_0(f) \right|^2 \gamma_{t_{sidereal}\Omega}(f) \; P_1(f) P_2(f)$$

Antenna pattern

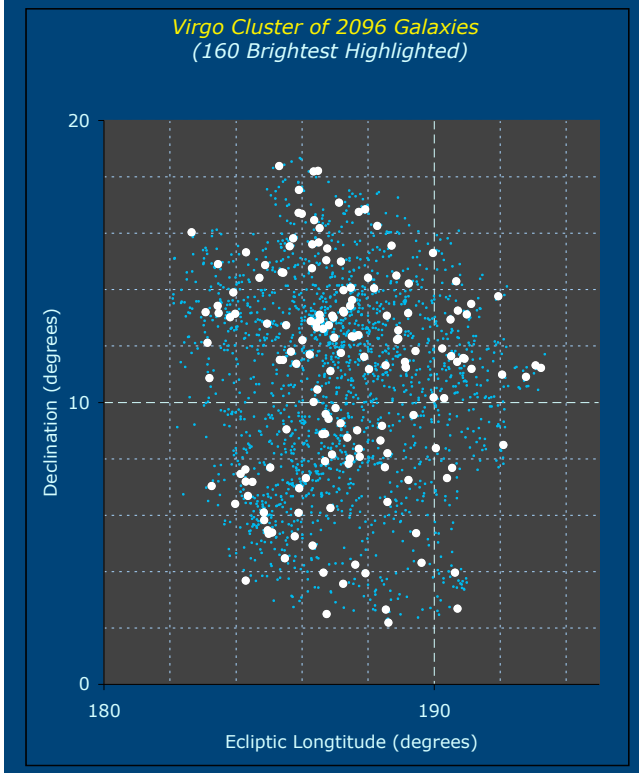
Source model

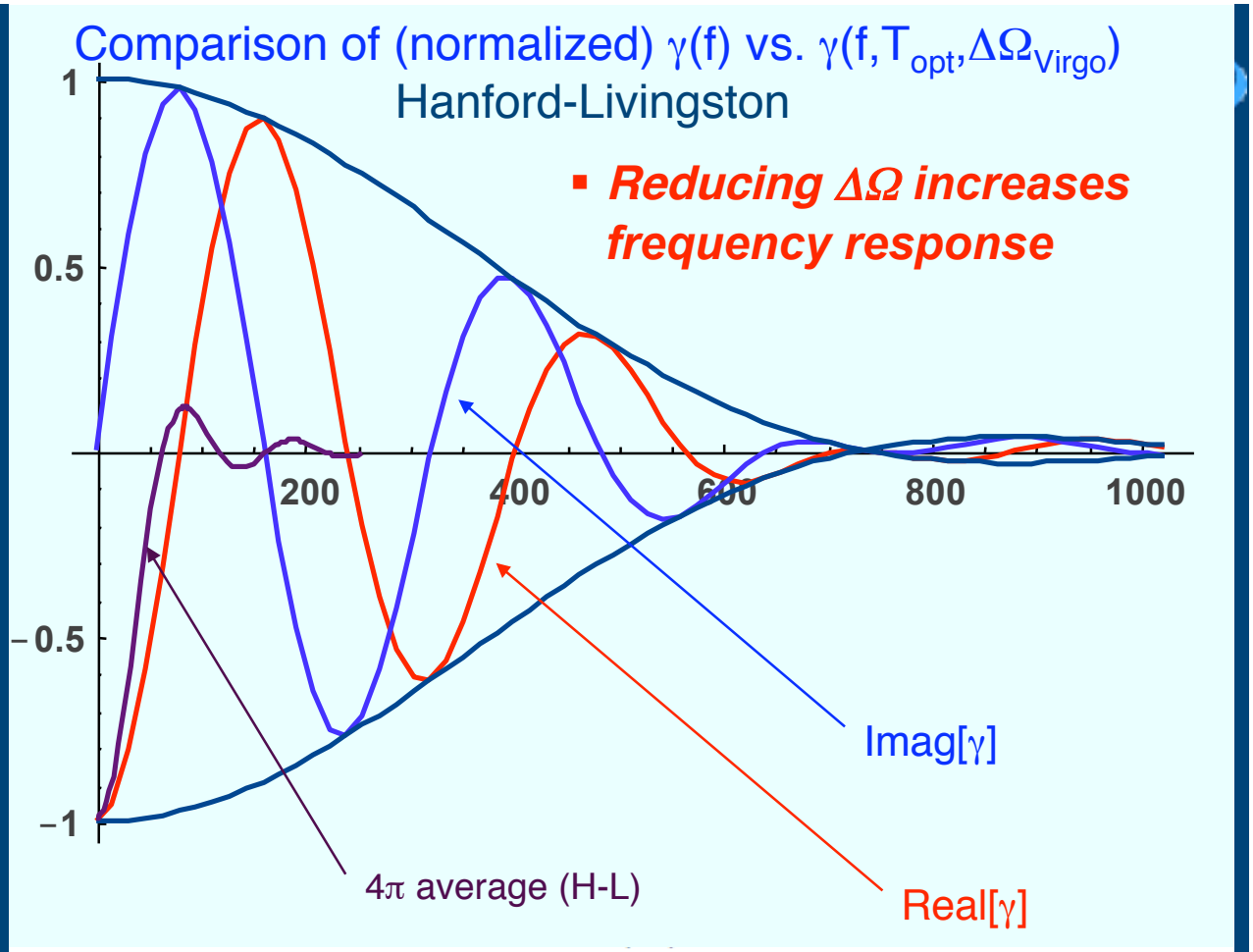
Detector sensitivity

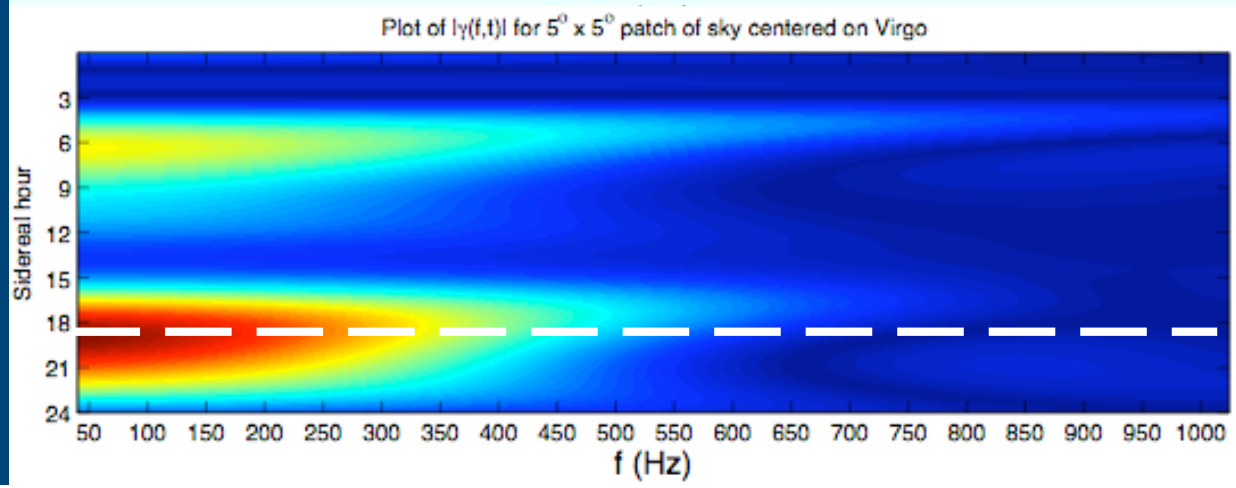
Following notation of gr-qc/9710117

