In- and out-of-equilibrium dynamics in integrable systems

Jean-Sébastien Caux
Universiteit van Amsterdam

Work done in collaboration with:

'Amsterdam integrable models group':
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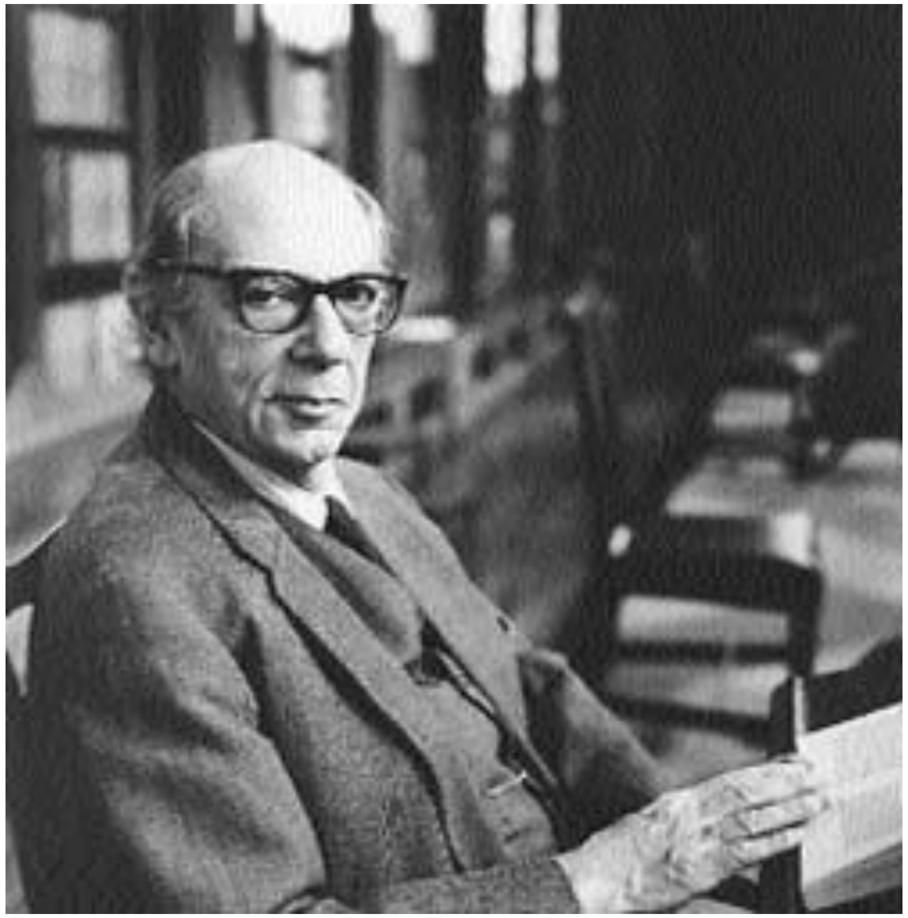
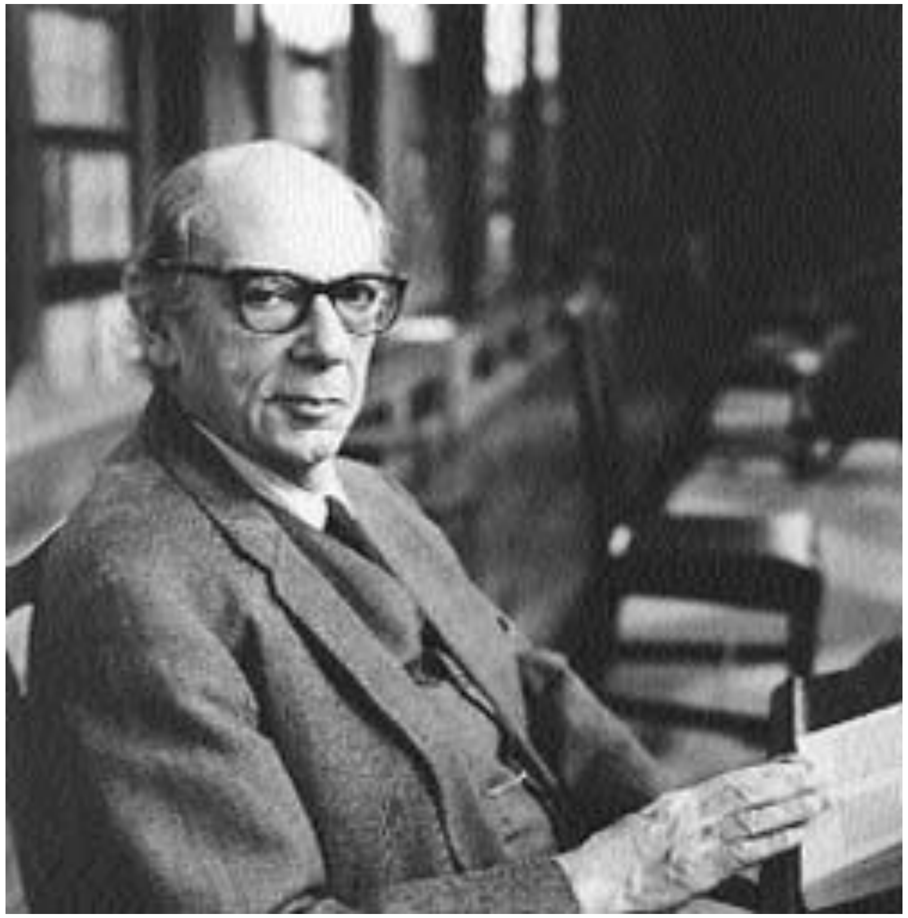
P. Calabrese, I. Pérez Castillo, A. Faribault, N. Slavnov



Isaiah Berlin

(June 6, 1909- Nov 5,1997)



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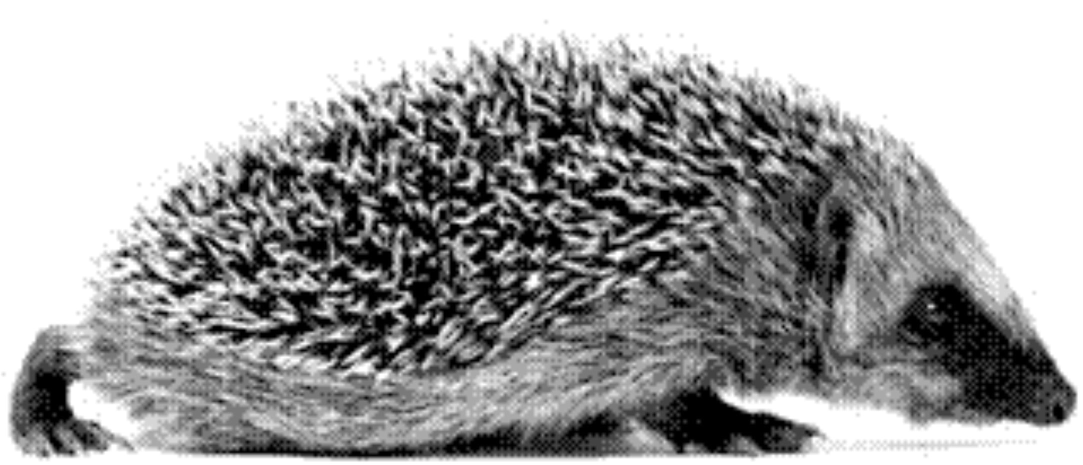

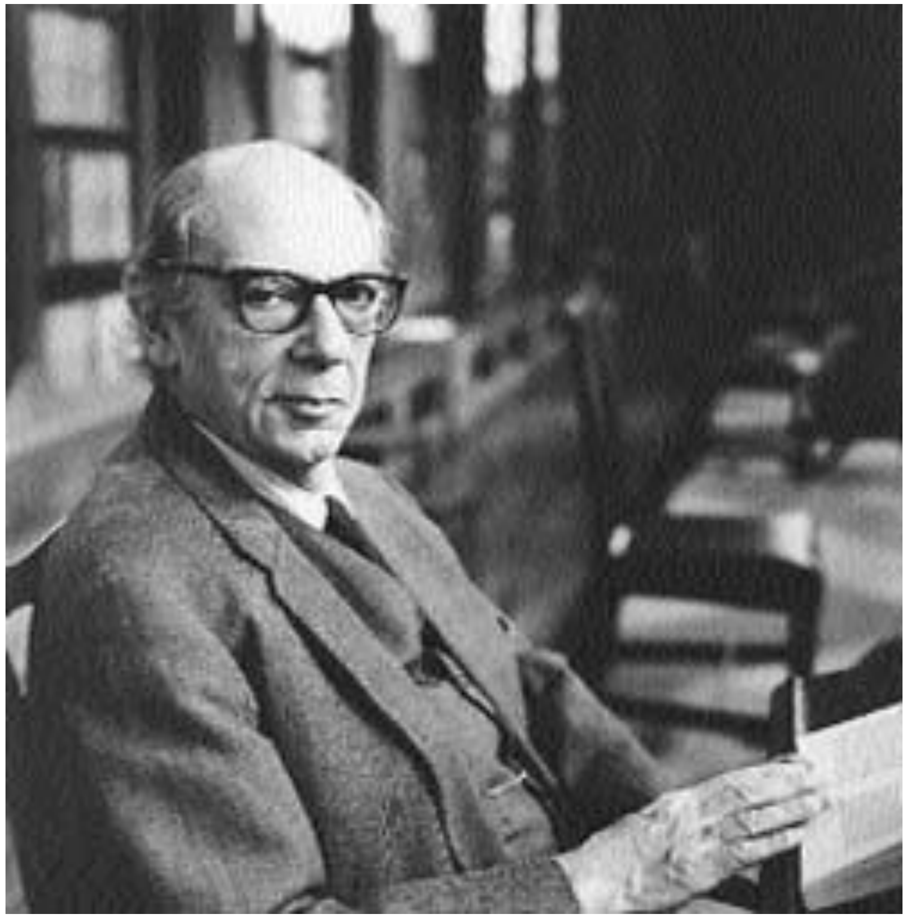

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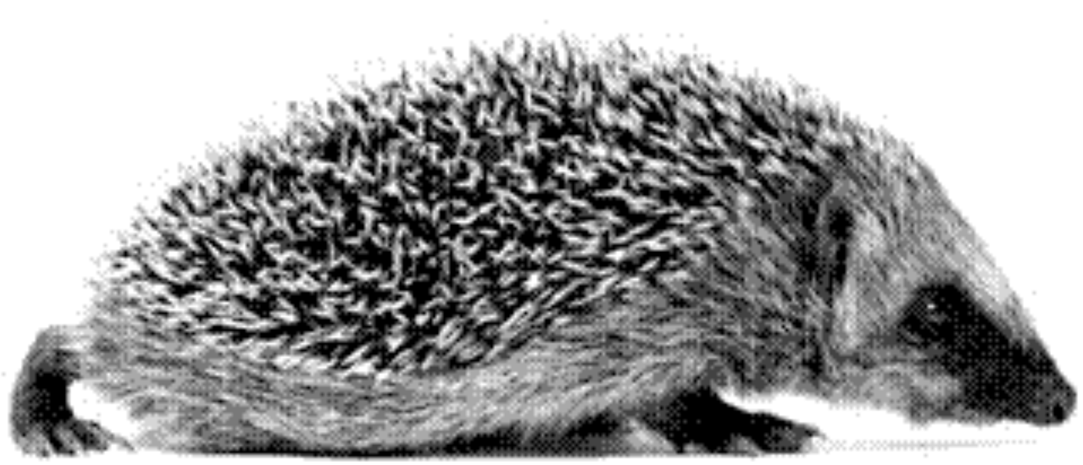

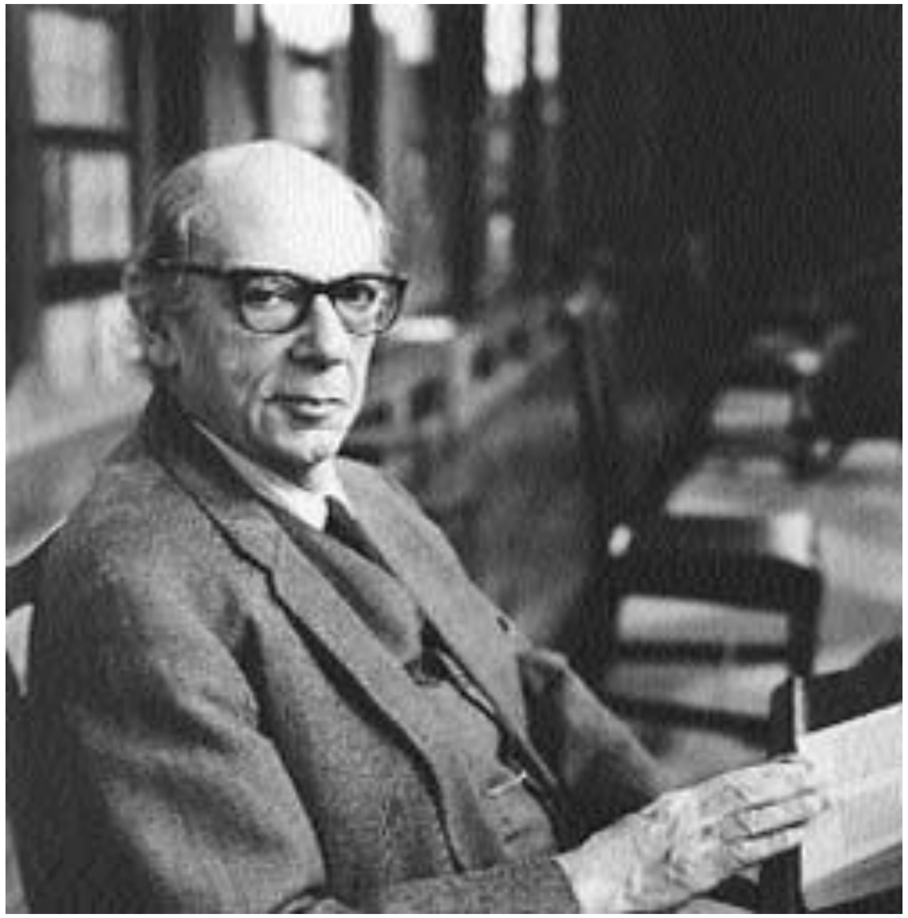

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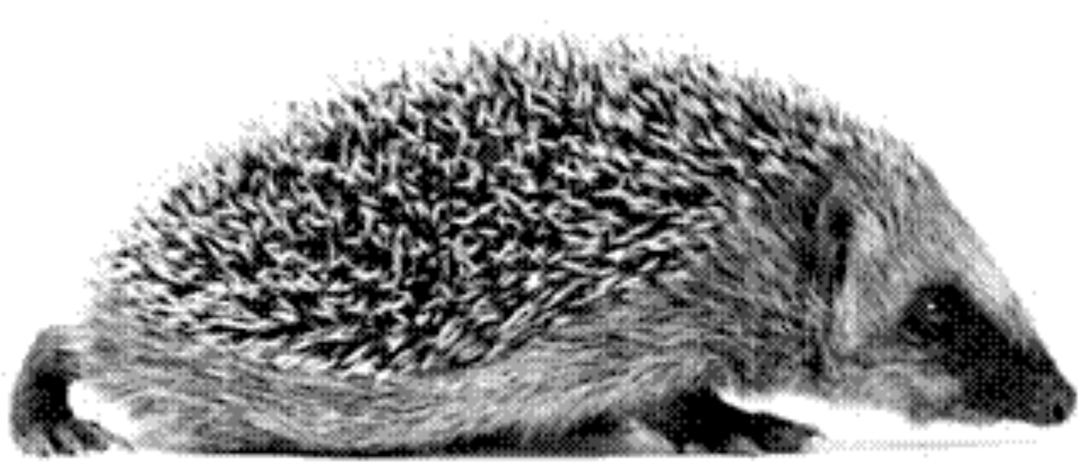

