Detecting Galactic Binaries with LISA.

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Data Analysis Challenges for LISA.

1. Large range of sources.

2. Elliminating confusion noise.

3. Likely need for a high number of templates to carry out matched filtering.

4. Need for fast data-analysis methods.

LISA.

1. Sensitive to frequencies in the range 10^{-4} to 10^{-1} Hz.
2. Arm Length of 5×10^6 km.
3. LISA can be thought of as two detectors. The configuration is equivalent to two 90^o interferometers where one is rotated by $\pi/4$ radians with respect to the other.
4. With LISA we can measure both polarizations of the GW.

Low Frequency Approximation.

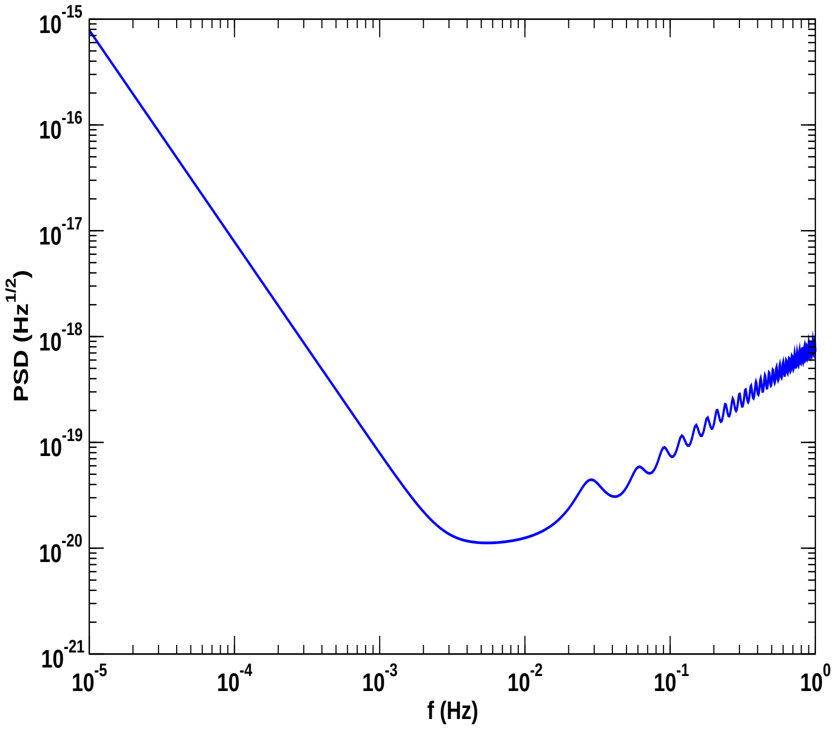

Figure 1: LISA PSD Curve.

1. Usually defined as $f \ll f_*$, where $f_* = (2\pi L)^{-1}$.
2. The arm Length is $L = 5 \times 10^6$ km.
3. $f_* \sim 10^{-2}$ Hz.
4. Transfer functions have a value of unity.

The Galactic Binary Waveform.

In the low frequency approximation (LFA), the waveform can be written as

$$h(t) = h_+(t)F^+(t) + h_\times(t)F^\times(t), \tag{1}$$

where

$$h_+(t) = A_0(1 + cos^2\iota)\,cos(\Phi(t) + \varphi_0) \tag{2}$$

$$h_\times(t) = A_0\,cos\iota\,sin(\Phi(t) + \varphi_0). \tag{3}$$

ι is the angle of inclination and the phase is defined by

$$\Phi(t) = 2\pi f_0 t + \pi \dot{f}_0 t^2 + 2\pi\left(f_0 + \dot{f}_0 t\right) AU\ sin\theta\,cos(2\pi f_m t - \phi), \tag{4}$$

where $f_m = 1/year$ is the modulation frequency. The beam pattern functions are given by

$$F^+ = \frac{1}{2}\left(cos2\psi D^+(t;\theta,\phi) - sin2\psi D^\times(t;\theta,\phi)\right) \tag{5}$$

$$F^\times = \frac{1}{2}\left(sin2\psi D^+(t;\theta,\phi) + cos2\psi D^\times(t;\theta,\phi)\right). \tag{6}$$

We can see that the waveform is a function of 8 parameters

$$h = h(t; A_0, \psi, \iota, \varphi_0, f_0, \dot{f}_0, \theta, \phi); \tag{7}$$

Data Analysis Review.

