Coherent Data Analysis Strategies Using a Network of Gravitational Wave Detectors

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Outline

- Overview
  - burst GW sources
- Motivation

- Our proposal
  - use detector network to construct null-stream (data of no GW signal)
  - application
- Examples
  - three detectors
  - two LIGO detectors
- Discussion

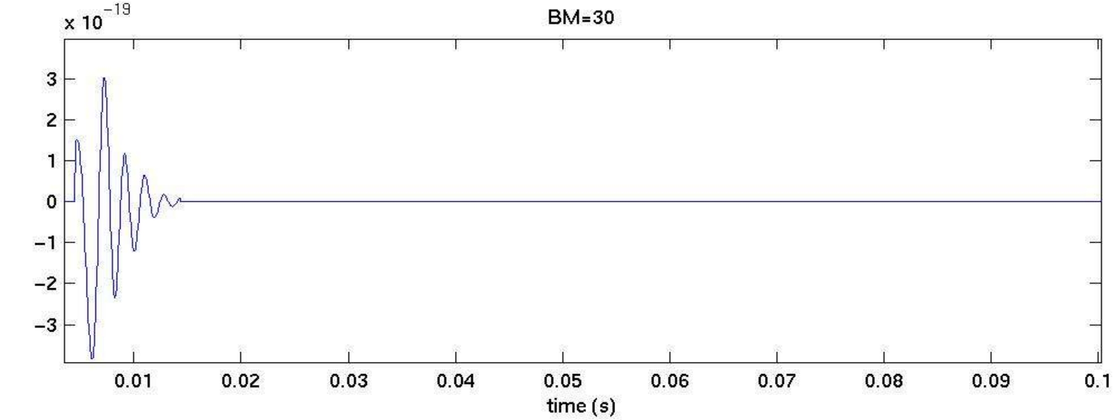
Burst GW Sources

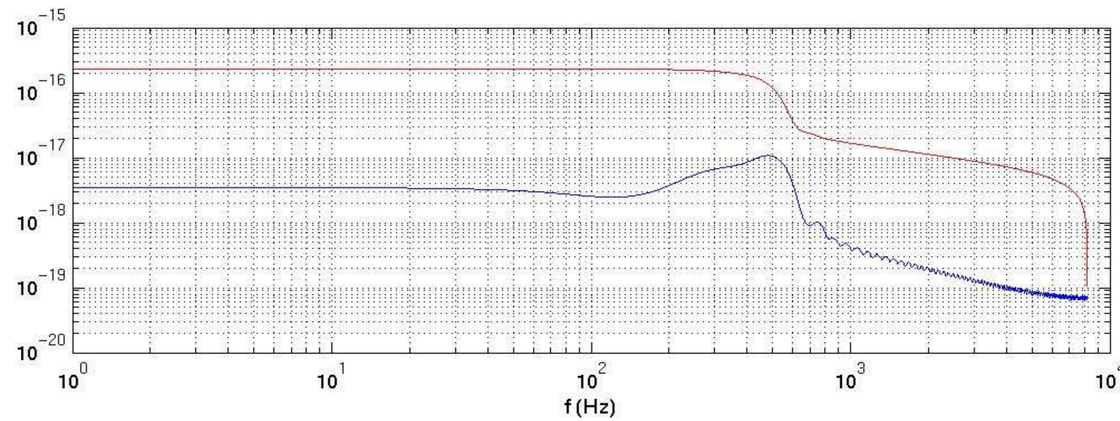
- Burst="unknown or only crudely modelled"
- "Popular" expectation
  - Short duration
    - ~1 -100 ms
  - High frequency
    - >~ 500 Hz

- Two numerical examples
  - rotational core collapse of massive stars
    - ~10 ms, ~500-1000 Hz
  - BH merger waveform (Lazarus project)
    - ~10 ms, ~500 Hz

Binary Black-Hole Merger Waveform

- T~10 ms
- at 1 Mpc
- fc~ 500 Hz





(Baker, et al. 2002, PRD)

astrogravs.gsfc.nasa.gov/docs/catalog.html#bhbh

www.ligo.caltech.edu/~ajw/bursts/burstsim.htm

- Burst GW signals and noise can look alike
    - detector glitches
    - local disturbance (e.g., fly-by plane)

- Vetoing techniques are crucial

- Positional determination helps cross-check with observations in other wavelengths

- multiple detector analysis is a requirement

A single GW observatory cannot convincingly detect a GW event.

Method

For triggered coincidence events:

- Construct "null stream" as a particular linear combination of time-series data from multiple detectors

  GW signals from a given direction will be cancelled out

- Construct a set of null streams as a function of presumed source positions

- Search the sky positions, look for the absence of the signal
  - if GW: verify the detection/localize source position
  - if noise: veto

- Our approach is equivalent to consistency check of data from multiple detectors
  - Amplitude
  - Phase/effective time delay
  - Coherent check of both

Principle of Null Stream Construction

Response of a detector to GW

$$h_i(t) = f_i^+ h_+(t) + f_i^\times h_\times(t)$$

$f_i^{+\times}$ is a function of ψ, α, δ, and detector orientation.

$$h_i(t) = A_i \sqrt{h_+^2(t) + h_\times^2(t)}\, \sin\left(2\psi + \xi_i + \xi^h(t)\right)$$

- two effects of antenna pattern in observed signals
  - amplitude modulation
  - phase/effective arrival time modulation