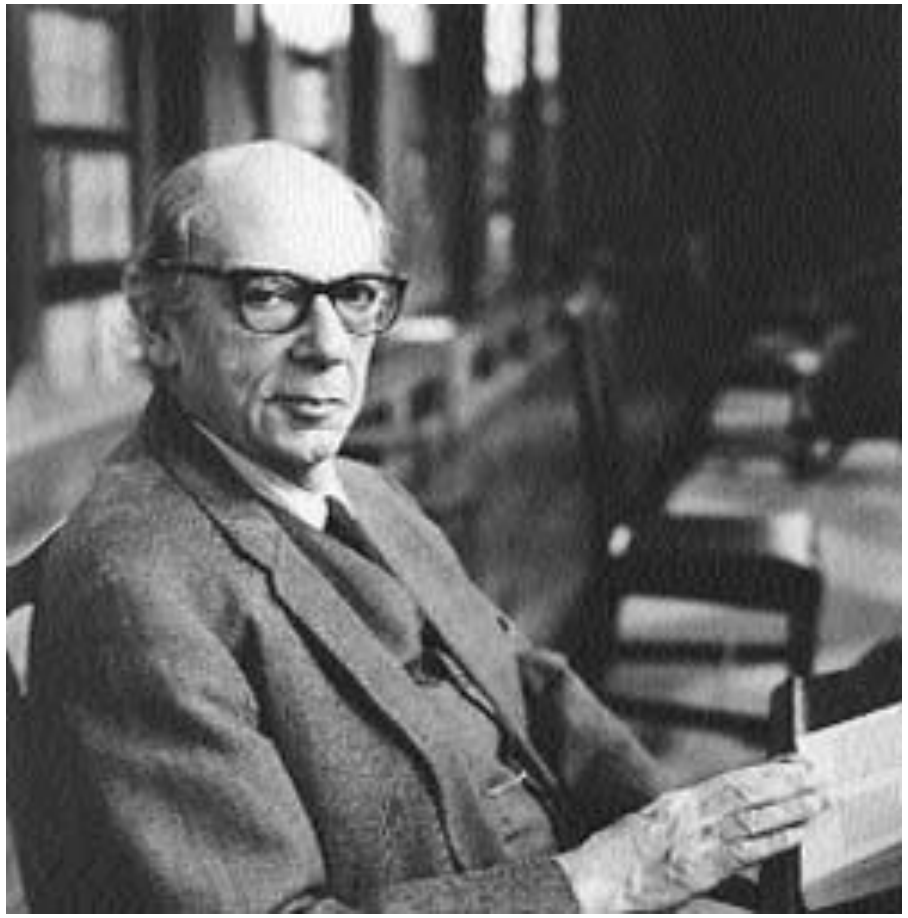

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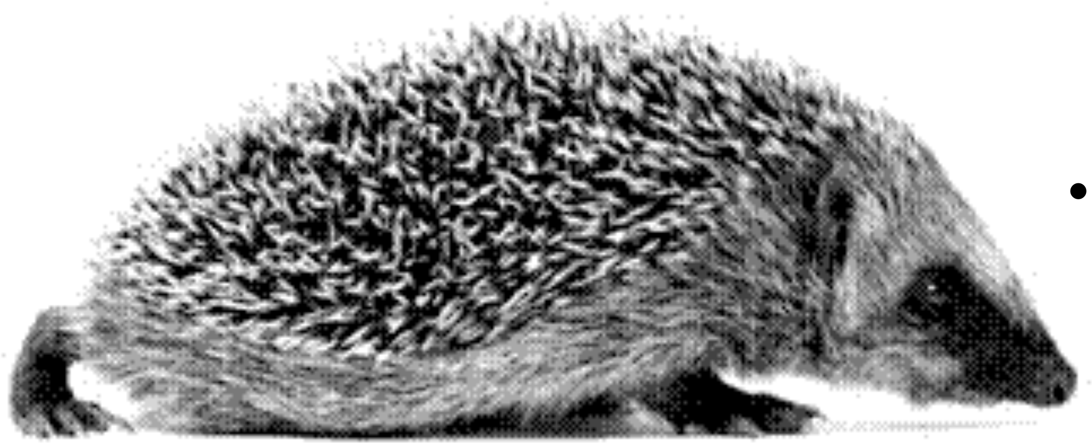
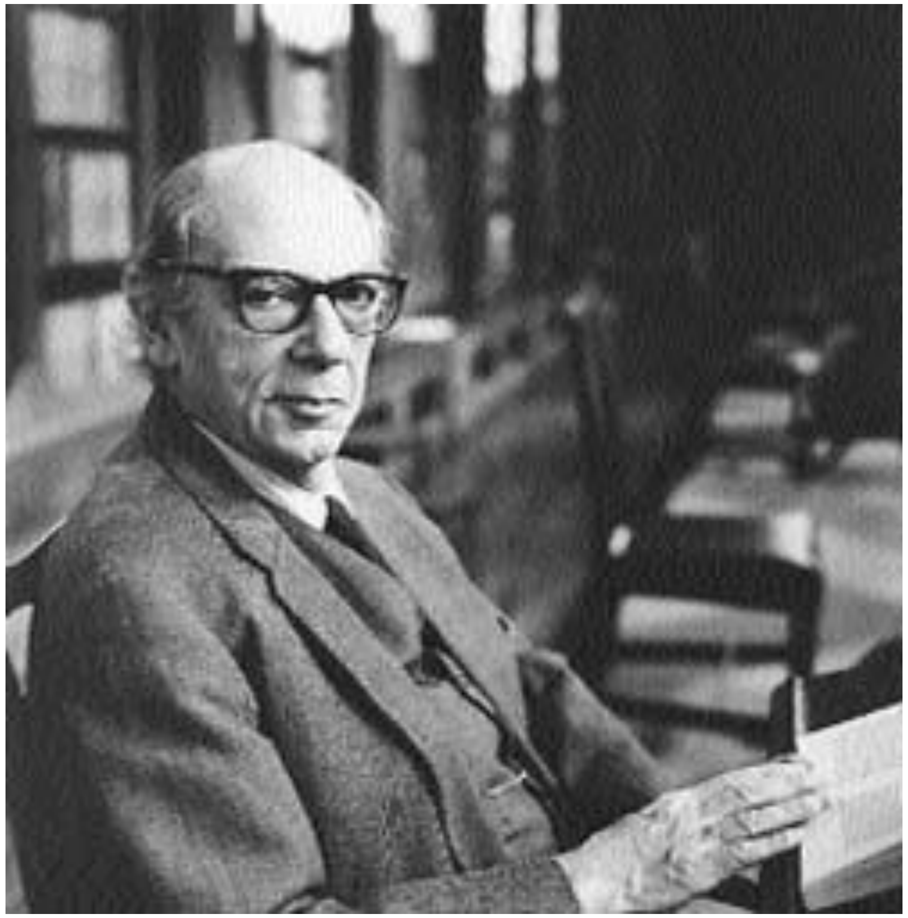


... but the hedgehog knows one big thing.

(Archilochus)



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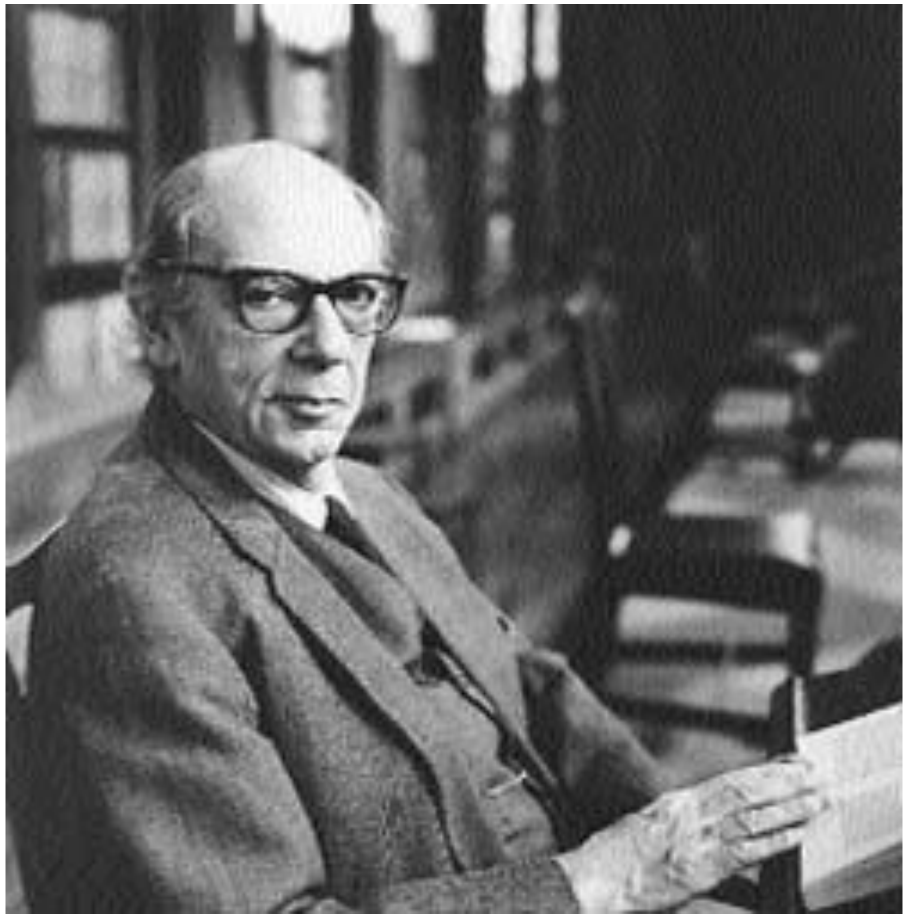

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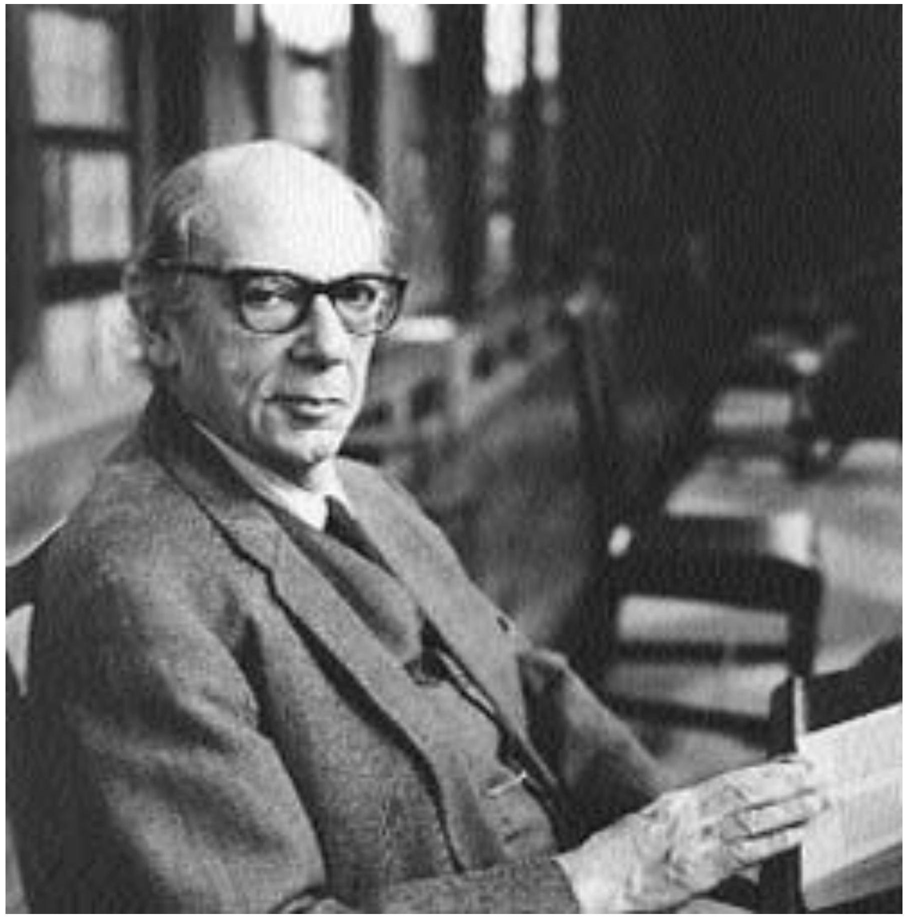

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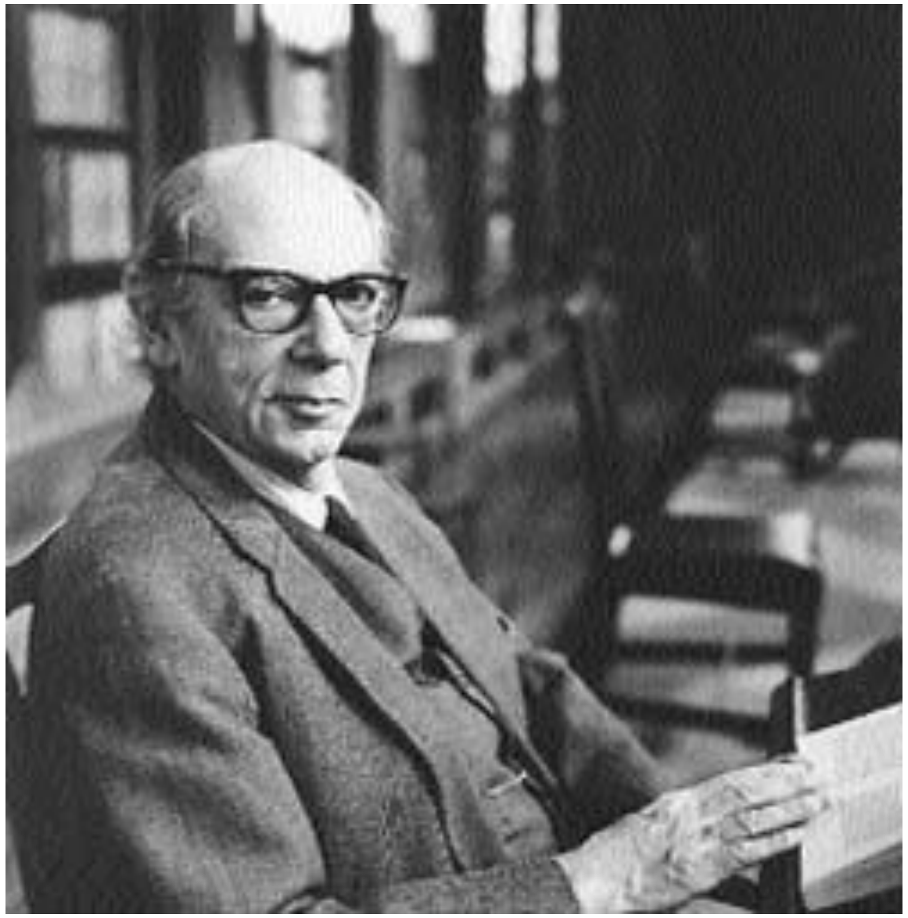

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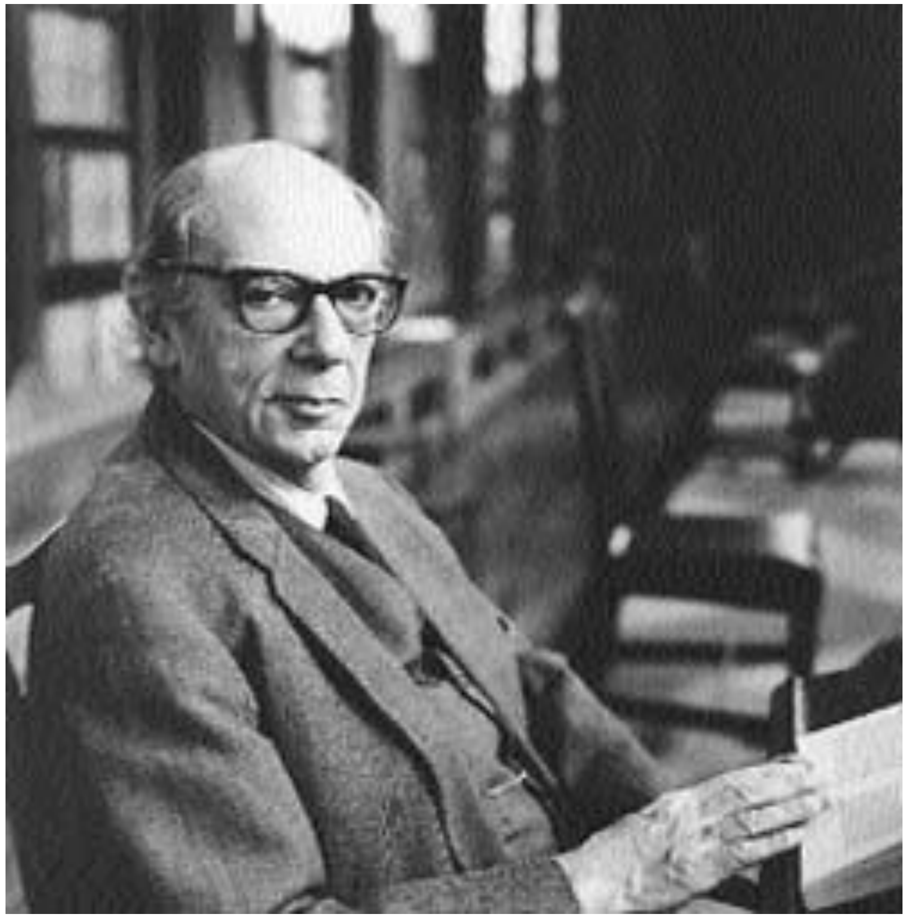

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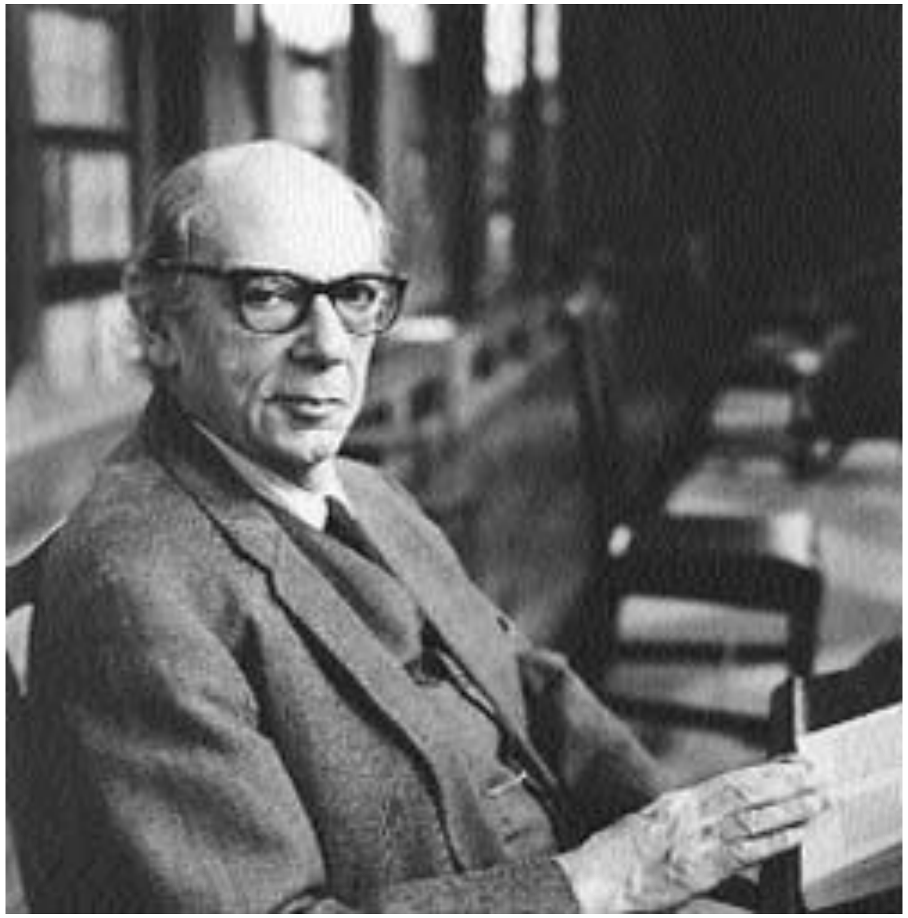