$\gamma_{t_{sidereal}, \Omega_{Virgo}}(f)$ for H-L

Frequency-time map of targeting Virgo cluster

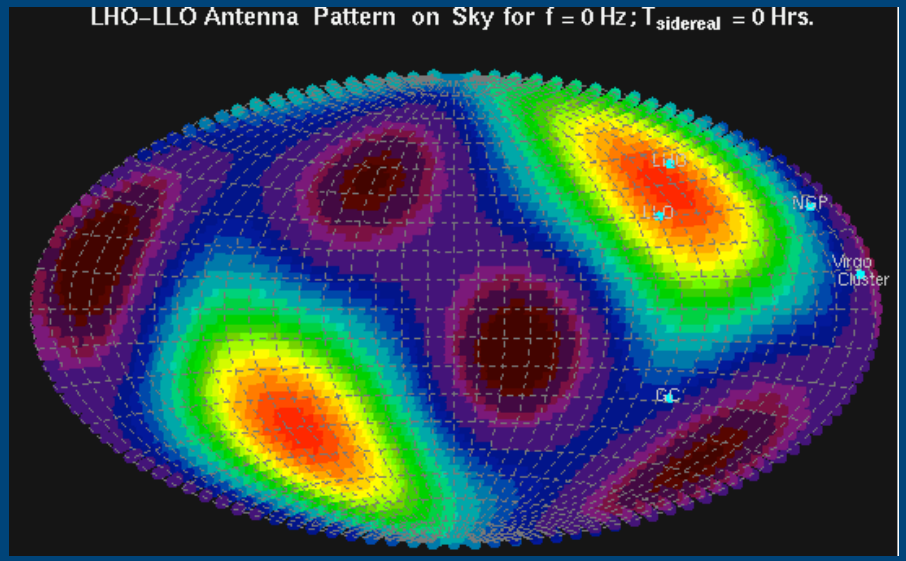




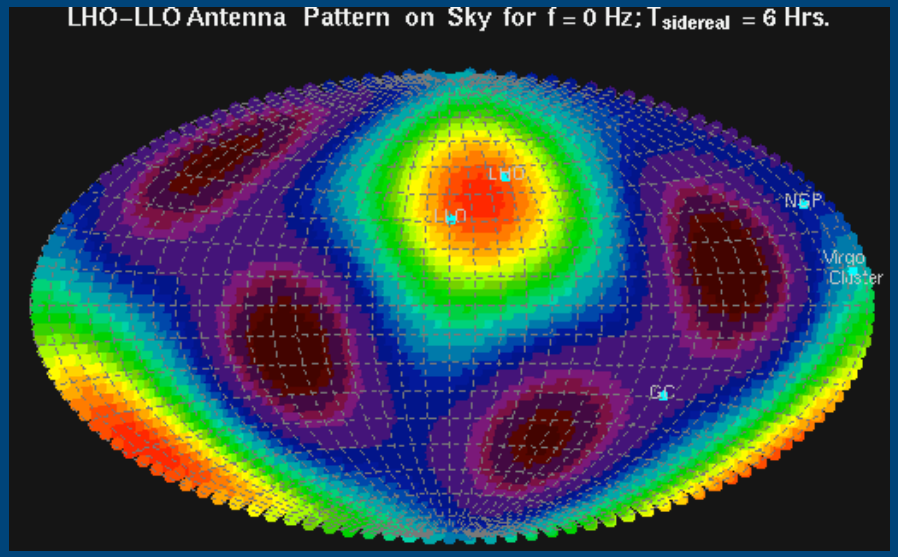


Antenna Pattern vs. Frequency

DC pattern vs. sidereal time



Pattern evolution with frequency



Matching pixel size to bandwidth, frequency

$$\Phi = 2\pi f\tau = \frac{2\pi c\tau}{\lambda} = \frac{2\pi D\cos\theta}{\lambda} \;;$$

$$\Delta\Phi = \frac{2\pi c\delta\tau}{\lambda} = \frac{2\pi D\sin\theta\,\delta\theta}{\lambda}$$