Antenna Pattern Modulation

- A_i, ξ_i:
  - polarization angle independent
  - known for given source location

$$A_i = \sqrt{f_i^{+2} + f_i^{\times 2}}$$

$$\xi_i = \tan^{-1} \frac{f^{+}_{i,\psi=0}}{f^{\times}_{i,\psi=0}}$$

$$\xi^h(t) = \tan^{-1} \frac{h_\times(t)}{h_+(t)}$$

- Effective time delay error:

$$\delta\tau_{ij} \sim \frac{|\xi_i - \xi_j|}{2\pi f_0} \leq \frac{1}{2 f_0}$$

Three-Detector Case

- Data

$$h_1(t) = f_1^+ h_+(t) + f_1^\times h_\times(t) + n_1(t)$$
$$h_2(t+\tau_{12}) = f_2^+ h_+(t) + f_2^\times h_\times(t) + n_2(t)$$
$$h_3(t+\tau_{13}) = f_3^+ h_+(t) + f_3^\times h_\times(t) + n_3(t).$$

- Null Stream=linear combination of data
  - signal exactly cancelled out (e.g., Guersel & Tinto 1989)
  - coefficients: polarization angle independent

$$A(\alpha,\delta,t) = A_{23} h_1(t) + A_{31} h_2(t+\tau_{12}) + A_{12} h_3(t+\tau_{13})$$

$$A_{ij} = (f_i^+ f_j^\times - f_j^+ f_i^\times).$$

Two -Detector Case

- Nearly perfectly aligned antenna beam pattern:

$$A_{12} = f_1^+ f_2^\times - f_2^+ f_1^\times \sim 0$$

- Null Stream (2-detector)
  - residual signal amplitude proportional to A_{12}
  - minimize residual signal variance at source direction

$$A(\alpha, \delta, t) = A_2 h_1(t) - \cos(\xi_1 - \xi_2) A_1 h_2(t + \tau_{12})$$

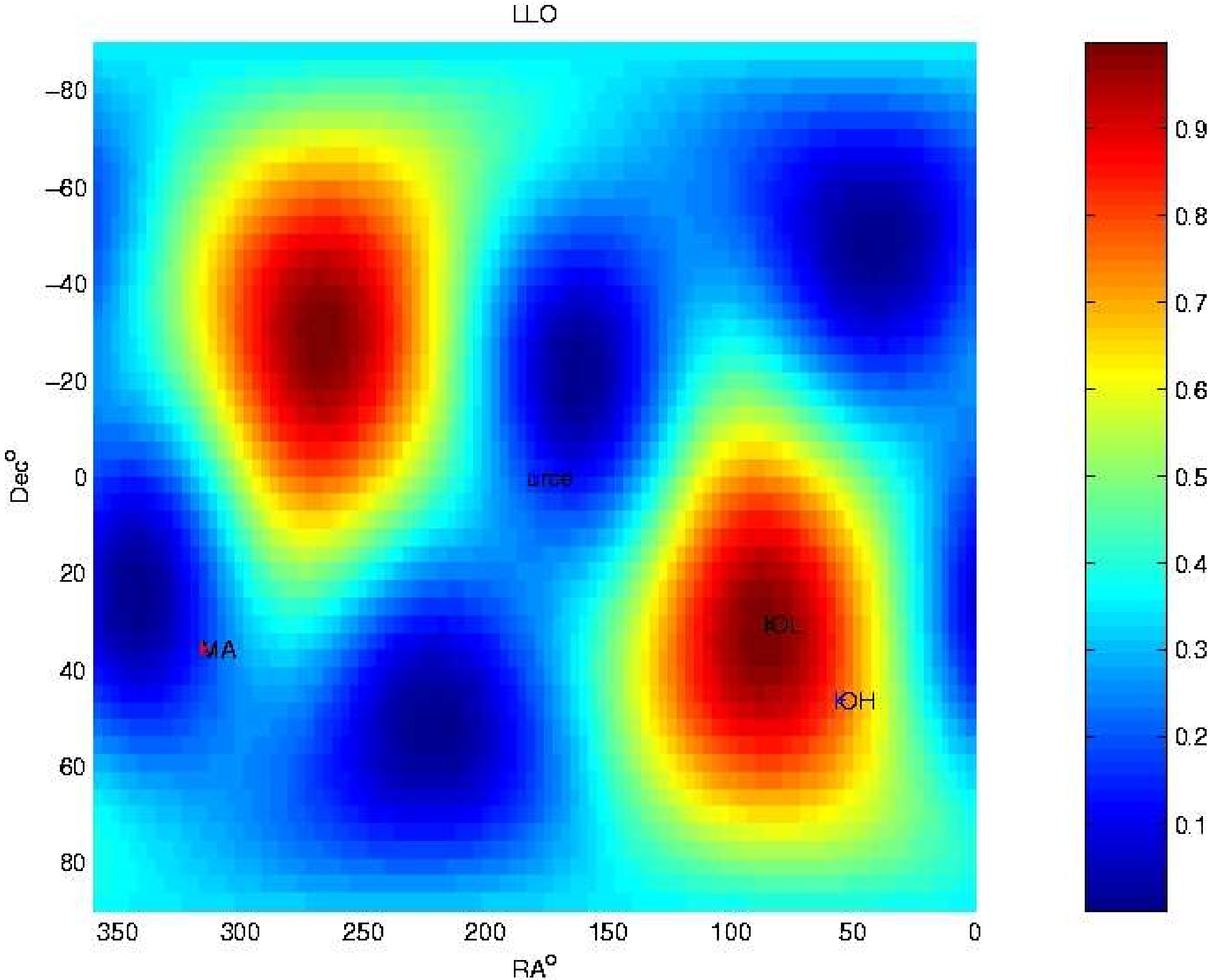
- Find minimum signal contribution in the null-stream
  - Calculate variance of normalized residual in f-domain
  - Compare it with expected noise distribution
    - ($P(\alpha, \delta)$ follows χ^2_{2N} distribution)

$$P(\alpha, \delta) = 2 \sum_{k=1}^{N} \frac{A_k^2(\alpha, \delta)}{\sigma_k^2}$$