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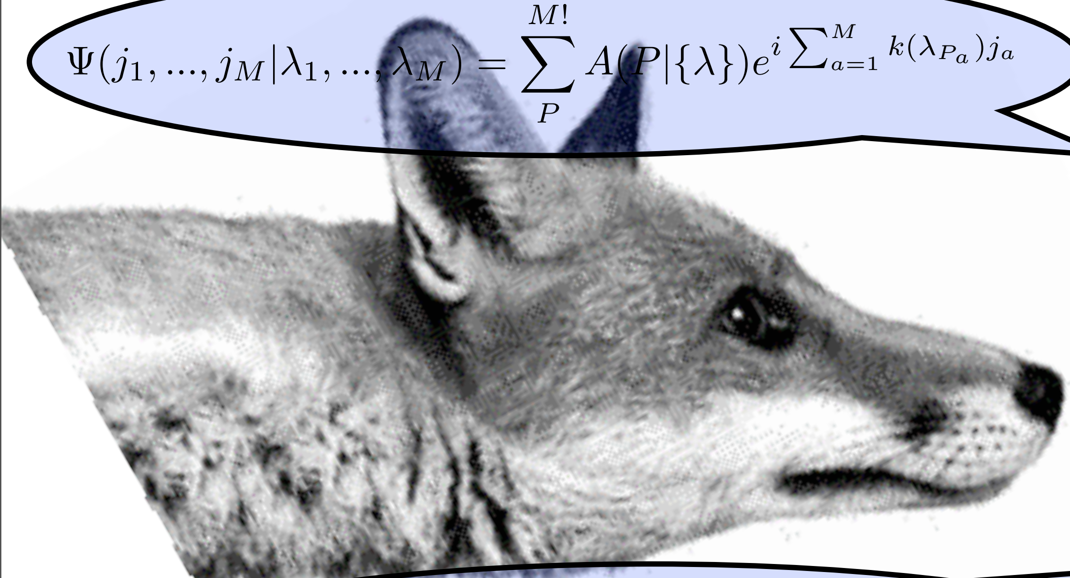

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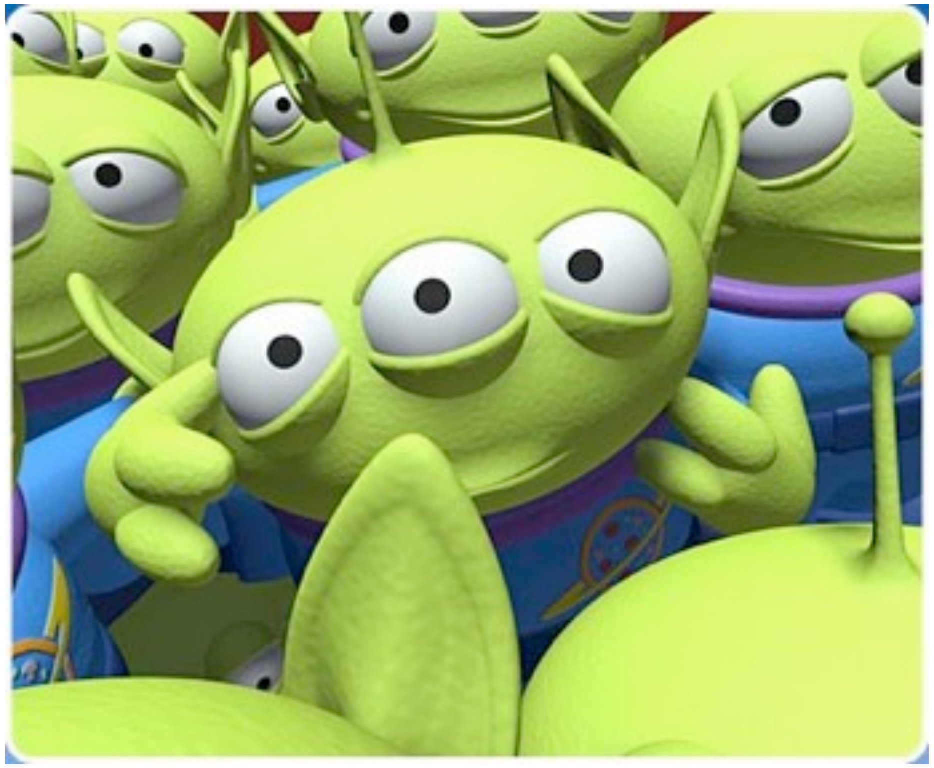

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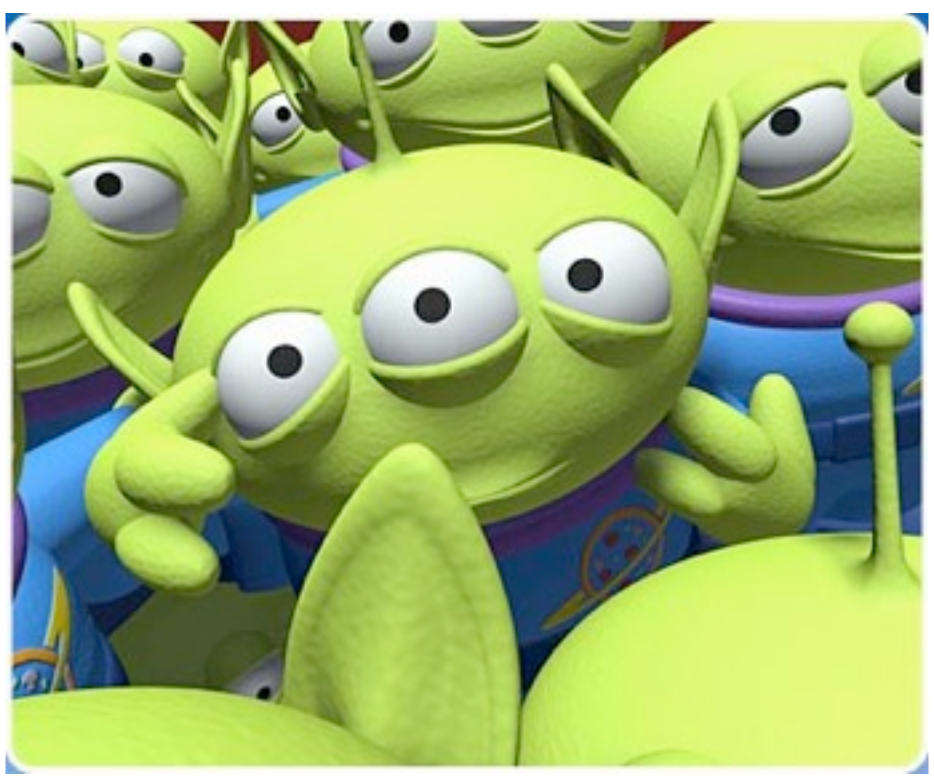

Computing nerd

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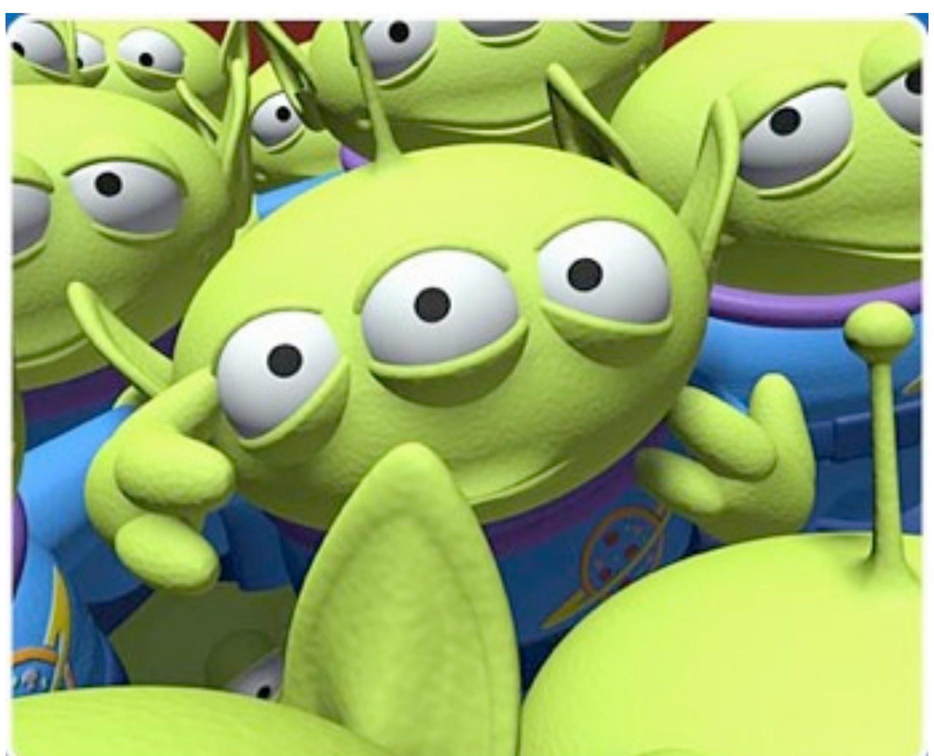


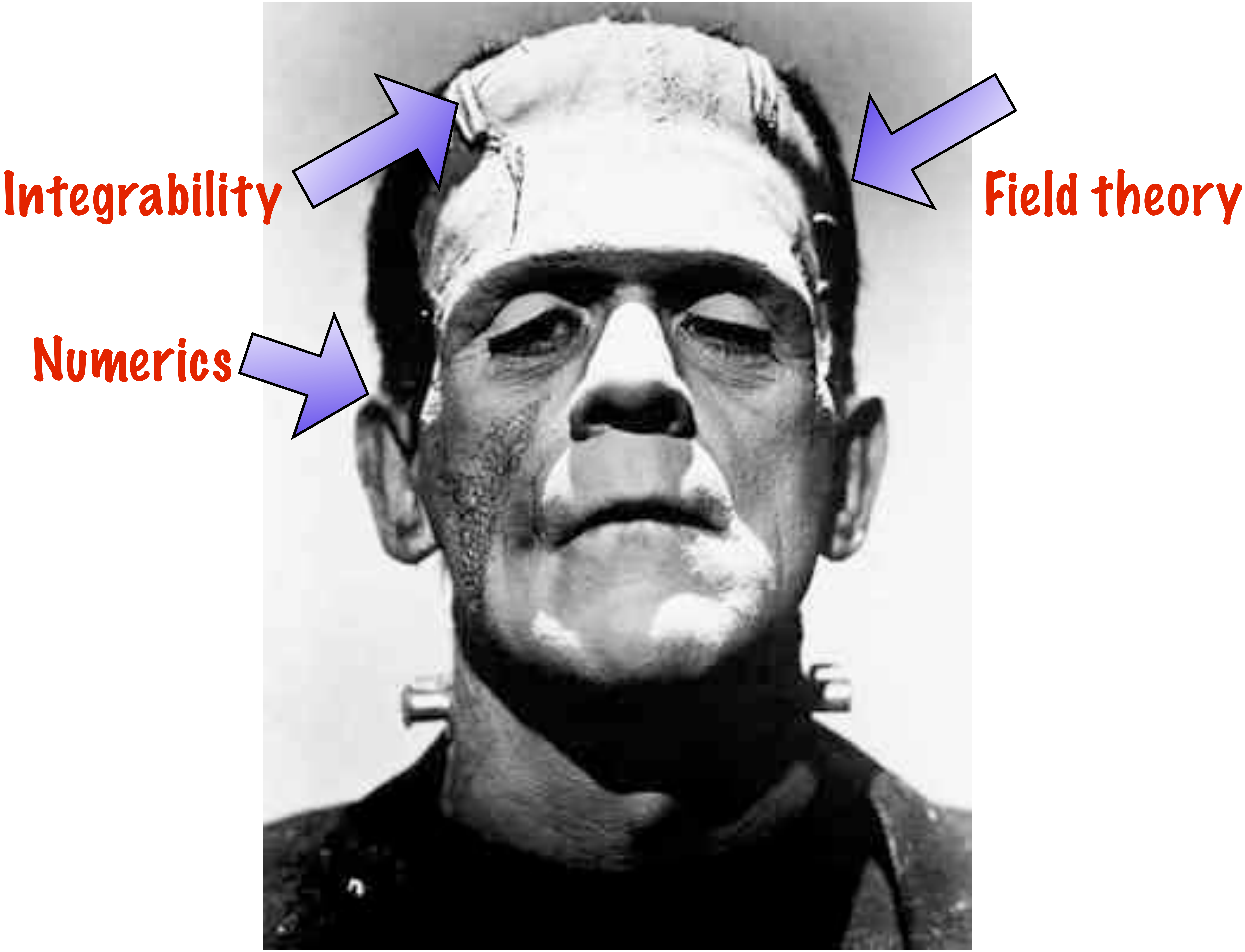
Computing nerd

Jack of all trades

Integrability

Integrability

Field theory


Integrability
Field theory
Numerics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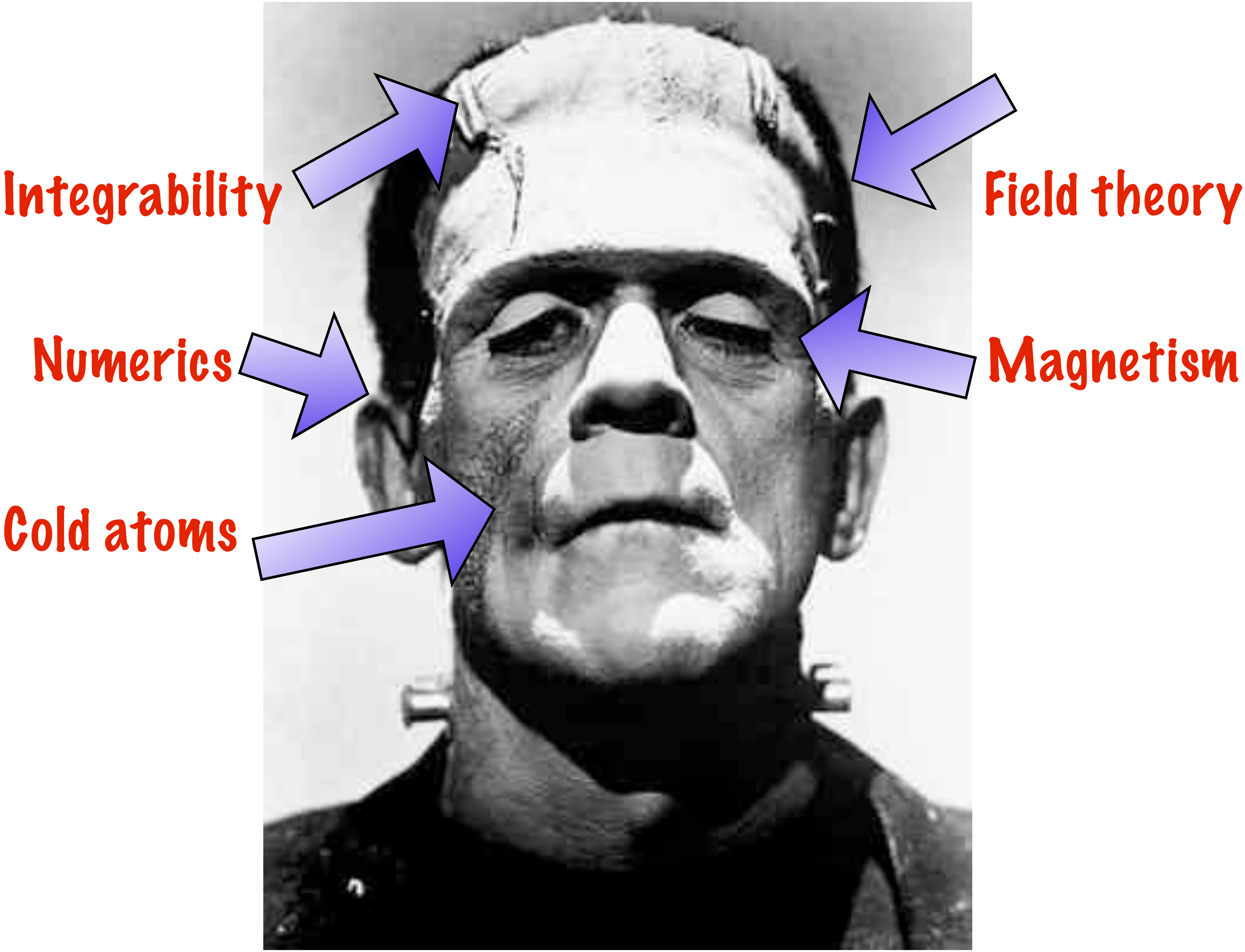
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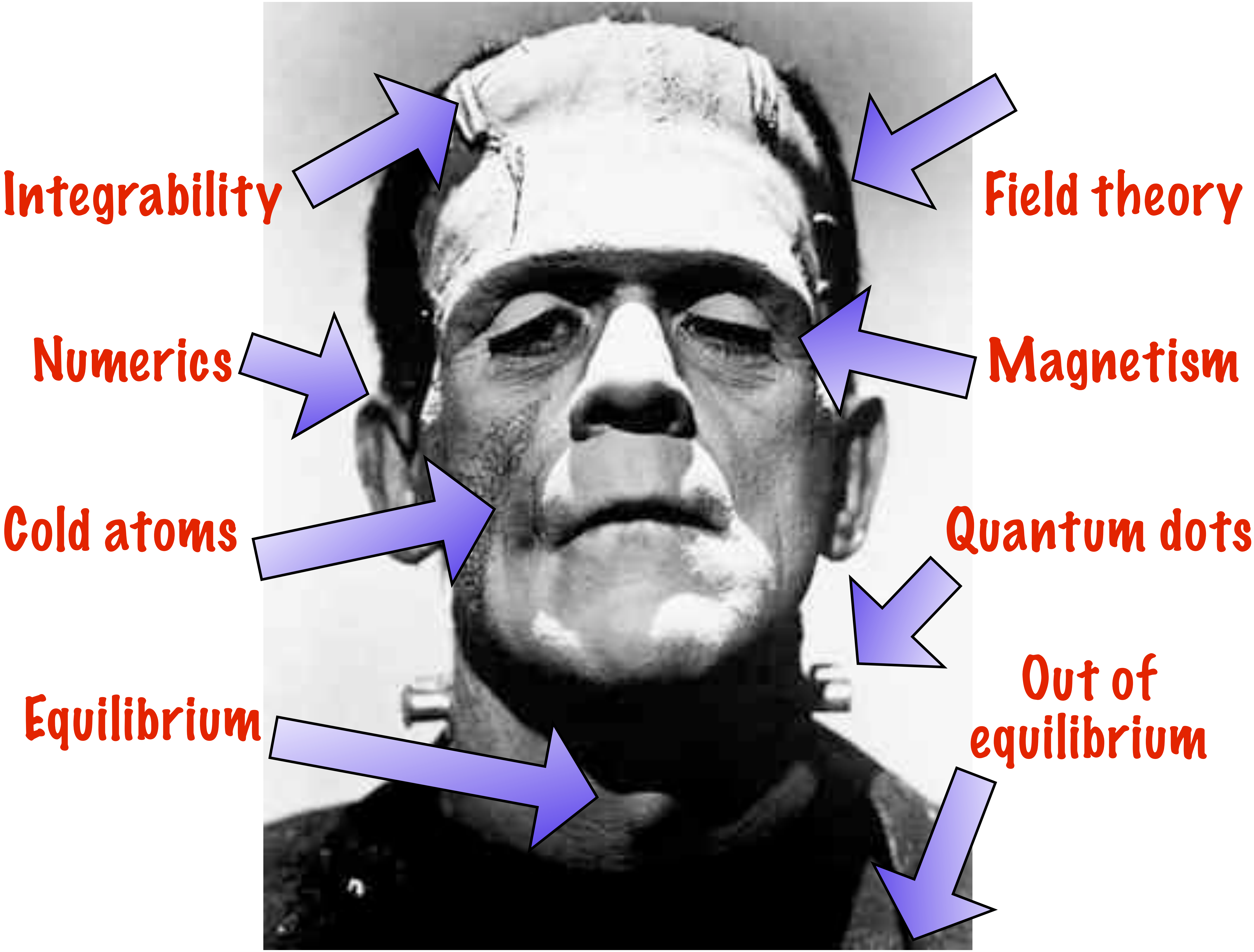
Integrability
Numerics
Cold atoms
Equilibrium
Field theory
Magnetism
Quantum dots
Out of equilibrium