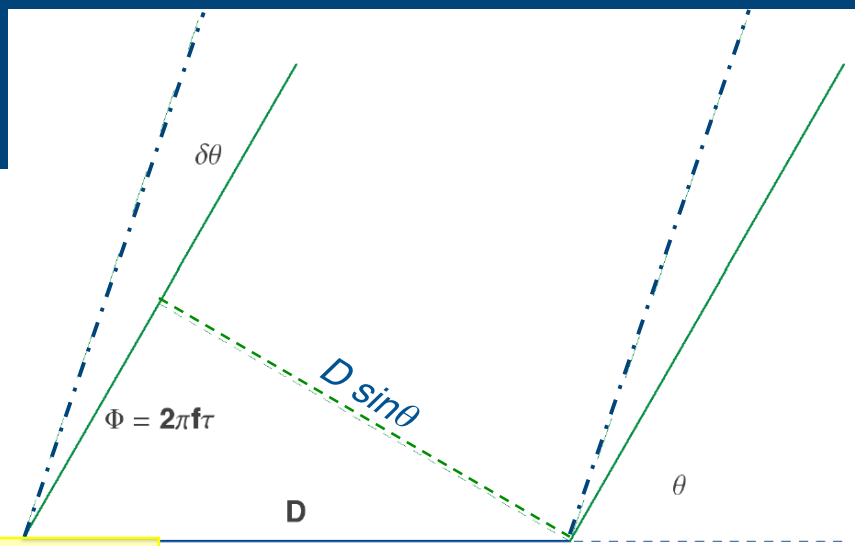
$$= 2\pi\delta f\tau = \delta f\,\frac{2\pi D\cos\theta}{c}$$

for $\Delta\Phi \sim \frac{\pi}{2}$

$$\delta\theta \sim \frac{\lambda}{4D\sin\theta}$$

$$\delta f = f\tan\theta\,\delta\theta$$

$$\delta f = f\tan\theta\sqrt{\frac{\Delta\Omega}{\pi}} = \frac{c}{4D\cos\theta}$$



← *Diffraction limit*

$N_{pixel} = 4\pi/\Delta\Omega$
$= 4\,(4D\sin\theta\, f/c)^2$
$= 256$ @ 200 Hz
$= 6400$ @ 1 kHz

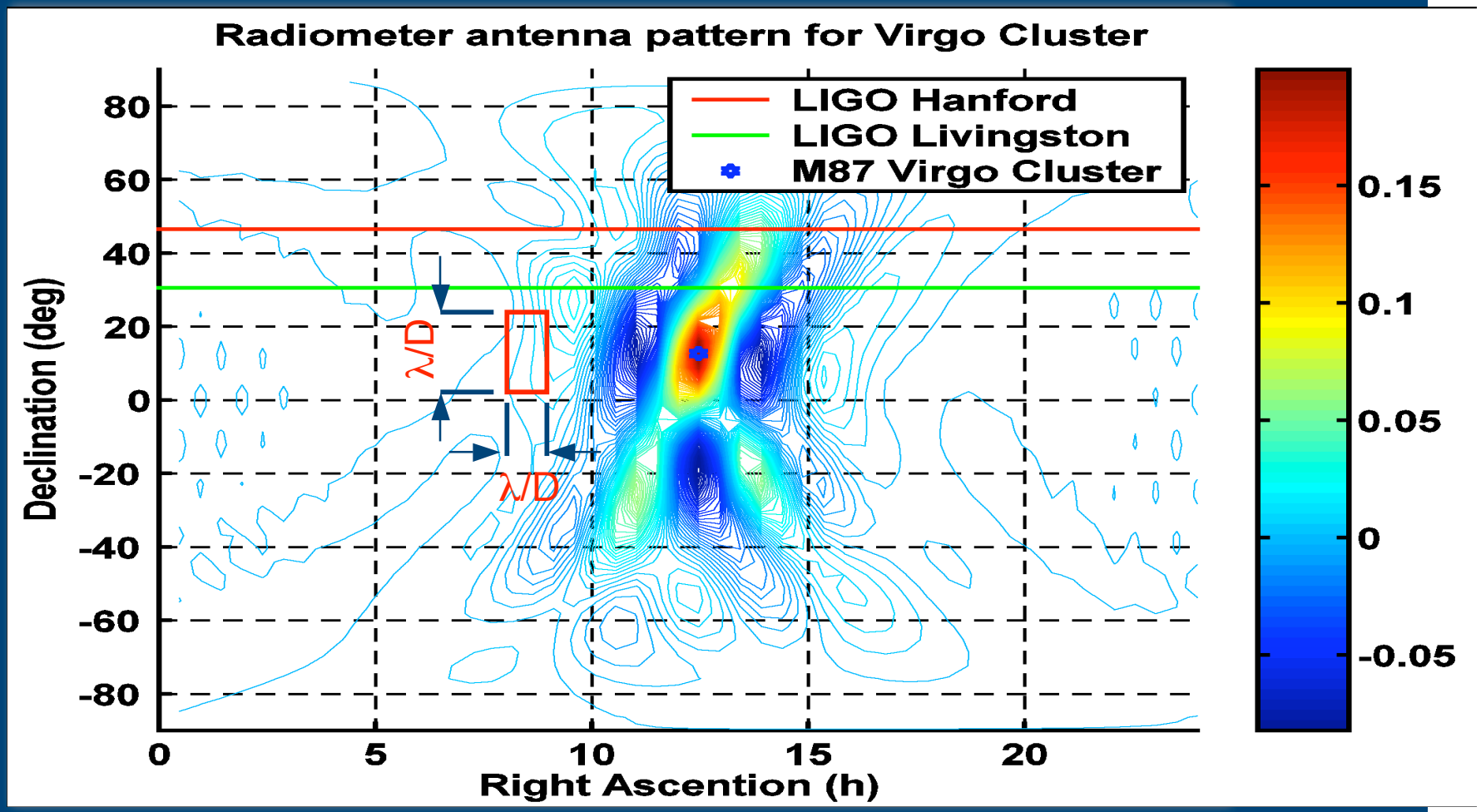
The antenna pattern for a flat $\Omega_{GW}(f)$ spectrum