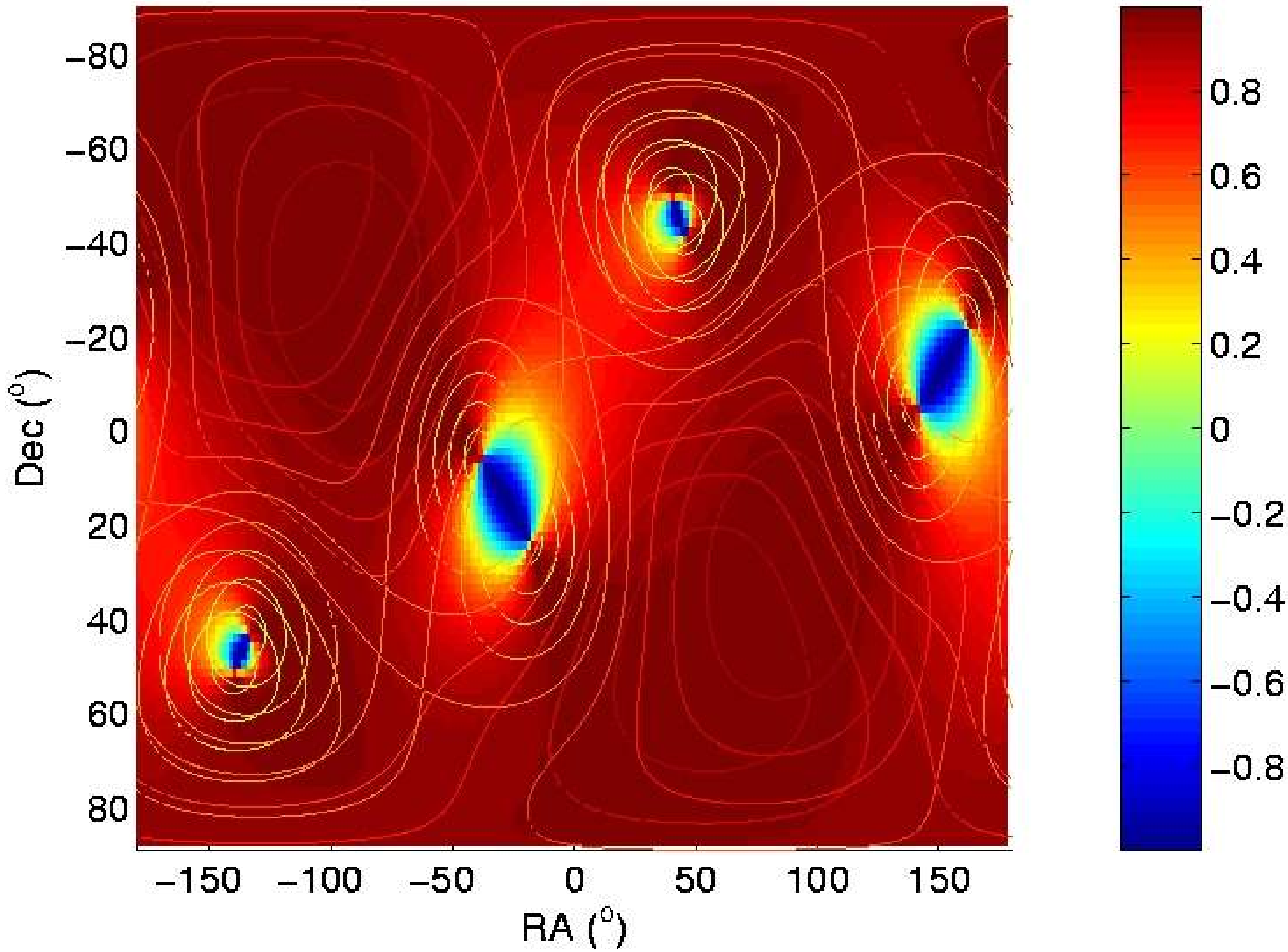
$$\text{(3-detector:)}\ \sigma_k^2 = A_{23}^2 \sigma_{1k}^2 + A_{31}^2 \sigma_{2k}^2 + A_{12}^2 \sigma_{3k}^2$$

$$\text{(2-detector):}\ \sigma_k^2 = A_2^2 \sigma_{1k}^2 + A_1^2 \cos^2(\xi_1 - \xi_2) \sigma_{2k}^2$$

A_1



$$-\cos(\xi_1 - \xi_2)$$



-80
-60
-40
-20
0
20
40
60
80
Dec (°)
-150
-100
-50
0
50
100
150
RA (°)
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8

Relevant Feature of LIGO,GEO and TAMA

- LIGO (Livingston, Hanford):
  - most sensitive, nearly perfectly aligned antenna pattern (H2 not considered yet)
  - h's out of phase for most part of the sky
- GEO/VIRGO
  - most sensitive around null of LIGO
  - GEO: long duty cycle
- TAMA
  - long time delay, i.e. good position/delay
  - long duty cycle