Outline of the talk

- Motivations
- Building blocks needed
- Part 1: equilibrium dynamics

  Lieb-Liniger, Heisenberg, Richardson
  Applications

- Part 2: quench dynamics

  Richardson, Heisenberg
  Geometric quench

Correlation functions and quantum quenches from integrability...

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- Way to reliably study quantum correlation effects in many-body systems (exotic excitations: transmutation, fractionalization, ...)
- There are some very good experimental realizations requiring phenomenology
- Great way to provide reliable beacons for other, more general methods (field theory-based, numerical)

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Reexpress the result in the basis of Bethe states:

$$\mathcal{O}|\{\lambda\}\rangle = \sum_{\{\mu\}} F^{\mathcal{O}}_{\{\mu\},\{\lambda\}} |\{\mu\}\rangle$$

using 'matrix elements' $F^{\mathcal{O}}_{\{\mu\},\{\lambda\}} = \langle\{\mu\}|\mathcal{O}|\{\lambda\}\rangle$

Bethe Ansatz (1931)

July 2, 1906 – March 6, 2005

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... made up of free waves ...

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... parametrized by rapidities...

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... and obeying some form of Pauli principle

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Imposing boundary conditions quantizes the allowable rapidities according to the Bethe equations

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Constructing all states in the Hilbert space ⟷ Obtaining all solutions to the Bethe equations

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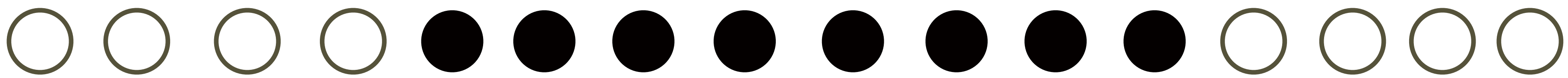

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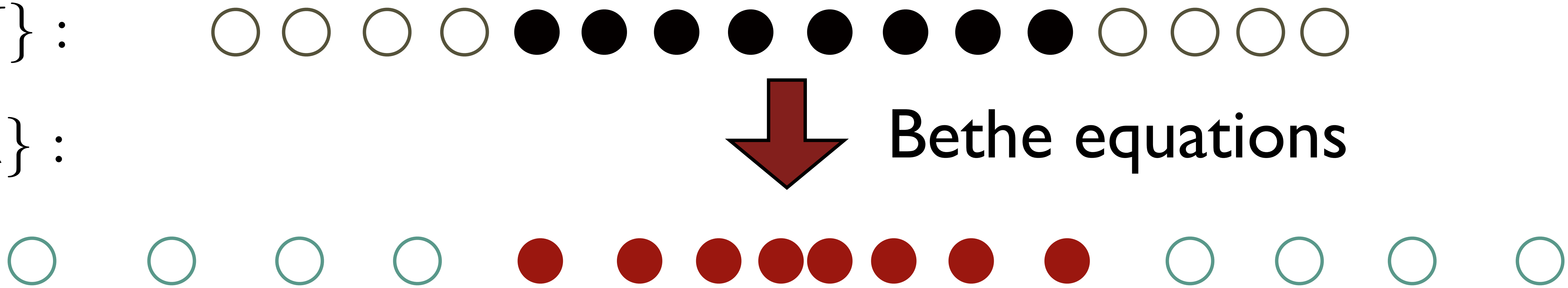

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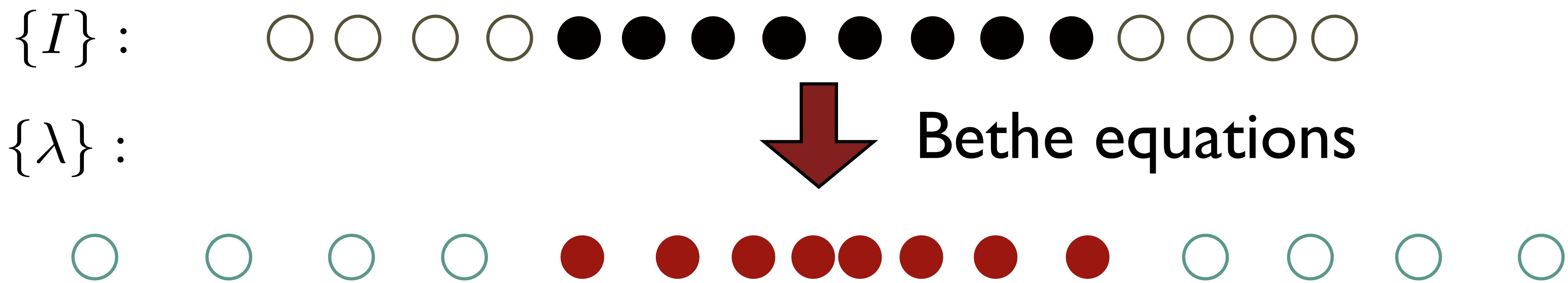

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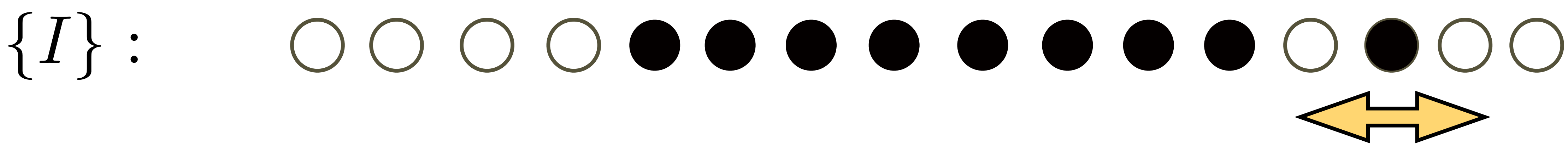

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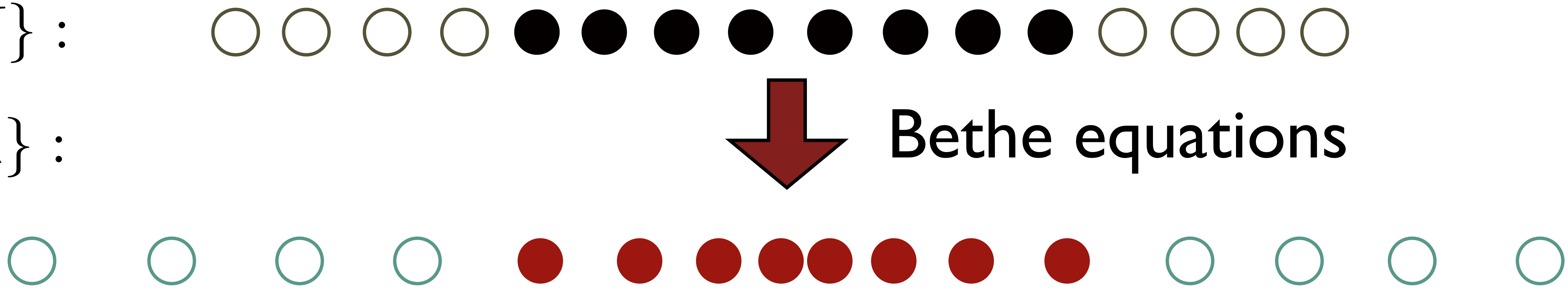

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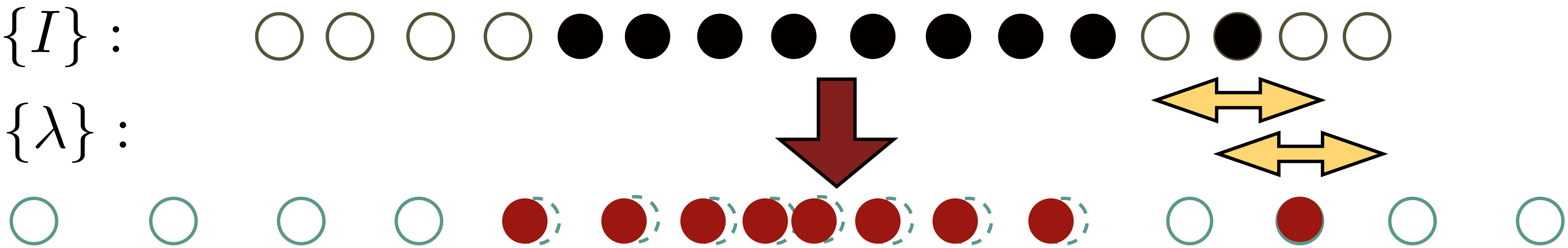

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$B(\lambda)$ creation operator, increasing particle number by 1

'Technology' needed: Algebraic BA

Like '2nd quantization' for Bethe Ansatz

Introduce family $A(\lambda), B(\lambda), C(\lambda), D(\lambda)$ of nonlocal operators which act in Hilbert space of model

$B(\lambda)$ creation operator, increasing particle number by 1

Wavefunctions: $|\Psi(\{\lambda\})\rangle = \prod_j B(\lambda_j)|0\rangle$

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State norms: Gaudin-Korepin formula

Scalar products: Slavnov's formula

$$S_M(\{\mu\},\{\lambda\}) = \langle 0| \prod_{j=1}^{M} C(\mu_j) \prod_{k=1}^{M} B(\lambda_k)|0\rangle$$

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where $T_{ab} = \frac{\partial}{\partial\lambda_a}\tau(\mu_b,\{\lambda\})$ (N.Slavnov, 1988)

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gives (at least in principle)
all matrix elements needed

Part I:

Equilibrium dynamics

Models which we treat:

Models which we treat:

- Heisenberg spin-1/2 chain

$$H = \sum_{j=1}^{N} \left[J(S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z) - H_z S_j^z \right]$$

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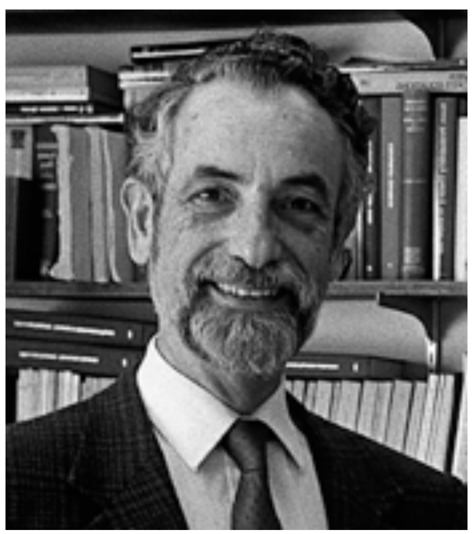

- Interacting Bose gas (Lieb-Liniger)

$$\mathcal{H}_N = -\sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2} + 2c \sum_{1 \le j < l \le N} \delta(x_j - x_l)$$

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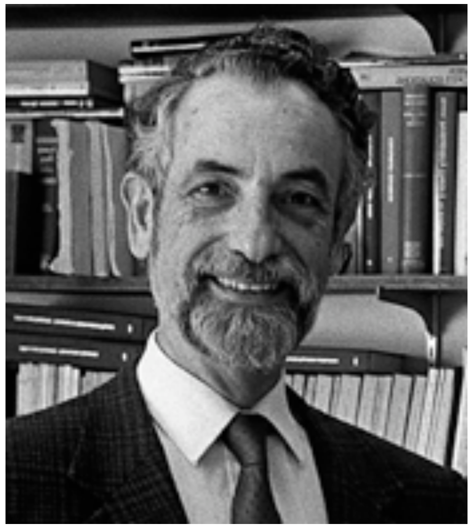

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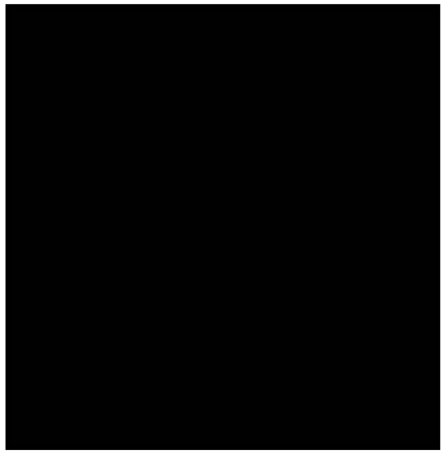

- Richardson model (+ Gaudin magnets)

$$H_{BCS}=\sum_{\substack{\alpha=1\\ \sigma=+,-}}^{N}\frac{\varepsilon_\alpha}{2}c^\dagger_{\alpha\sigma}c_{\alpha\sigma}-g\sum_{\alpha,\beta=1}^{N}c^\dagger_{\alpha+}c^\dagger_{\alpha-}c_{\beta-}c_{\beta+}$$

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- DYNAMICAL STRUCTURE FACTOR

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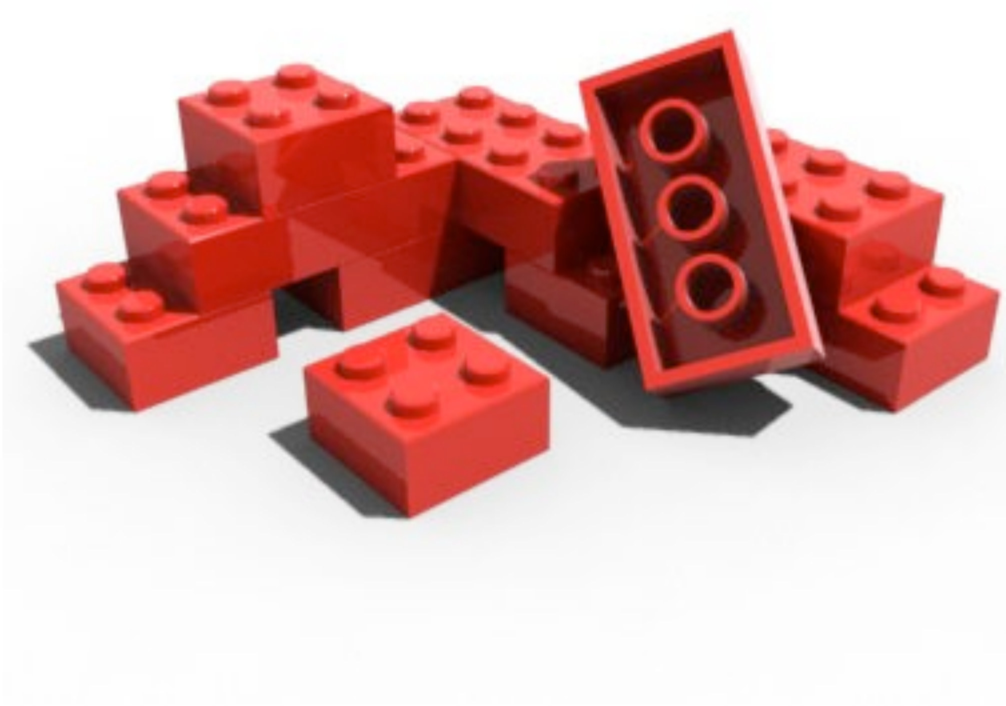
→ ***Bragg spectroscopy, interference experiments, ...***

(zero temperature only (for now !))