- Optimal filter (and therefore spatial resolution in the sky) depends on:
  - Detector power spectra (known)
  - Frequency content of expected signal (unknown)

$$\tilde{Q}_{t_{sidereal}\Omega}(f) = \lambda \cdot \frac{\left|\tilde{h}_0(f)\right|^2 \gamma_{t_{sidereal}\Omega}(f)}{P_1(f)P_2(f)}$$

- Source modeling possible -- simplest assumption is a flat signal spectrum
  - For this case and assuming the LIGO design h[f] curve (shape only) we can calculate the full antenna pattern.

The antenna pattern for a flat $\Omega_{GW}(f)$ spectrum from a spatially finite source



The sensitivity

- For a flat $\Omega_{GW}(f)$ spectrum:

$$h_0 = \left(\frac{SNR}{2\sqrt{T}\gamma_{\Omega}^{DC}\left(\int df \frac{1}{P_1(f)P_2(f)} \right)^{1/2}} \right)^{1/2}$$

- For 2 interferometers at the current H1 sensitivity this is

$$h_0 = 3\times 10^{-24}\, Hz^{-1/2} (SNR)^{1/2} \left(T/1yr \right)^{-1/4}$$

To be done

- Full optimal filter for all sky
  - Declination generated by time shifting same filter
- Inversion of data into sky map
  - Bayesian maximum energy methods
  - Direct inversion (notoriously not robust)
  - Deconvolution
    - e.g., *CLEAN algorithm used in VLBI radio measurements*
- Simulations