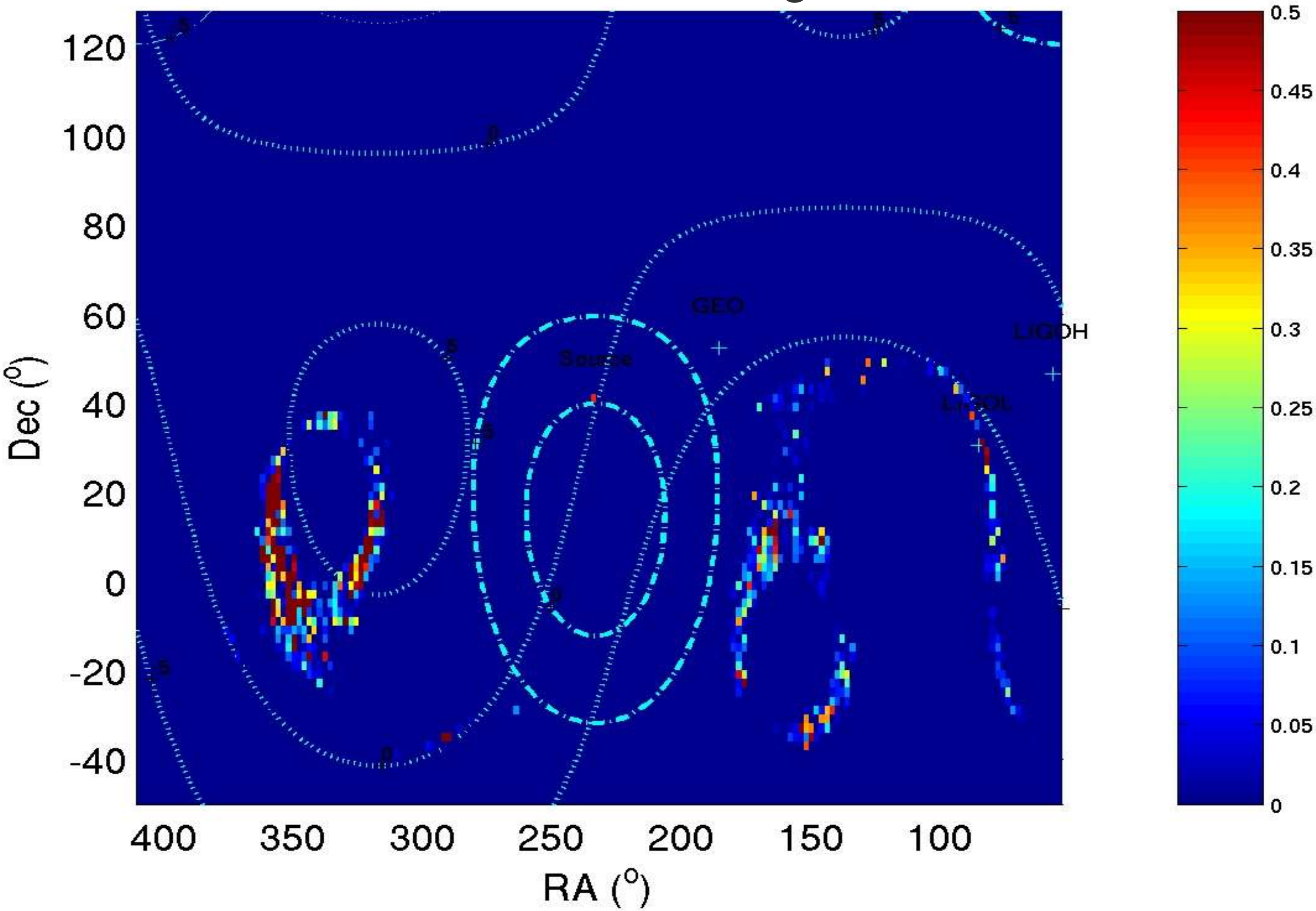
Monte-Carlo Simulation in Progress

- Detectors:
  - 2 LIGO+GEO and 2 LIGO+TAMA
  - 2 LIGOs+VIRGO
- Noise: use projected S(f):
  - Noise: stationary Gaussian. Initial LIGO
- Signal: use numerical BH merger waveform from Lazarus project
  - $M_{total} = 20 \ M_{sun}$
- Assuming time dependence of antenna pattern negligible compared with signals
  - when calculating expected noise