Building correlation functions piece by piece



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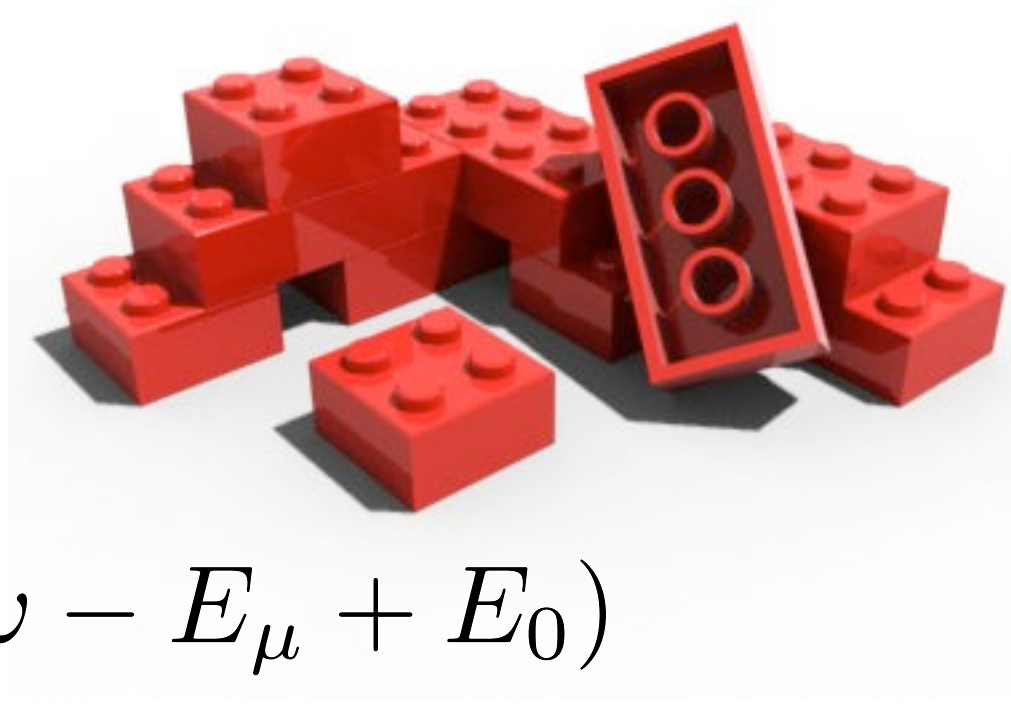

Our needed building blocks are:

$$S^{a,\bar{a}}(q,\omega) = 2\pi \sum_{\mu} |\langle 0|\mathcal{O}_q^a|\mu\rangle|^2 \delta(\omega - E_\mu + E_0)$$

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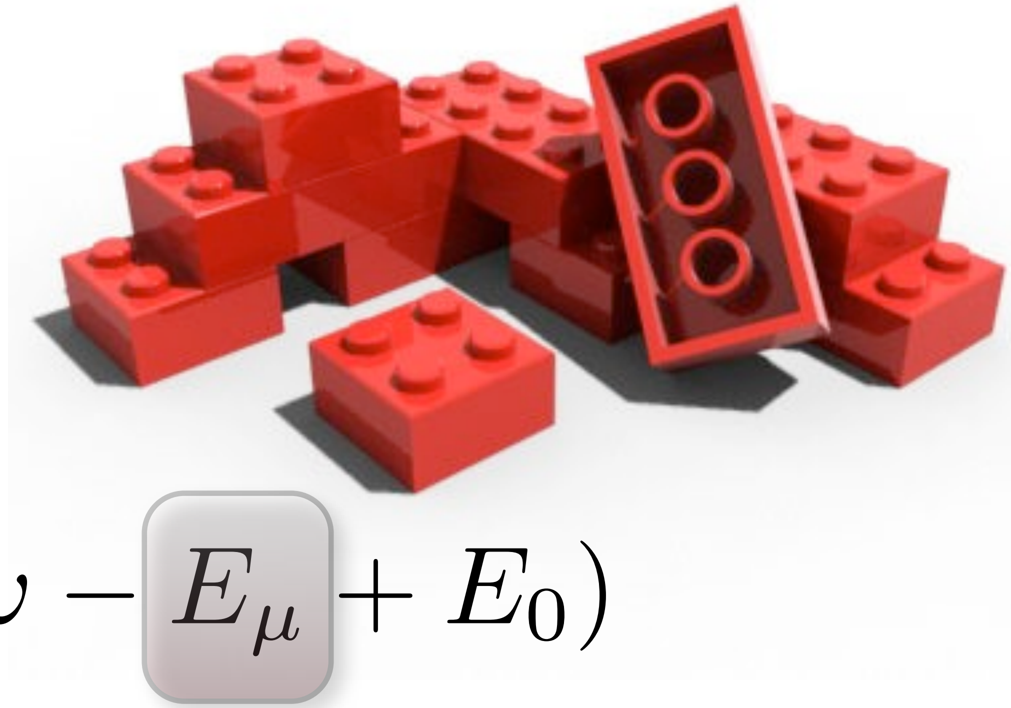

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→ ***Bethe Ansatz; quantum groups***

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→ ***Algebraic Bethe Ansatz; q. groups***

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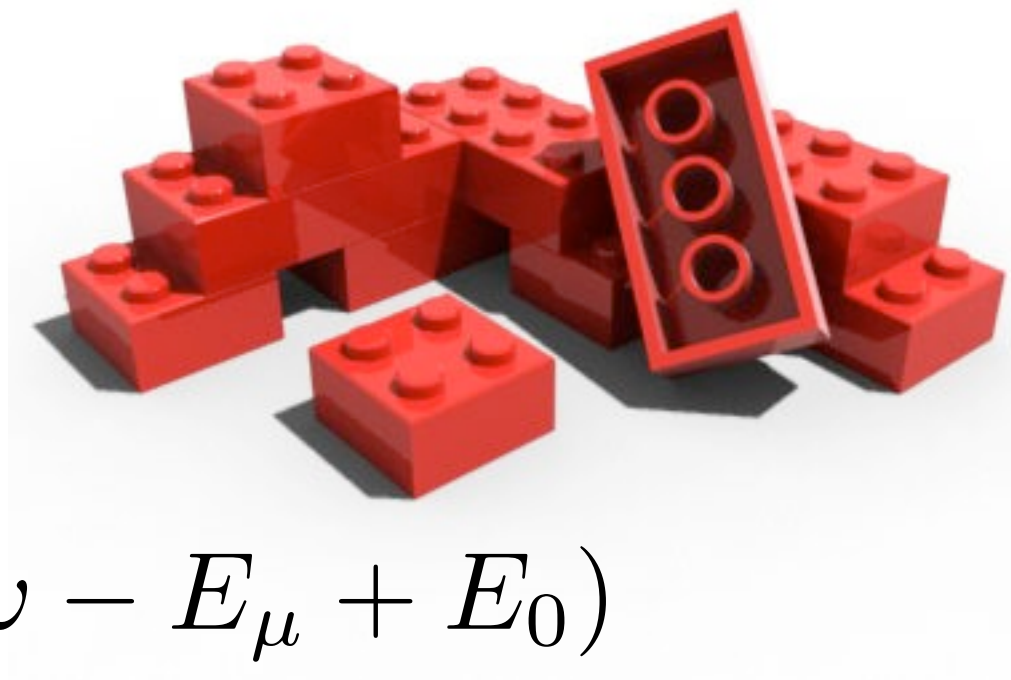

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→ ***Numerics (ABACUS); analytics***

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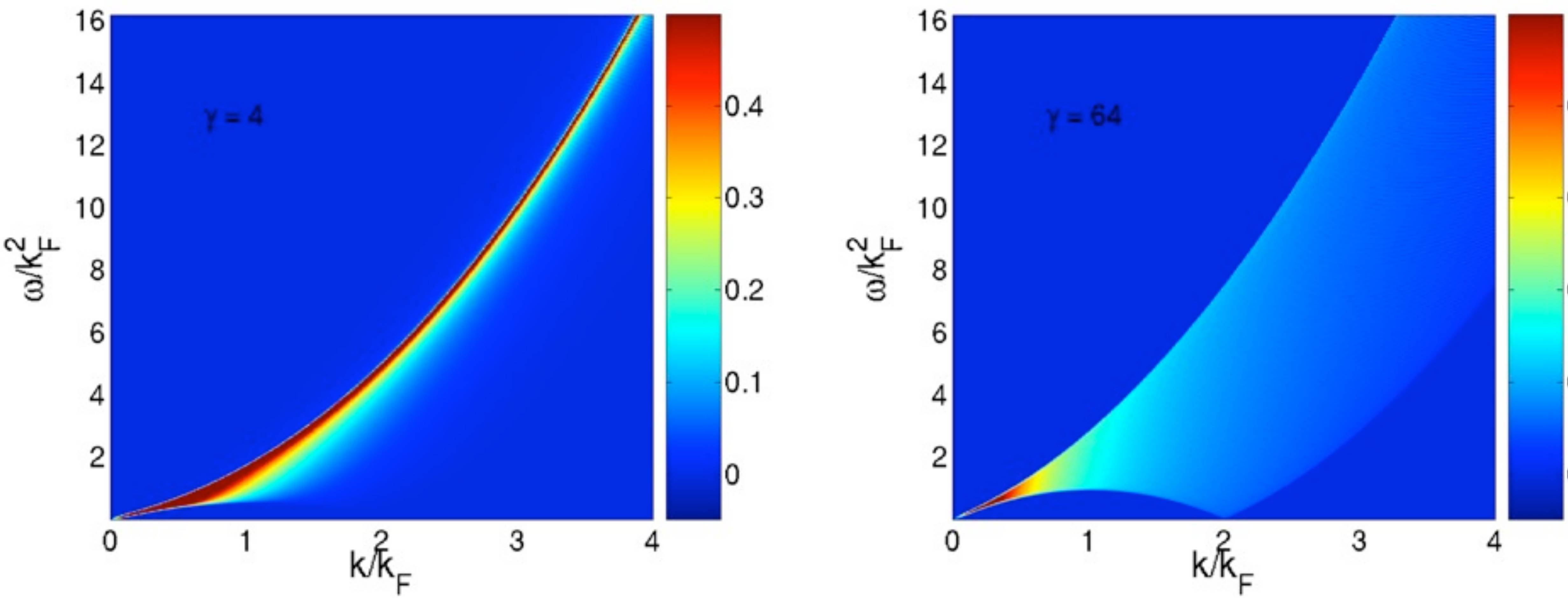
Lieb-Liniger Bose gas

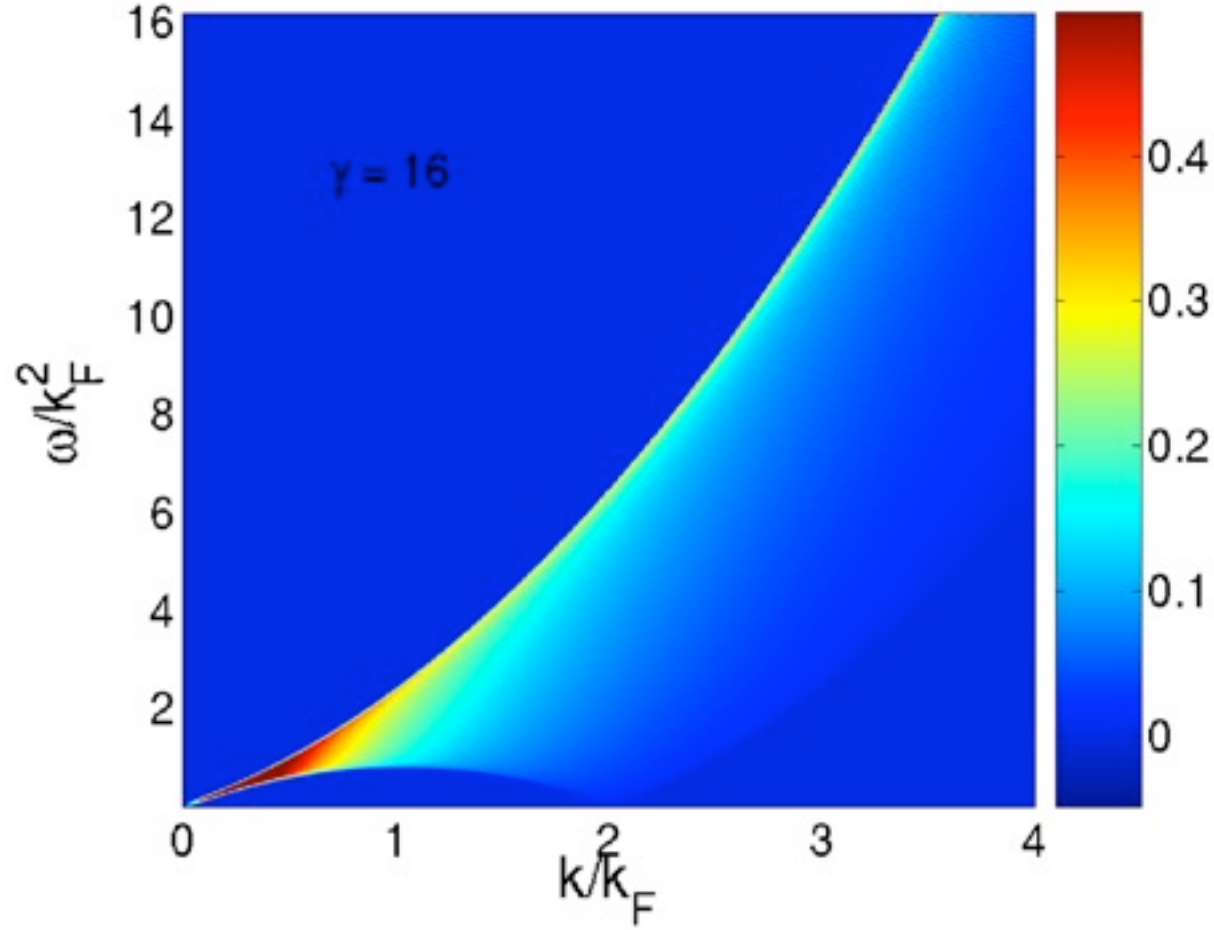
Density-density (dynamical SF)

(J-S C & P Calabrese, PRA 2006)

$$S(k,\omega) = \frac{2\pi}{L}\sum_{\alpha} |\langle 0|\rho_k|\alpha\rangle|^2 \delta(\omega - E_\alpha + E_0)$$



Correspondence with excitations



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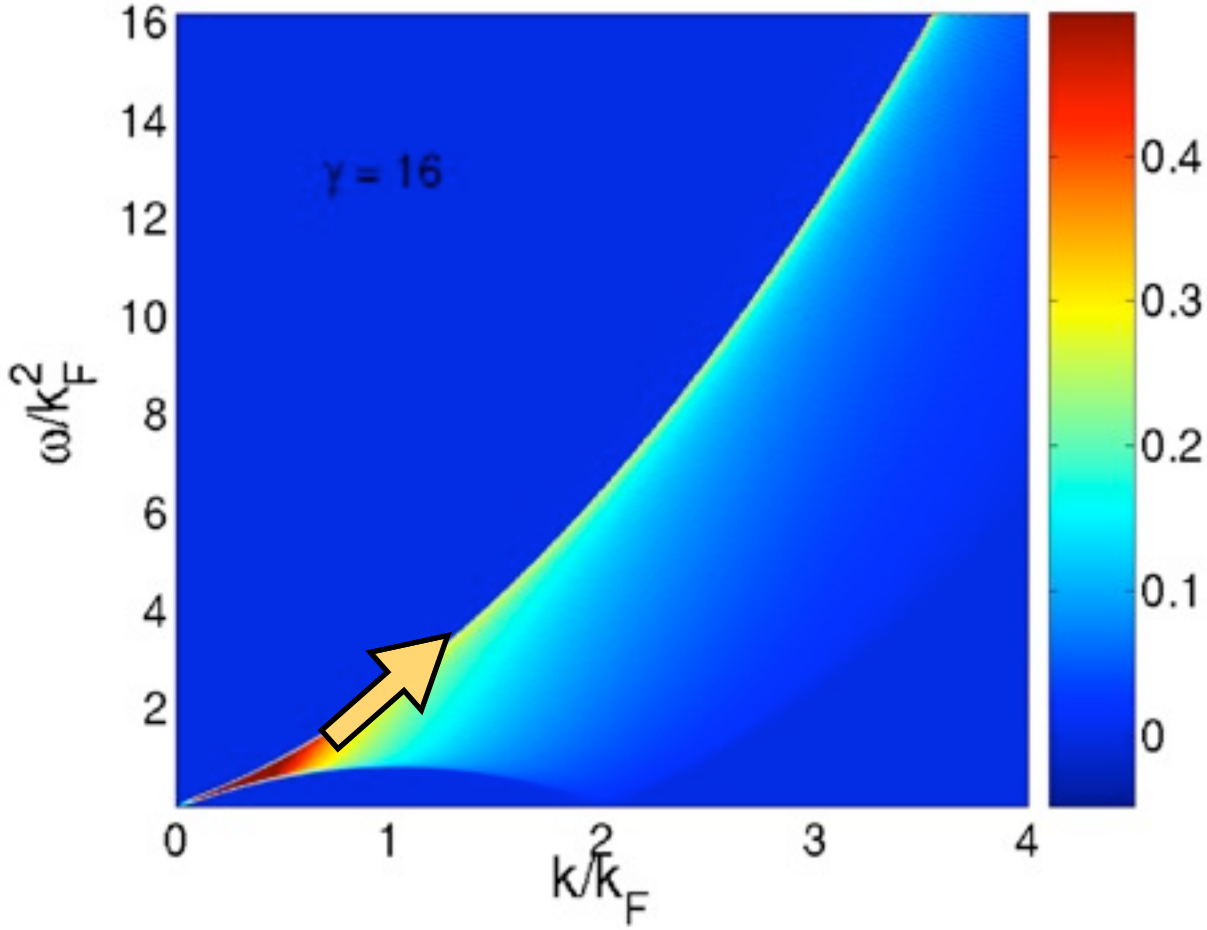
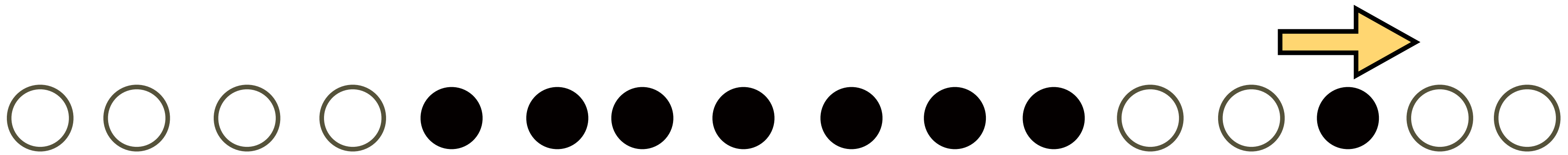


