Fisher Information Matrix

Given the 'true parameters' $\tilde{\theta}^a$ and best-fit values $\tilde{\theta}^a + \Delta\theta^a$, at large SNR, errors $\Delta\theta^a$ obey the PDF

$$p(\Delta\theta^a) = p^{(0)} e^{-\frac{1}{2}\Gamma_{ab}\Delta\theta^a\Delta\theta^b},$$

$p^{(0)}$ is a constant; Γ_{ab} the *Fisher information matrix*:

$$\Gamma_{ab} = 2\int_0^\infty \frac{\tilde{h}_a^*(f)\tilde{h}_b(f) + \tilde{h}_a(f)\tilde{h}_b^*(f)}{S_h(f)}\, df,$$

where $h_a \equiv \partial h/\partial\theta^a$.

Parameter Estimation

Errors in measurement are given by the covariance matrix:

$$\Sigma^{ab} \equiv \langle \Delta\theta^a \Delta\theta^b \rangle = (\Gamma^{-1})^{ab},$$

where $\langle \cdot \rangle$ is average over the PDF. RMS error σ_a and correlation coefficient c^{ab}

$$\sigma_a = \langle (\Delta\theta^a)^2 \rangle^{1/2} = \sqrt{\Sigma^{aa}}, \quad c^{ab} = \frac{\langle \Delta\theta^a \Delta\theta^b \rangle}{\sigma_a \sigma_b} = \frac{\Sigma^{ab}}{\sqrt{\Sigma^{aa}\Sigma^{bb}}}.$$

Signal Model

Fourier domain restricted waveform is given by

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} e^{i\psi(f)},$$

where $\mathcal{A} \propto \mathcal{M}^{5/6} Q(\text{angles})/D$, and to 3.5PN order the phase of the Fourier domain waveform is given by

$$\psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\,\eta\,v^5} \sum_{k=0}^{N} \alpha_k\, v^k,$$

$v = (\pi M f)^{1/3}$, $M = m_1 + m_2$, $\eta = m_1 m_2 / M^2$ and D is the distance to binary.

α coefficients

$$
\begin{aligned}
\alpha_0 &= 1, \quad \alpha_1 = 0, \quad \alpha_2 = \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right), \quad \alpha_3 = -16\pi \\
\alpha_4 &= 10\left(\frac{3058673}{1016064} + \frac{5429}{1008}\,\eta + \frac{617}{144}\,\eta^2\right) \\
\alpha_5 &= \frac{38645\,\pi}{756} + \frac{38645\,\pi}{252}\,\log\left(\frac{v}{v_{lso}}\right) + \frac{5\,\pi}{3}\eta\left[1 + 3\log\left(\frac{v}{v_{\rm lso}}\right)\right] \\
\alpha_6 &= \left(\frac{11583231236531}{4694215680} - \frac{640\,\pi^2}{3} - \frac{6848\,\gamma}{21}\right) \\
&+ \eta\left(-\frac{15335597827}{3048192} + \frac{2255\,\pi^2}{12} - \frac{1760\,\theta}{3} + \frac{12320\,\lambda}{9}\right) \\
&+ \frac{76055}{1728}\eta^2 - \frac{127825}{1296}\eta^3 - \frac{6848}{21}\log\left(4\,\frac{v}{v_{\rm lso}}\right) \\
\alpha_7 &= \frac{77096675\,\pi}{254016} + \frac{1014115\,\pi}{3024}\,\eta - \frac{36865\,\pi}{378}\,\eta^2
\end{aligned}
$$