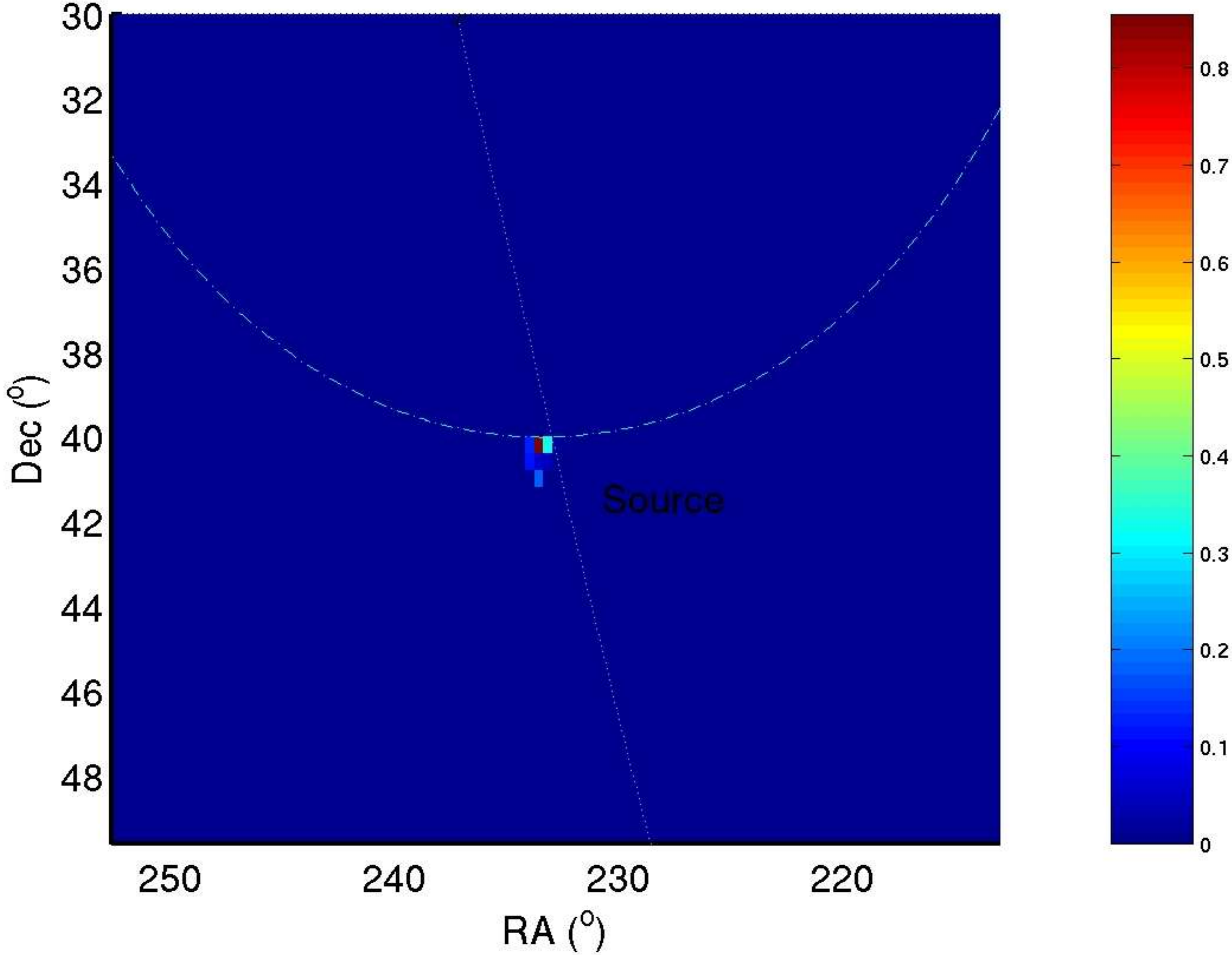
Examples

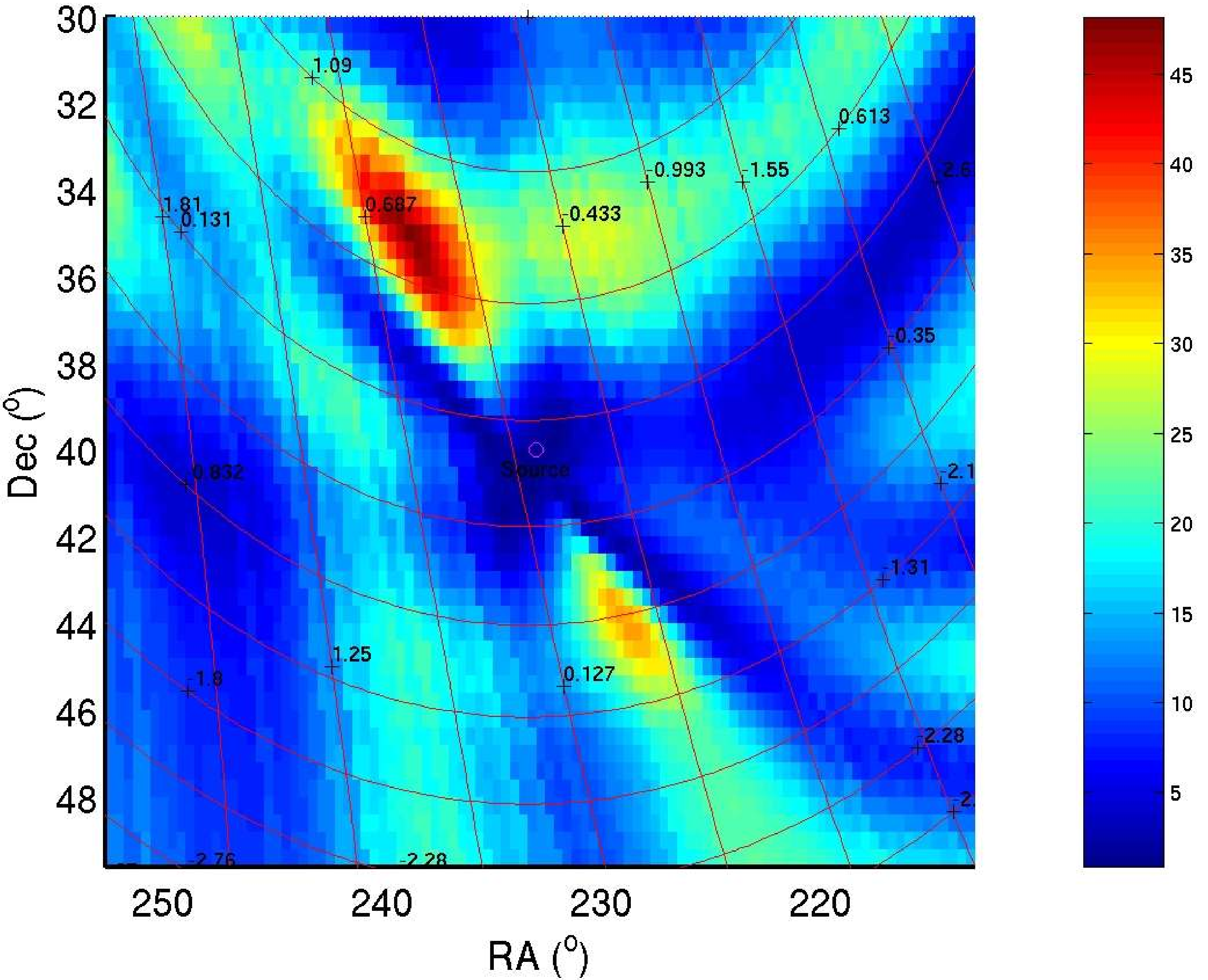
χ^2 test

10-10 Ms merger, d=1 Mpc, snr=93, LLO-LHO-GEO, near null of LIGO, 2-deg resolution