Particle-like



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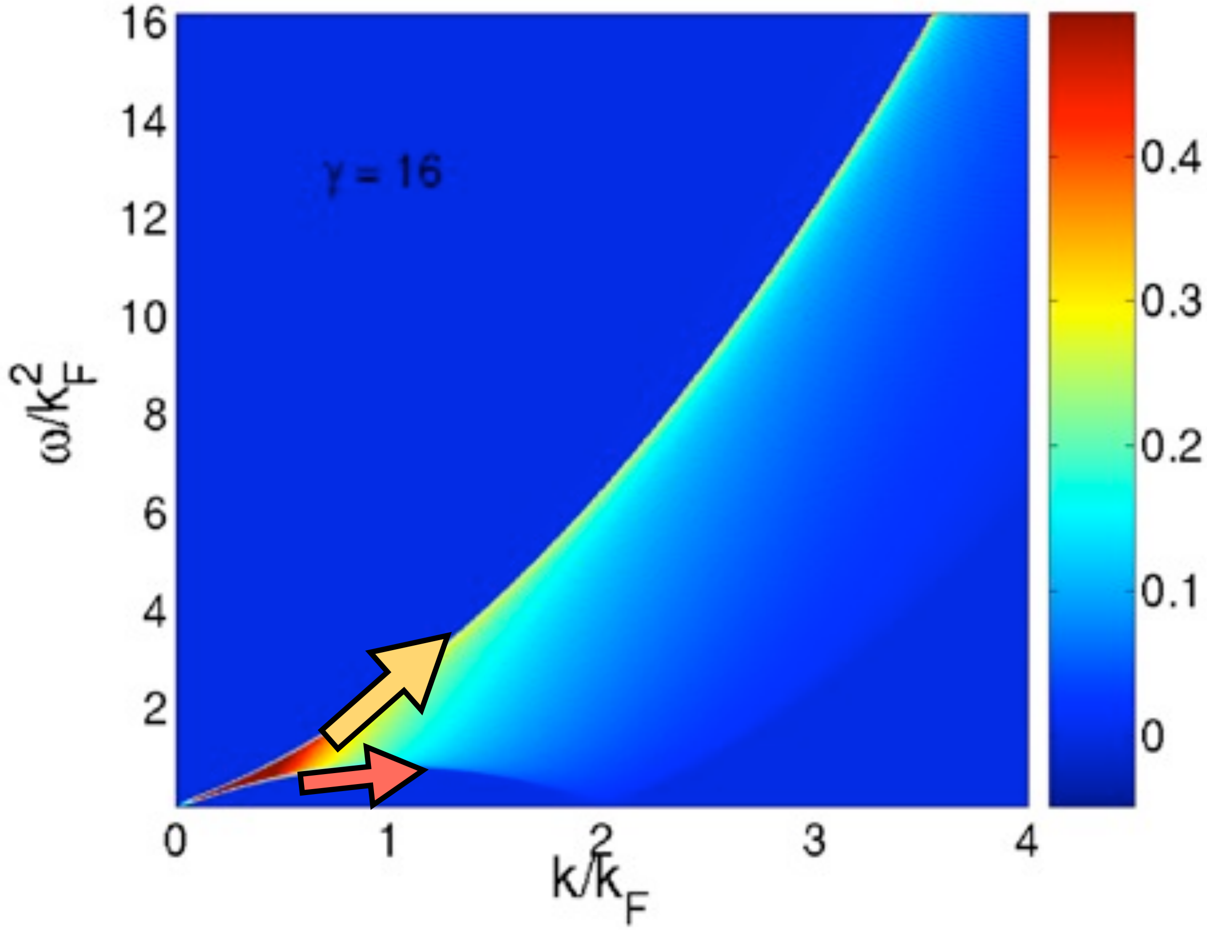

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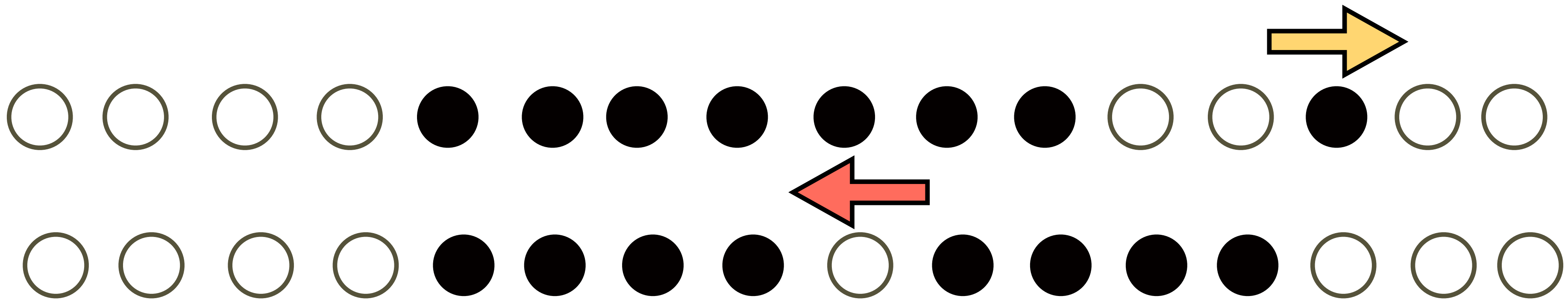

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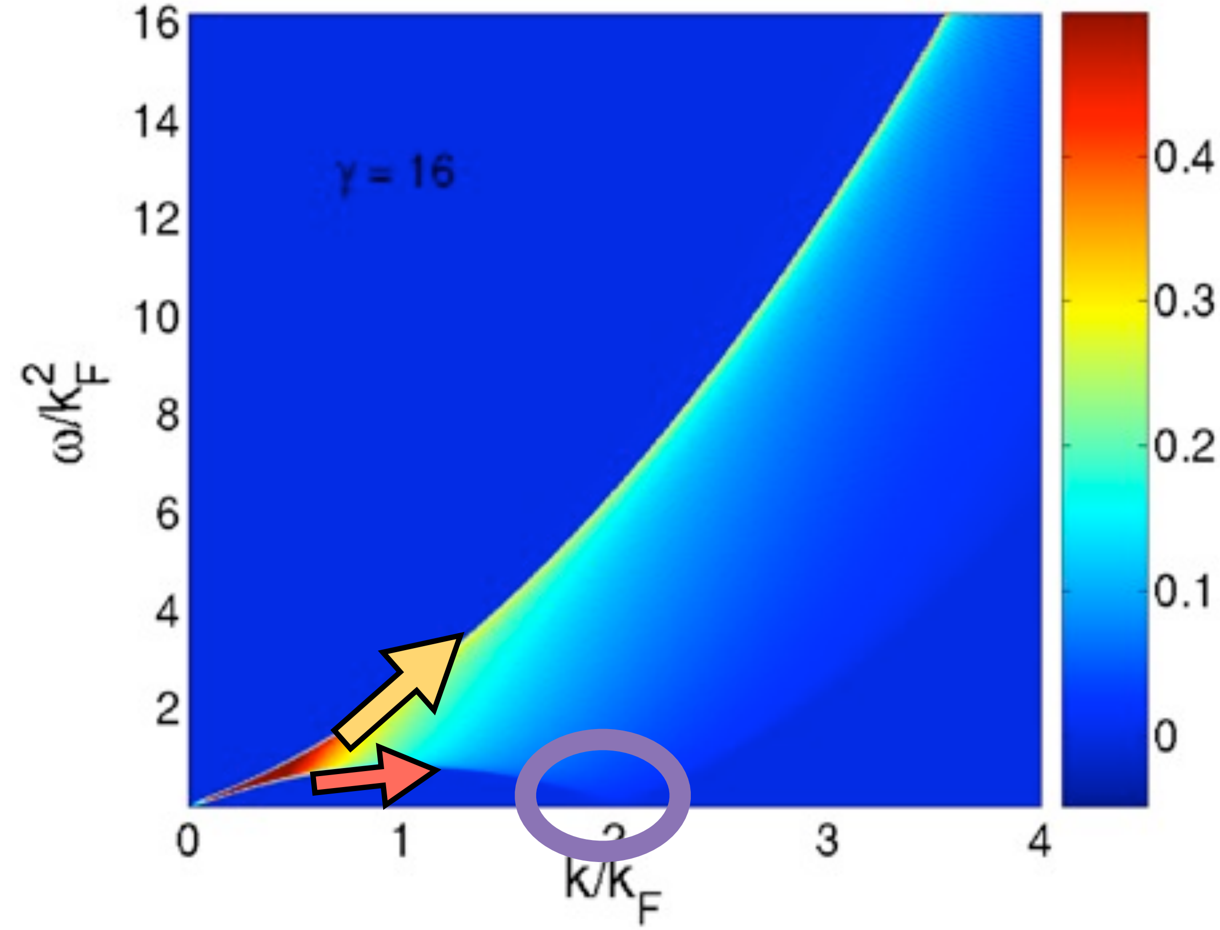

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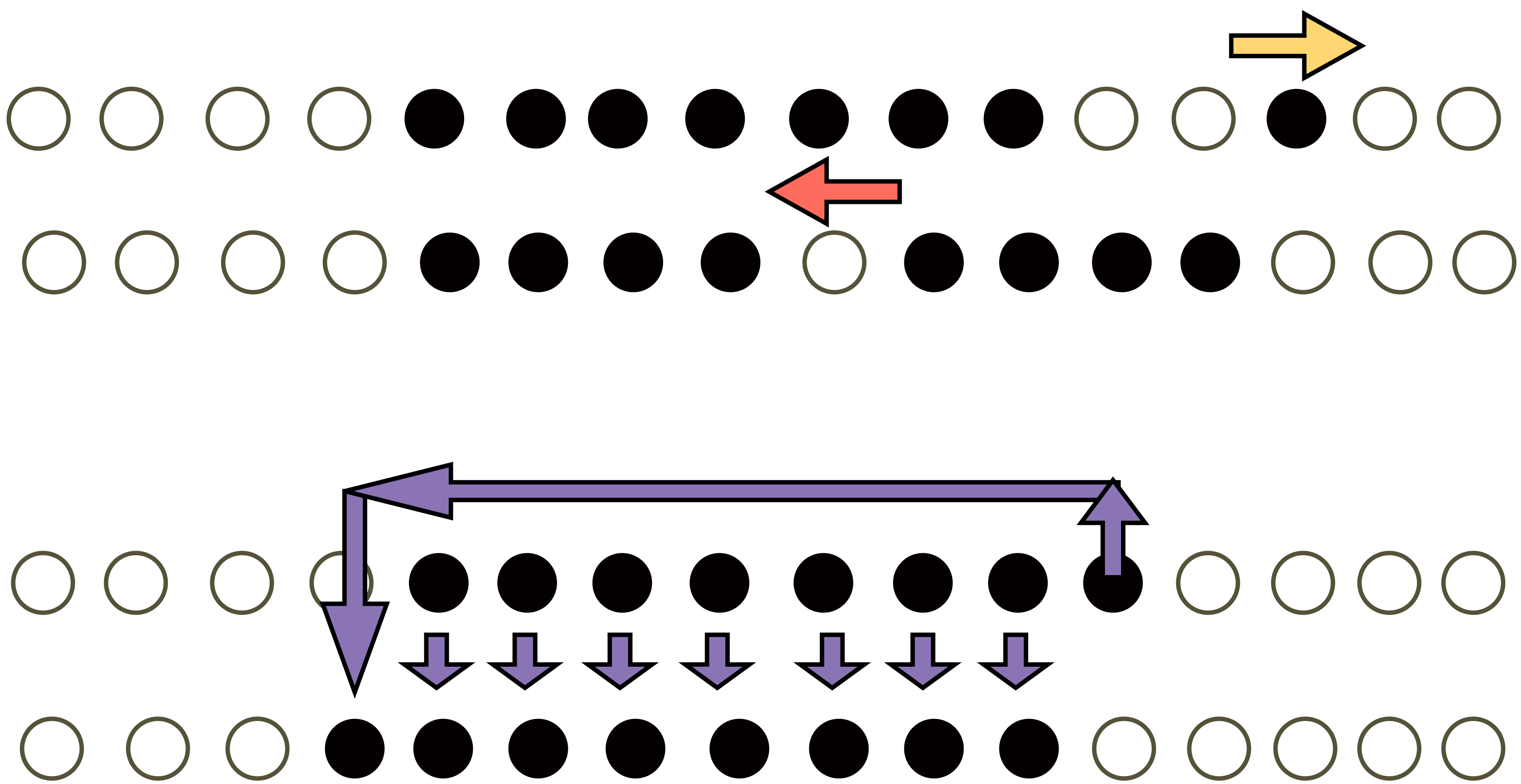
Umklapp