We define the scalar product of two waveforms h and g by

$$\langle a \,|b\rangle = 2\int_0^\infty \frac{df}{S_h(f)}\left[\tilde{h}(f)\tilde{g}^*(f) + \tilde{h}^*(f)\tilde{g}(f)\right], \tag{8}$$

where the * denotes complex conjugate and $\tilde{h}(f)$, $\tilde{g}(f)$ are the Fourier transforms of $h(t)$, $g(t)$. The (square) of the signal-to-noise ratio obtained by matched filtering a signal h with a template g is given by

$$\rho^2 \equiv \left(\frac{S}{N}\right)^2 = \frac{\langle h\,|g\rangle^2}{\langle g\,|g\rangle}. \tag{9}$$

If the template perfectly matches the signal then the SNR is simply $\rho^2 = \langle h,\, h\rangle$. Given two waveforms h and g, not necessarily belonging to the same family of approximants, their overlap is defined as

$$O \equiv \frac{\langle h\,|g\rangle}{\sqrt{\langle h\,|h\rangle\,\langle g\,|g\rangle}}, \tag{10}$$

where, the inner-product between two real functions $h(t)$ and $g(t)$ is defined by Eq. (8).

The total waveform is a linear combination of the sum of the two individual signals, i.e.

$$h(t) = h_I(t) + h_{II}(t). \tag{11}$$

We can define the metric tensor by

$$G_{\mu\nu} = \frac{\langle \partial_\mu h\,|\partial_\nu h\rangle}{\langle h\,|h\rangle} - \frac{\langle h\,|\partial_\mu h\rangle\,\langle h\,|\partial_\nu h\rangle}{\langle h\,|h\rangle^2}. \tag{12}$$

If we had pre-normalised the waveforms, the right hand side would disappear.

Template Number.

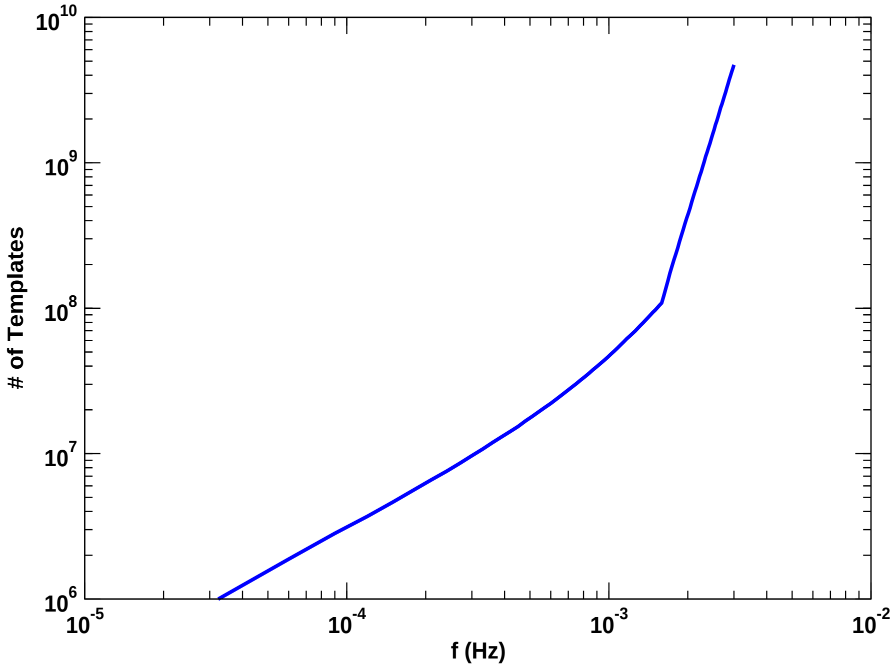

Figure 2: Template Number.

1. We project from the initial 8d space to the supspace defined by $\lambda = \{f_0, \dot{f}_0, \theta, \phi\}$.
2. We assume that our template is a hypercube.
3. If the proper distance in the $\dot{f}_0$ direction is not comparable to the proper distance between two templates, we neglect this parameter.
4. $\dot{f}_0$ becomes important at $f \sim 1.6 \times 10^3$ Hz.

Fast LISA Analysis.

1. Use a WKB method for calculating metric tensor and placing templates.
2. Each template is useful over a number of bins.
3. Computational cost for a one signal search is on the order of a Teraflop.
4. 20 mins on todays machines.
5. Teraflop computers when LISA is launched?