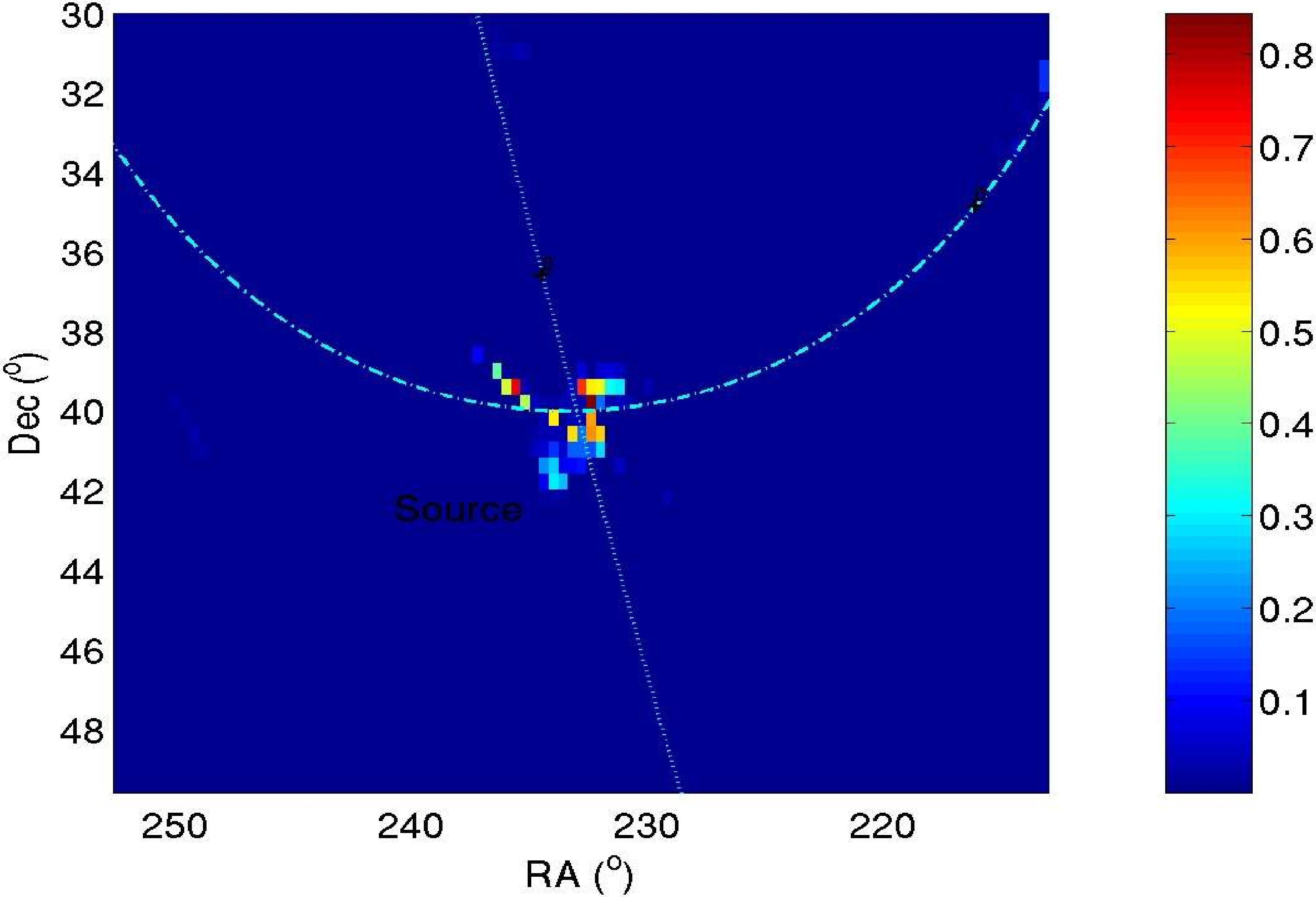
d=1 Mpc, close-up: positional determination ~10' (Searched with 15' resolution)