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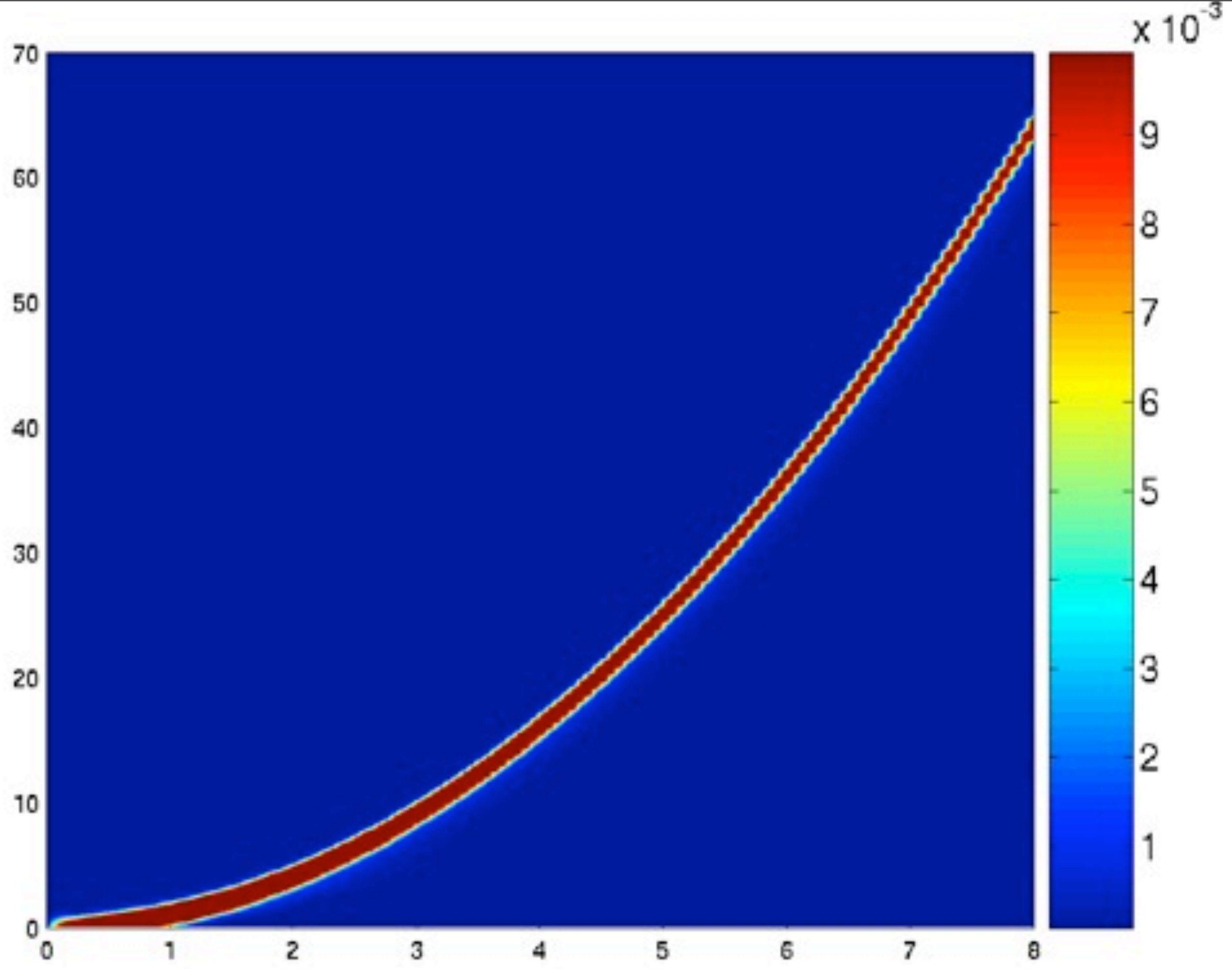
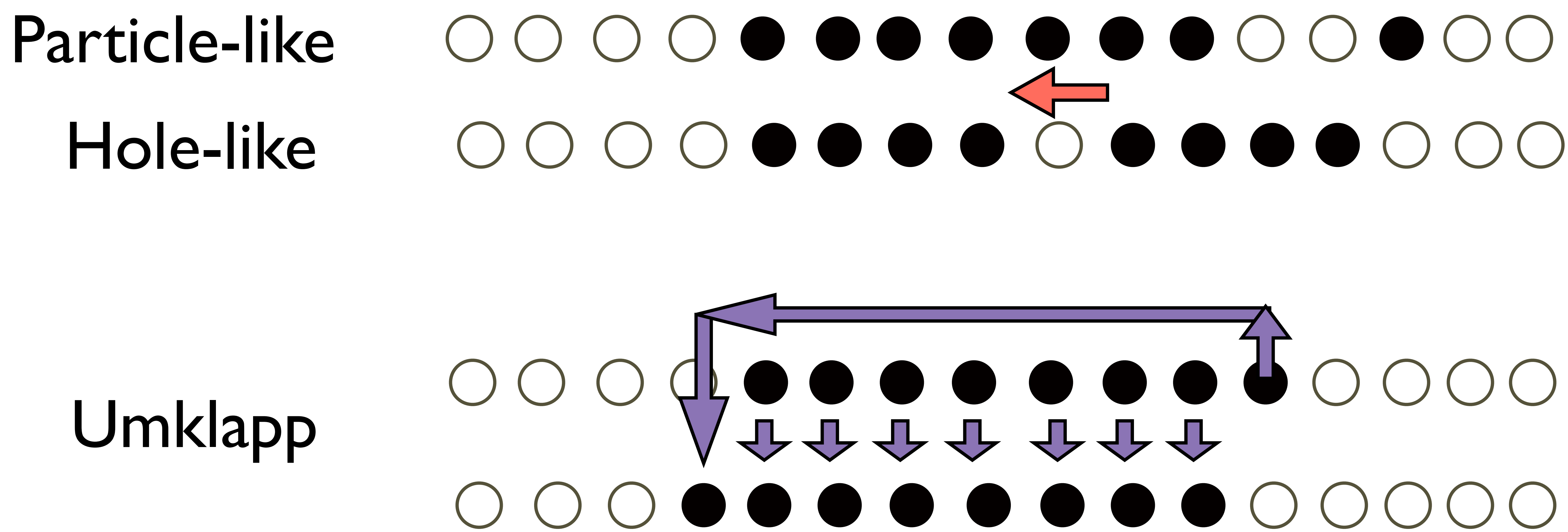


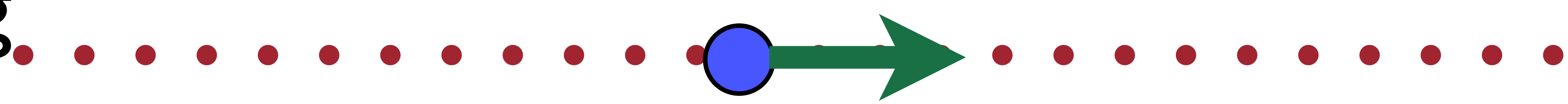


Drag force on impurity in 1d BG: superfluidity revisited

(A. Yu. Cherny J.-S.C & J. Brand, PRA 2009)

Drag force on impurity in 1d BG: superfluidity revisited

`Impurity' moving through gas



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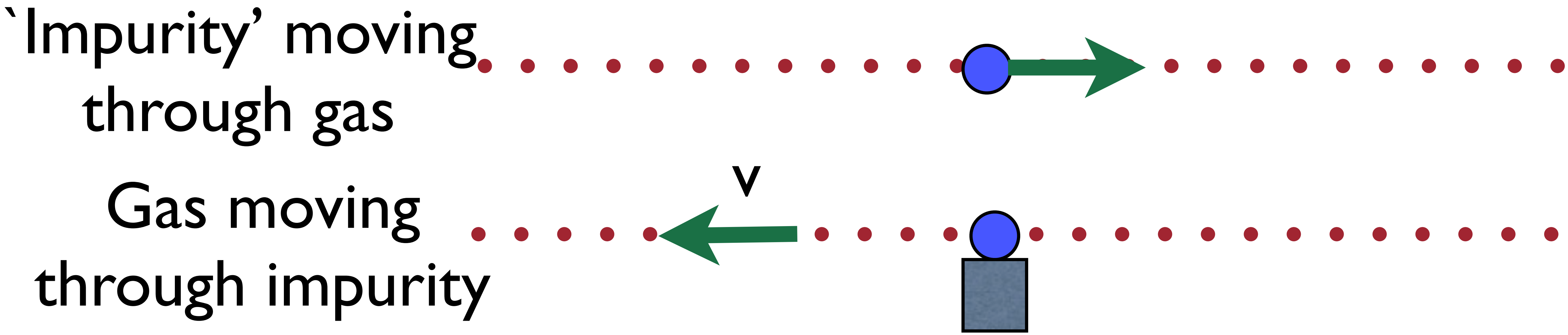

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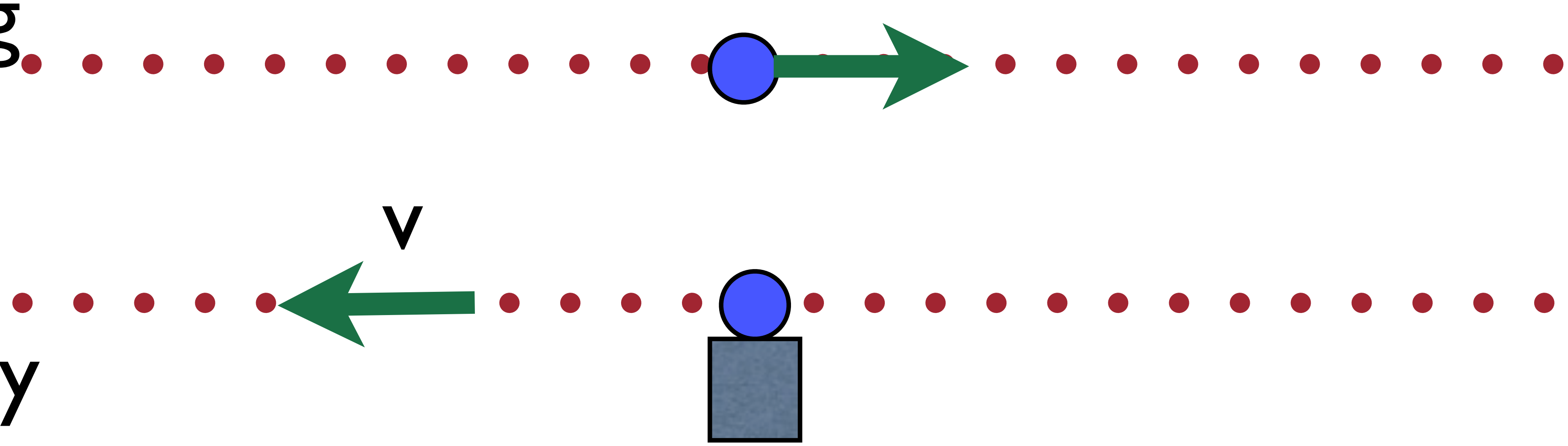
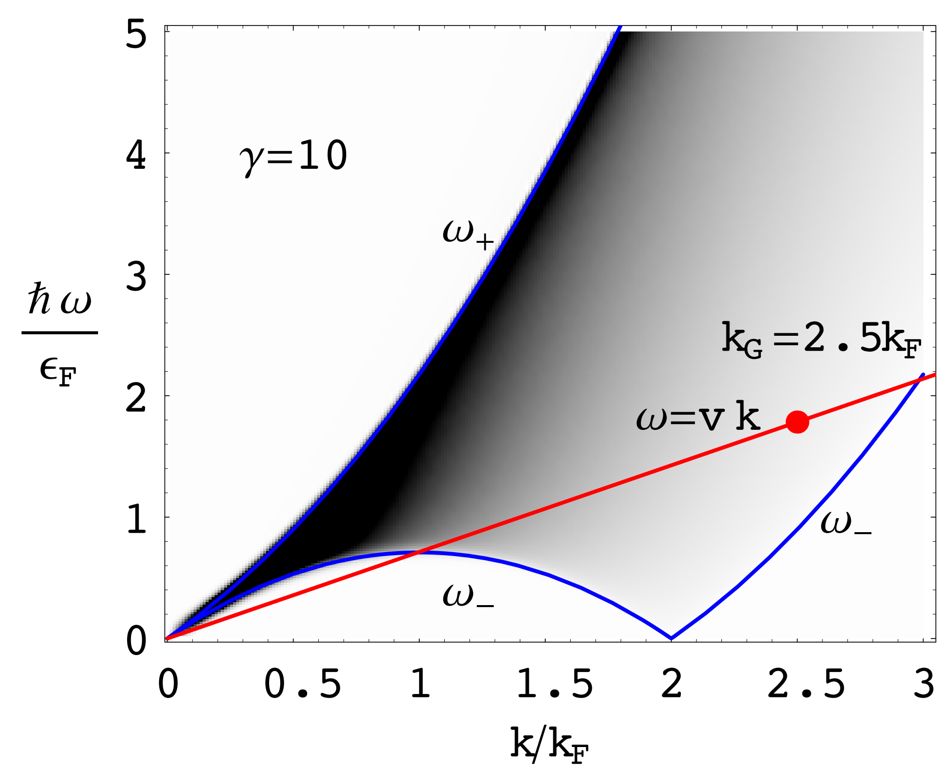
Gas moving through impurity





Drag force is given in linear response theory by integral over structure factor:

$$F_{\mathrm{v}}(v) = \int_0^{+\infty} dkk|\tilde{V}_{\mathrm{i}}(k)|^2 S(k, kv)/L$$

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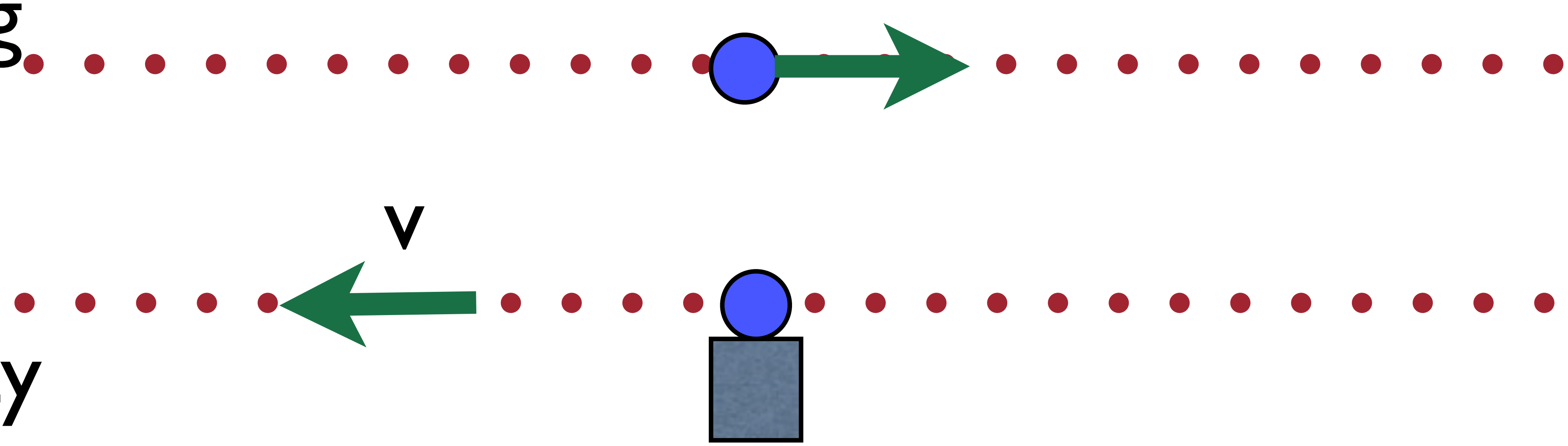

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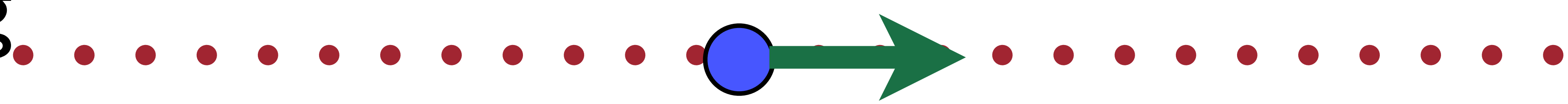

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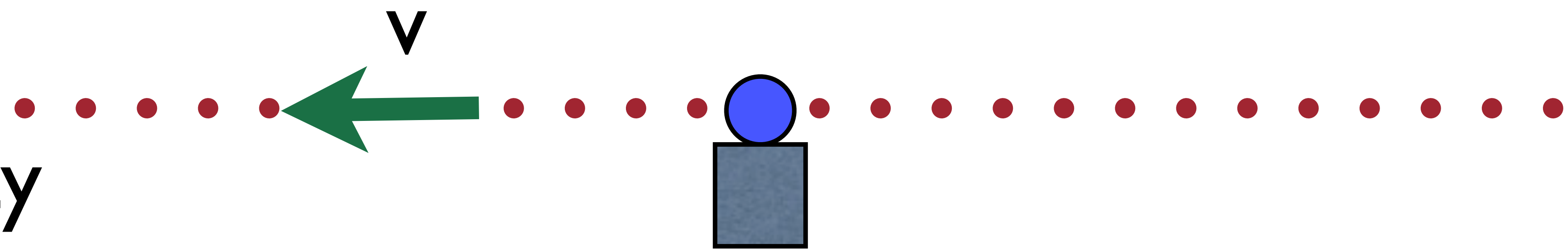

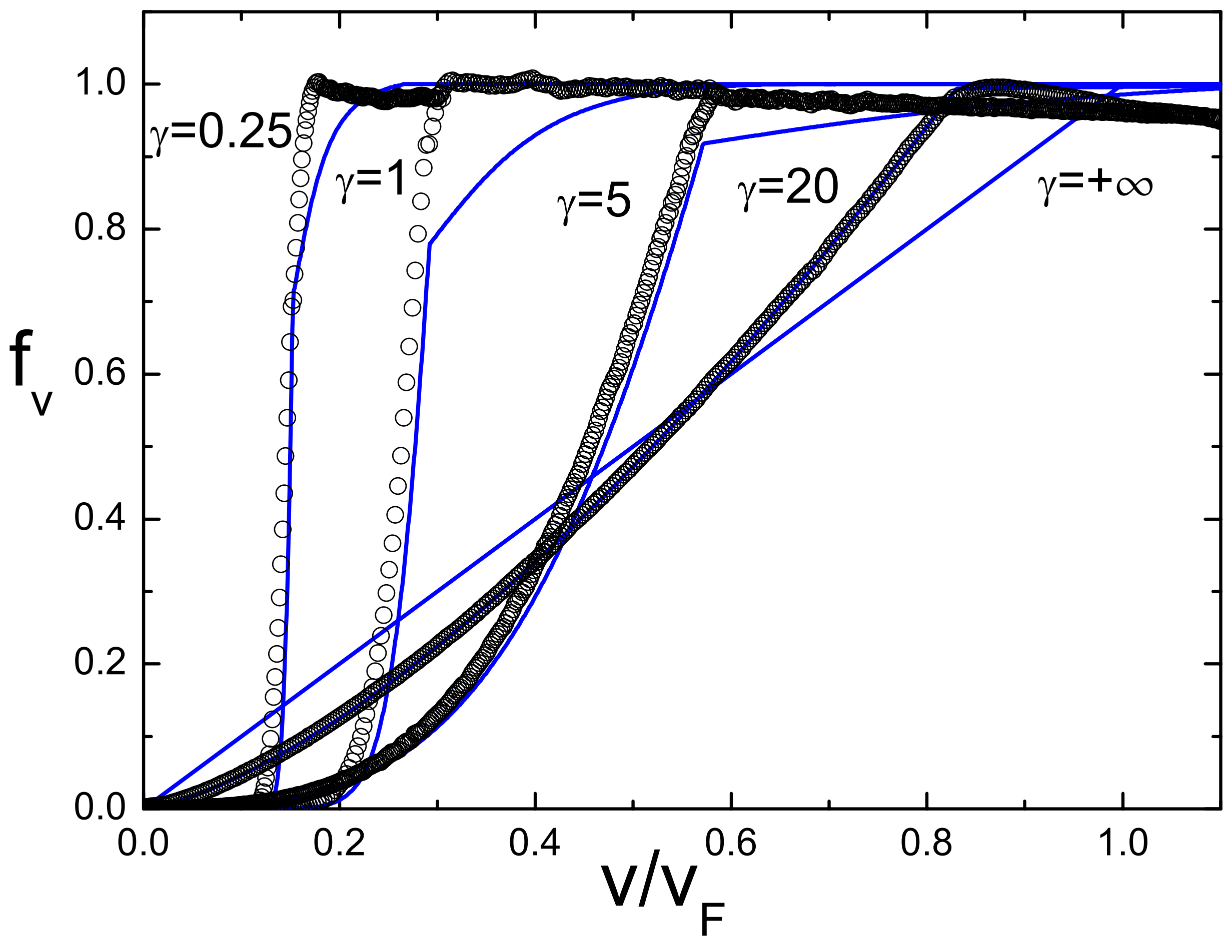

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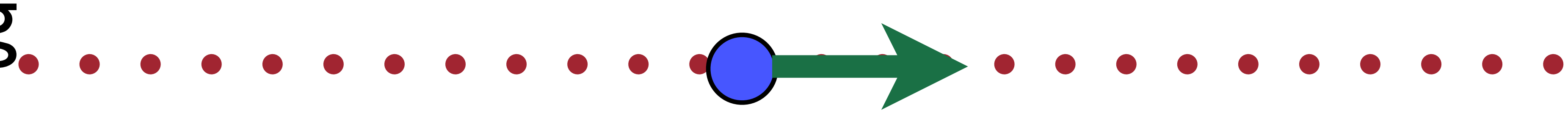