FINIS

The antenna pattern DC part

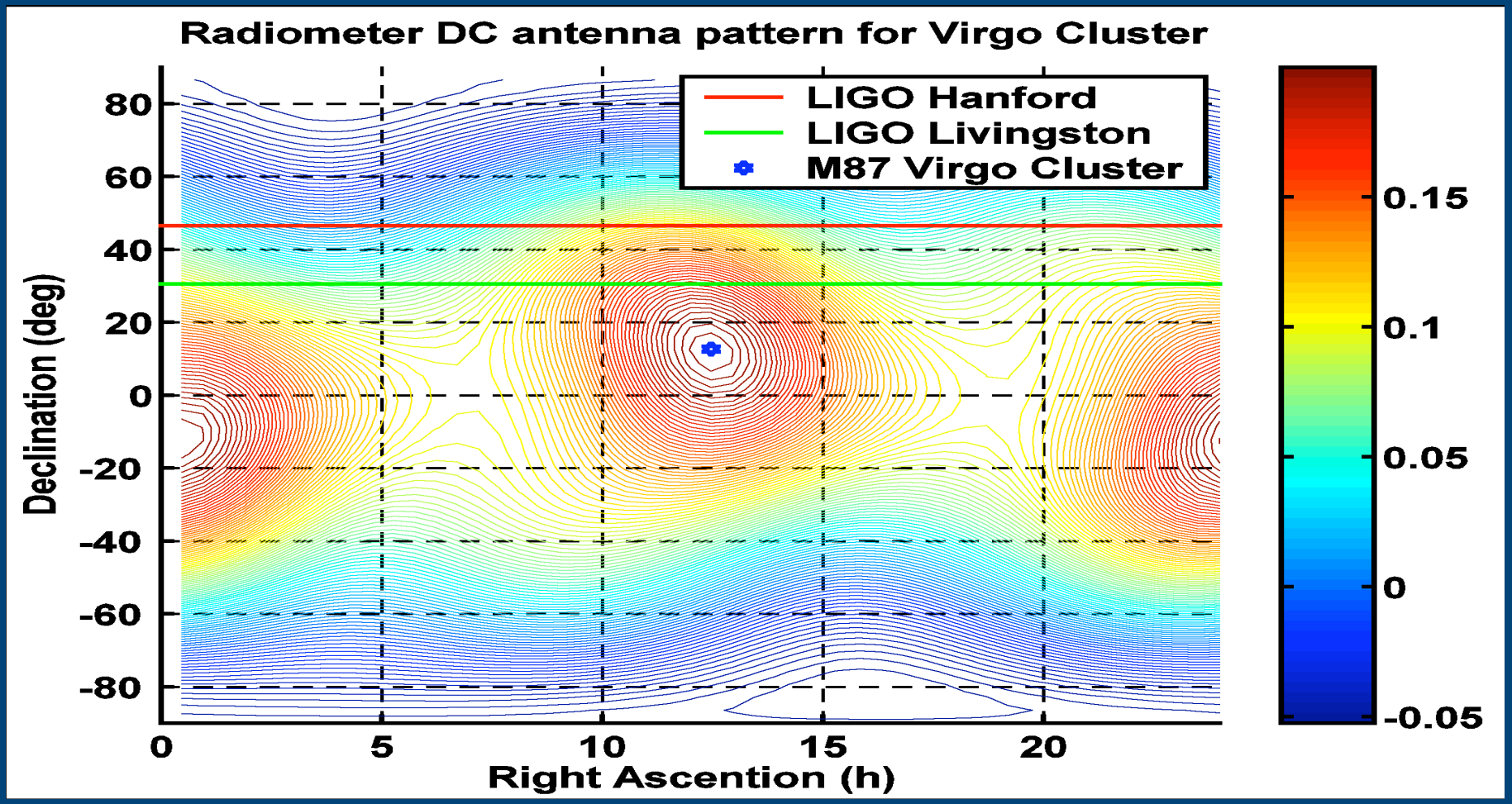
- Look at frequency independent (geometric) part first:

$$\frac{\mu}{\sigma} \propto \gamma_{DC}^{\Omega} := \left(\frac{1}{T_{sidereal}} \int dt \left(F_{1,t}^{+}(\Omega) F_{2,t}^{+}(\Omega) + F_{1,t}^{\times}(\Omega) F_{2,t}^{\times}(\Omega) \right)^2 \right)^{1/2}$$

- Interpretation:
  - Power coupling loss relative to 2 permanently optimally aligned detectors

- How bad is it for existing interferometers?