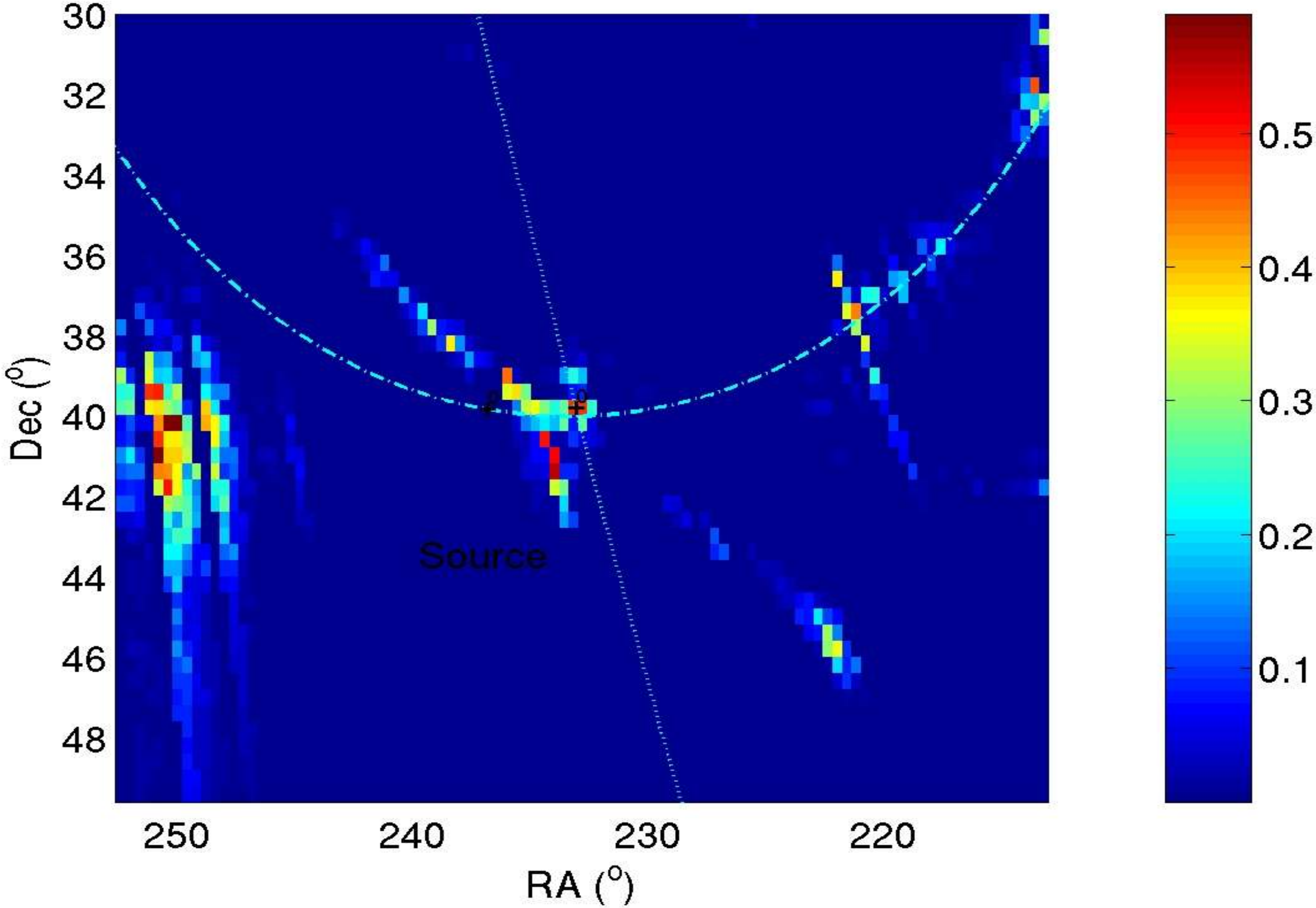
Dec (°)
RA (°)
30
32
34
36
38
40
42
44
46
48
250
240
230
220
45
40
35
30
25
20
15
10
5
1.09
0.613
-0.993
-1.55
1.81
0.131
0.687
-0.433
-0.35
0.832
Source
-2.1
-1.31
1.25
1.8
0.127
-2.28
-2.76
-2.28

d=3 Mpc



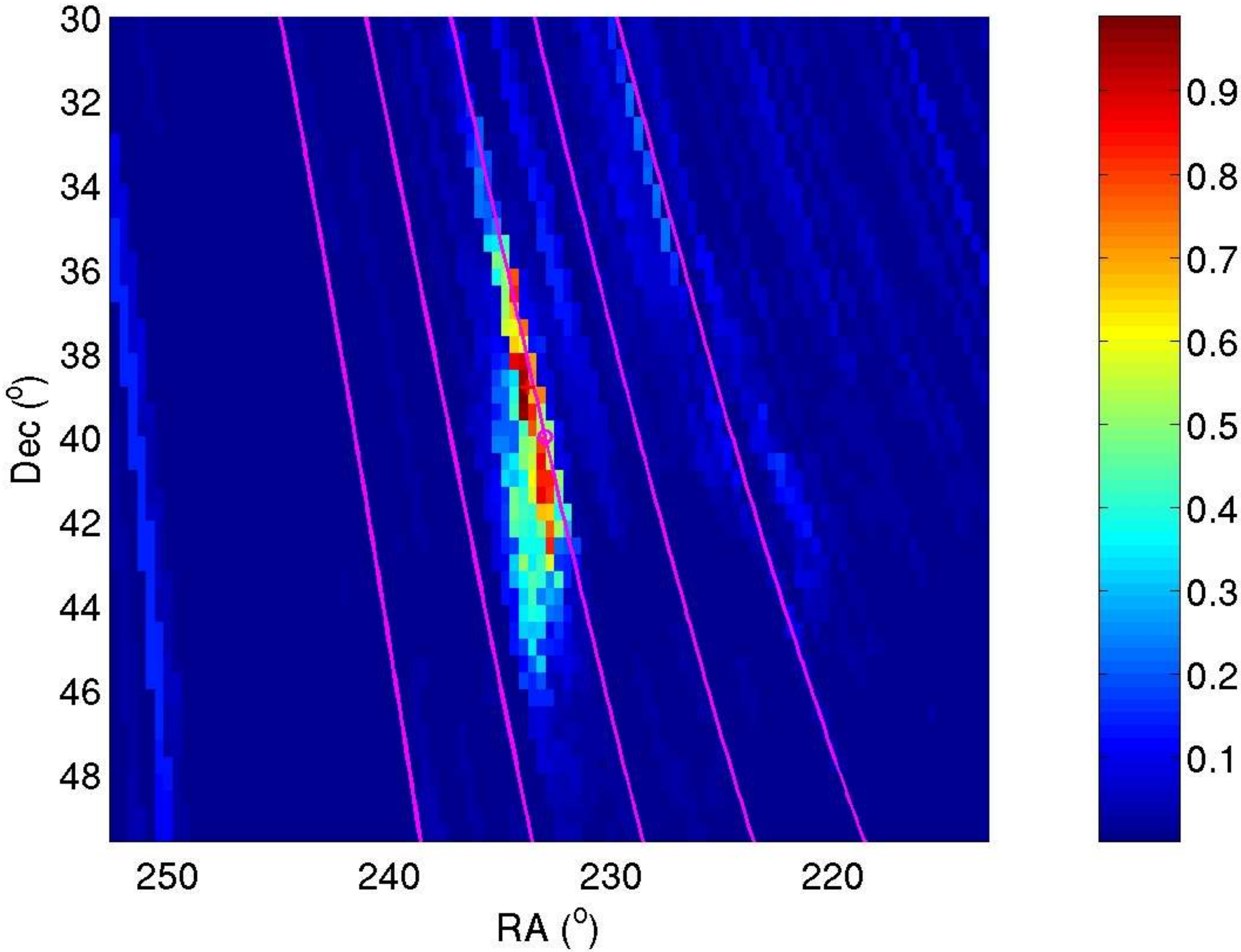
30
32
34
36
38
40
42
44
46
48
Dec (°)
Source
250
240
230
220
RA (°)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1