NS-NS Binary

PN Order	Δt_c	$\Delta \phi_c$	$\Delta \mathcal{M}/\mathcal{M}$	$\Delta \eta/\eta$
Advanced LIGO				
1PN	0.3977	0.9256	0.0267%	4.656%
1.5PN	0.4668	1.474	0.0142%	1.638%
2PN	0.4623	1.392	0.0143%	1.764%
2.5PN	0.5090	1.354	0.0134%	1.334%
3PN	0.4830	1.419	0.0136%	1.383%
3.5PN	0.5098	1.363	0.0134%	1.352%
Initial LIGO				
1PN	0.3598	1.238	0.0771%	9.792%
1.5PN	0.4154	1.942	0.0419%	2.768%
2PN	0.4109	1.816	0.0423%	3.007%
2.5	0.4605	1.642	0.0384%	2.129%
3PN	0.4271	1.778	0.0394%	2.248%
3.5PN	0.4643	1.666	0.0388%	2.171%
VIRGO				
1PN	0.1363	0.5134	0.0183%	3.044%
1.5PN	0.1578	0.7981	0.0098%	1.004%
2PN	0.1562	0.7515	0.0098%	1.085%
2.5PN	0.1743	0.7015	0.0091%	0.7957%
3PN	0.1624	0.7489	0.0093%	0.8334%
3.5PN	0.1756	0.7094	0.0092%	0.8091%

NS-BH Binary

PN Order	Δt_c	$\Delta \phi_c$	$\Delta \mathcal{M}/\mathcal{M}$	$\Delta \eta/\eta$
Advanced LIGO				
1PN	0.5959	1.261	0.1420%	7.059%
1.5PN	0.7394	2.091	0.0763%	2.316%
2PN	0.7208	1.848	0.0773%	2.669%
2.5PN	0.9000	1.213	0.0686%	1.515%
3PN	0.7466	1.454	0.0713%	1.683%
3.5PN	0.9521	1.204	0.0691%	1.551%
Initial LIGO				
1PN	0.9550	2.510	0.5217%	20.06%
1.5PN	1.182	4.135	0.2850%	5.410%
2PN	1.148	3.597	0.2903%	6.316%
2.5	1.467	1.964	0.2491%	3.305%
3PN	1.163	2.549	0.2628%	3.778%
3.5PN	1.587	1.945	0.2513%	3.400%
VIRGO				
1PN	0.4906	1.069	0.1134%	5.782%
1.5PN	0.6069	1.763	0.0603%	1.923%
2PN	0.5918	1.561	0.0611%	2.215%
2.5PN	0.7384	1.035	0.0541%	1.263%
3PN	0.6123	1.235	0.0563%	1.401%
3.5PN	0.7816	1.027	0.0545%	1.293%

BH-BH Binary

PN Order	Δt_c	$\Delta \phi_c$	$\Delta \mathcal{M}/\mathcal{M}$	$\Delta \eta/\eta$
Advanced LIGO				
1PN	1.162	1.974	1.041%	59.88%
1.5PN	1.441	3.188	0.6115%	9.609%
2PN	1.404	2.850	0.6240%	10.79%
2.5PN	1.819	1.555	0.5300%	5.934%
3PN	1.376	2.184	0.5631%	6.889%
3.5PN	1.977	1.652	0.5367%	6.167%
Initial LIGO				
1PN	2.406	5.038	4.750%	216.2%
1.5PN	2.986	8.143	2.781%	28.81%
2PN	2.900	7.179	2.851%	32.82%
2.5	3.836	3.070	2.351%	16.48%
3PN	2.734	4.920	2.541%	19.85%
3.5PN	4.292	3.298	2.381%	17.18%
VIRGO				
1PN	1.621	1.854	0.8745%	52.12%
1.5PN	1.430	2.972	0.5095%	8.586%
2PN	1.395	2.667	0.5199%	9.625%
2.5PN	1.787	1.527	0.4417%	5.370%
3PN	1.378	2.087	0.4688%	6.194%
3.5PN	1.928	1.616	0.4474%	5.576%

Error at a Fixed Distance

Errors associated with the parameter estimation are inversely related to SNR ($\sigma \propto 1/\rho$). Given the error σ_0 corresponding to an SNR ρ_0, the error σ at SNR ρ corresponding to a fixed distance, say, 300 Mpc, is $\sigma = \rho_0\sigma_0/\rho$:

$$\sigma(D_L) = \rho_0\sigma_0\pi^{2/3}D_L\left[\frac{2\,\eta\,M^{5/3}}{15}\int_{f_s}^{f_{\rm lso}(M)}\frac{f^{-7/3}}{S_h(f)}\,df\right]^{-1/2}.$$