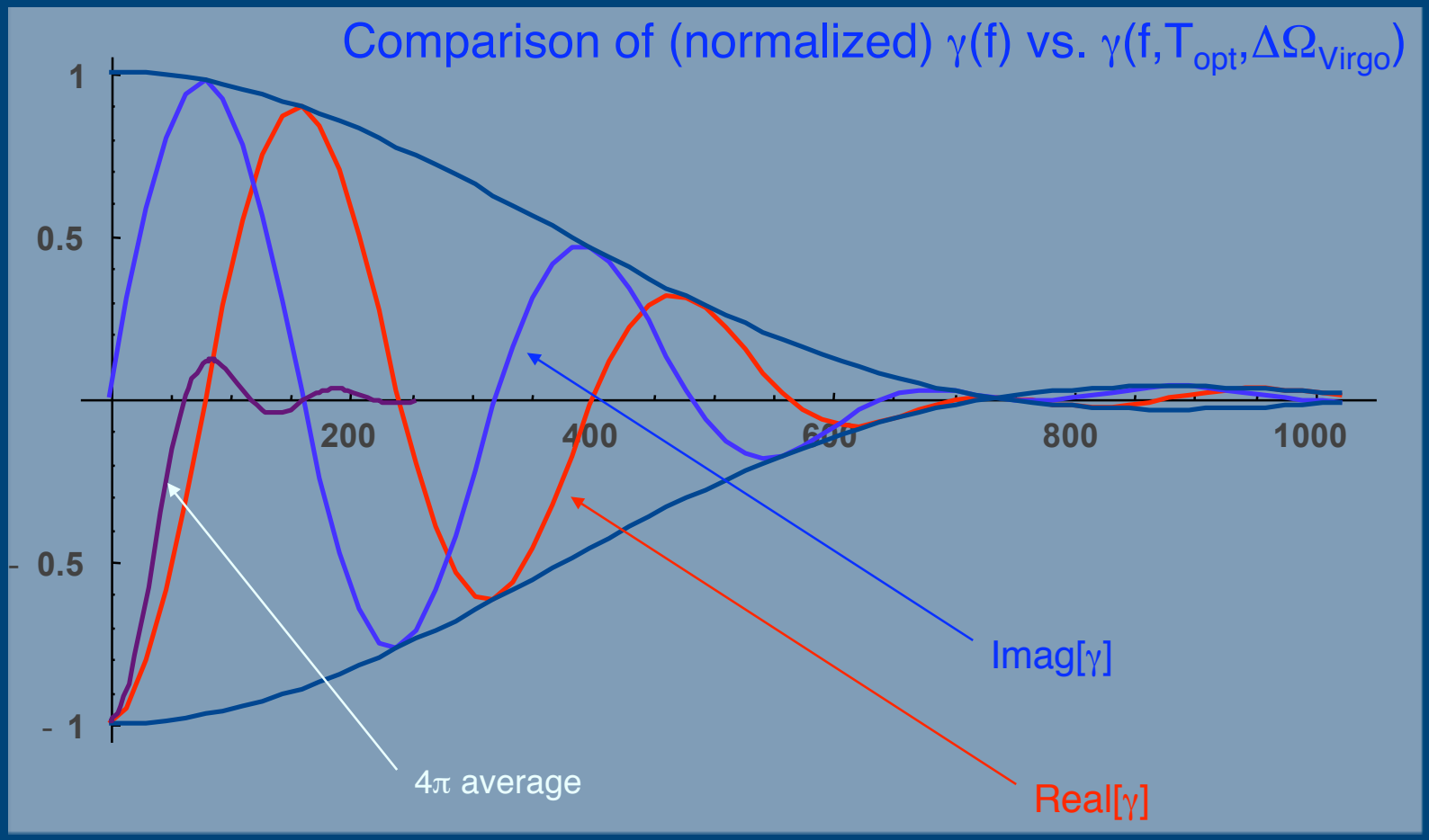
Coupling from other sky directions
Antenna pattern, DC part



γ_{DC} for various targets

Object	LHO-LLO	LHO-VIRGO	LLO-VIRGO
Galactic center	0.189	0.256	0.251
M31 Andromeda	0.158	0.296	0.231
M81	0.064	0.465	0.114
M87 Virgo cluster	0.202	0.252	0.262

Reducing angular size of patch on sky increases f-response



The work to do

Implementation

- Almost identical to current Stochastic pipeline
  - Only modification: time-dependent overlap reduction function