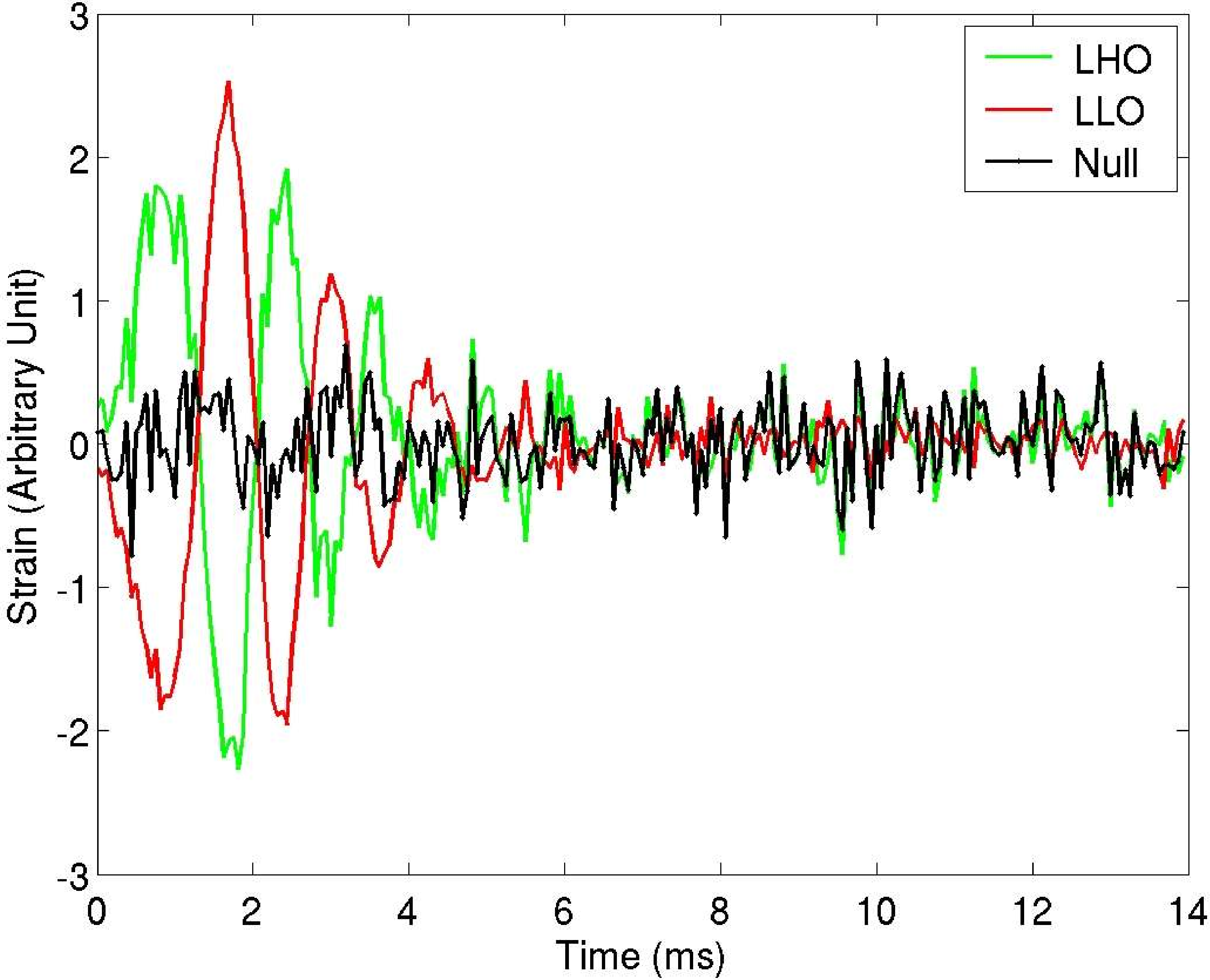
d=5 Mpc (failed)



2-detector (LLO-LHO)

d=1 Mpc, snr=91, identical white noise





LHO
LLO
Null
Strain (Arbitrary Unit)
Time (ms)
3
2
1
0
-1
-2
-3
0
2
4
6
8
10
12
14

Monte-Carlo Simulation
(in progress)

Summary

- Discussed null-stream construction for 2,3 detectors
    - Work on a list of triggered “coincident” events from multiple detectors
    - application to positional determination
    - implication to veto
    - works for almost any waveforms
        - ignore arrival time change due to Earth rotation
        - Antenna pattern : time-dependence negligible when calculating expected noise

Future work

- Optimal positional determination ?
- Implication to Veto
- Dependence on snr of each detector, source direction, observable signal properties
- include bars, realistic noise, other type of waves (e.g. inspiral waves), etc.