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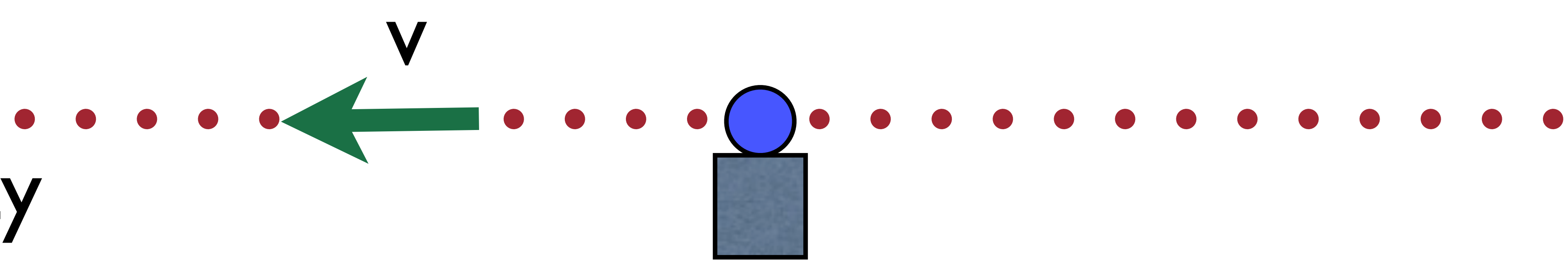

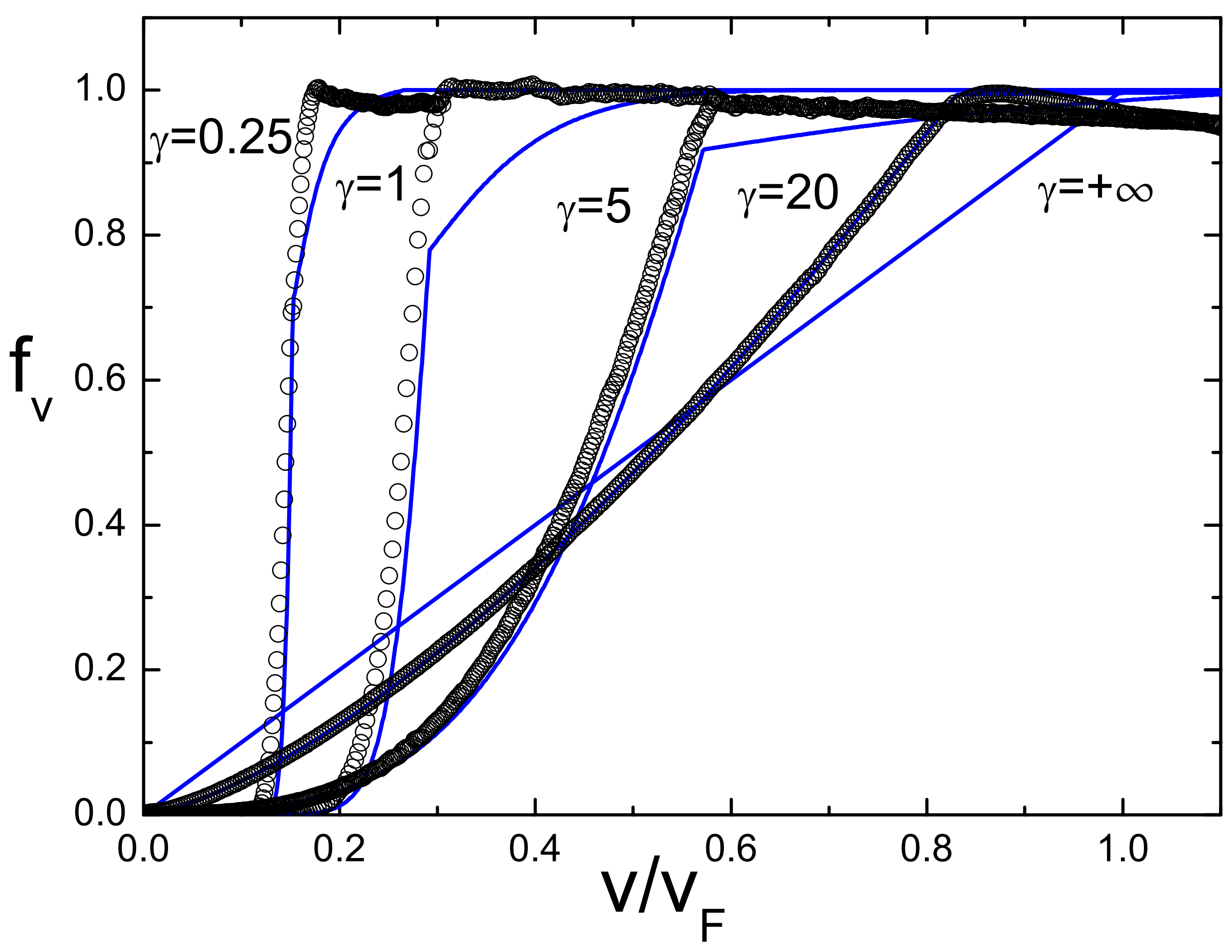

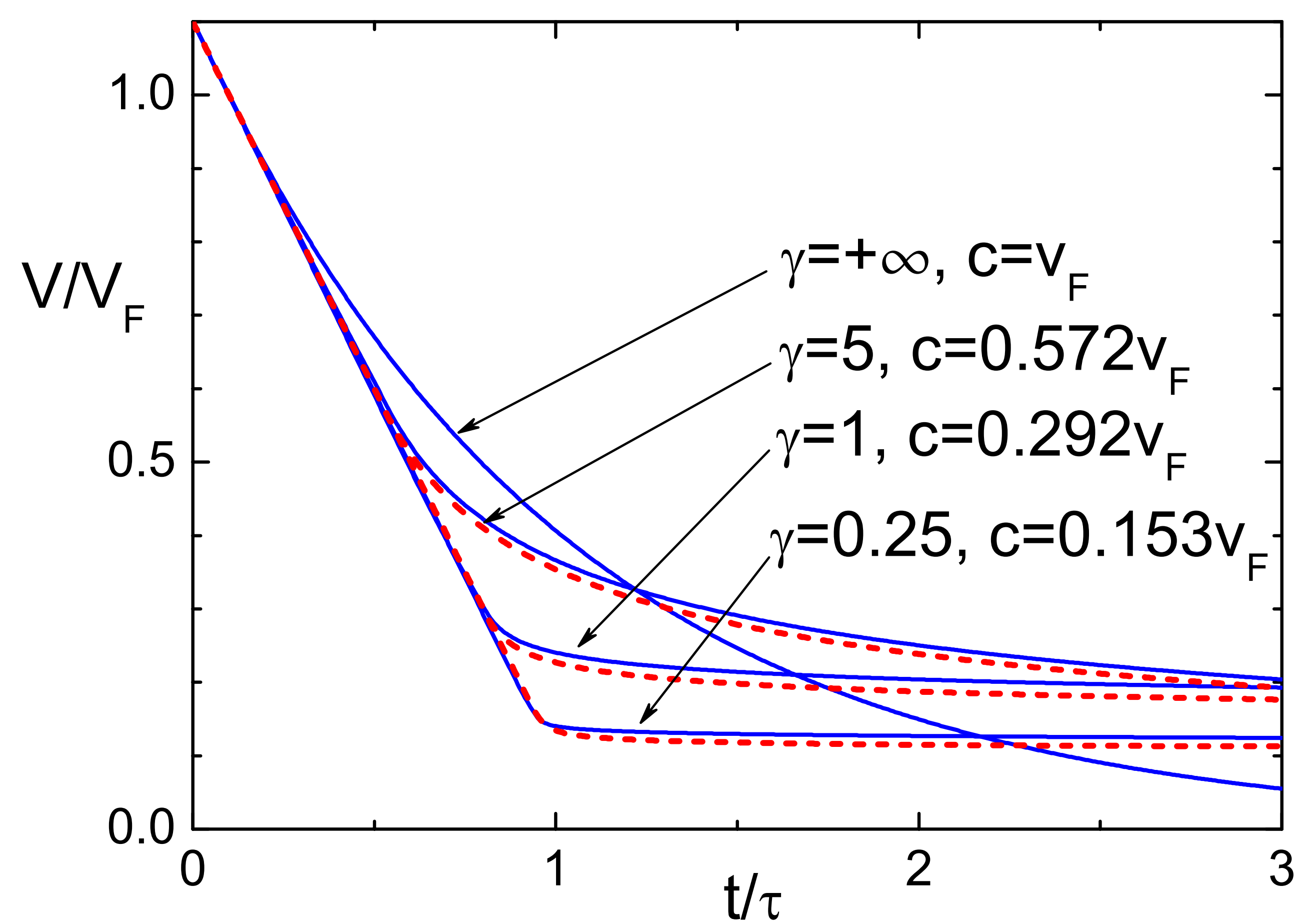

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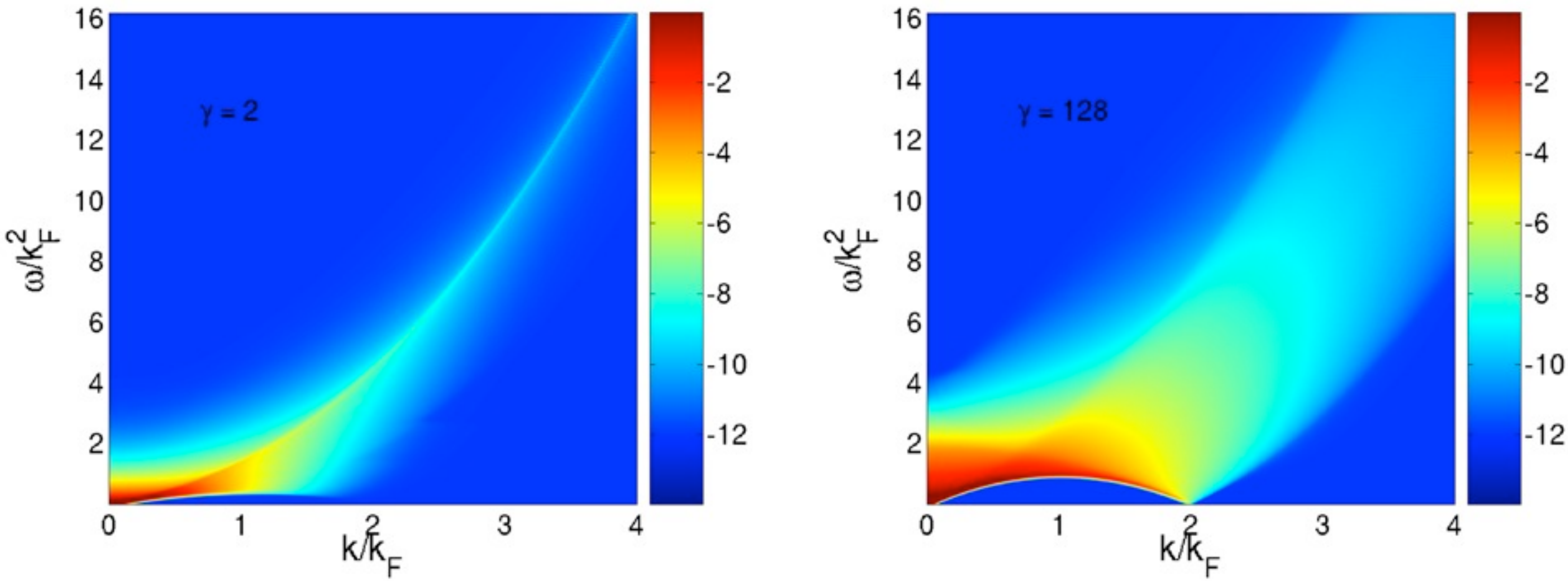
One-particle dynamical function

$$G_2(x,t) = \langle \Psi^\dagger(x,t)\Psi(0,0)\rangle_N$$

(J-S C, P Calabrese & N Slavnov, JSTAT 2007)



The attractive Lieb-Liniger model: analytical solution

$$H = -\frac{\hbar^2}{2m}\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2} - 2\bar{c}\sum_{\langle i,j\rangle}\delta(x_i - x_j)$$

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bound state solutions: strings

$$\lambda_\alpha^{j,a} = \lambda_\alpha^j + \frac{i\bar{c}}{2}(j + 1 - 2a) + i\delta_\alpha^{j,a}\,.$$

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bound state solutions: strings

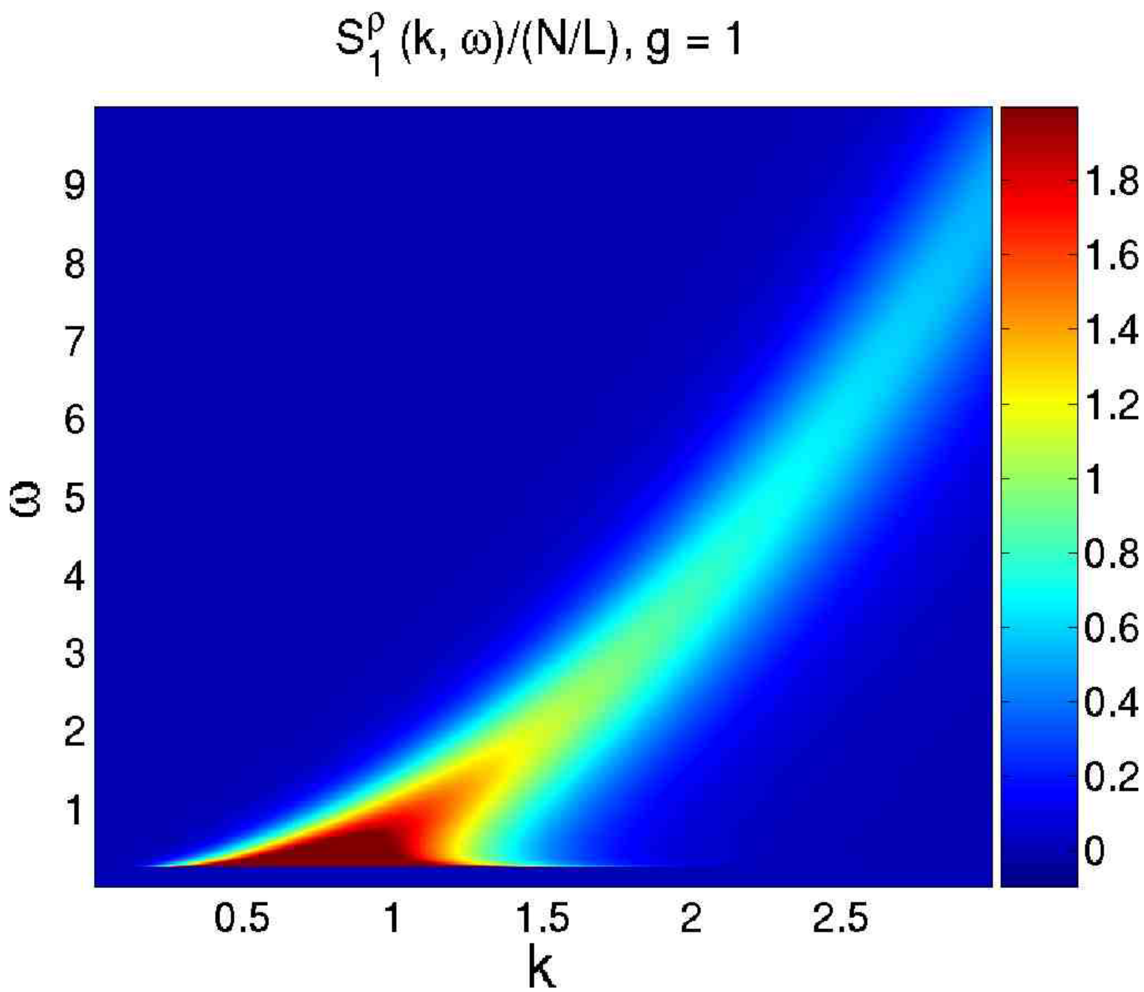
$$\lambda_\alpha^{j,a} = \lambda_\alpha^j + \frac{i\bar{c}}{2}(j + 1 - 2a) + i\delta_\alpha^{j,a}\,.$$

(J. B. McGuire, 1964; F. Calogero & A. DeGasperis, 1975; Y. Castin & C. Herzog, 2001)

Attractive Lieb-Liniger: analytical solution for CFs

(J.-S.C & P. Calabrese PRL 2007; JSTAT 2007)

Single-particle coherent part + two-particle continuum



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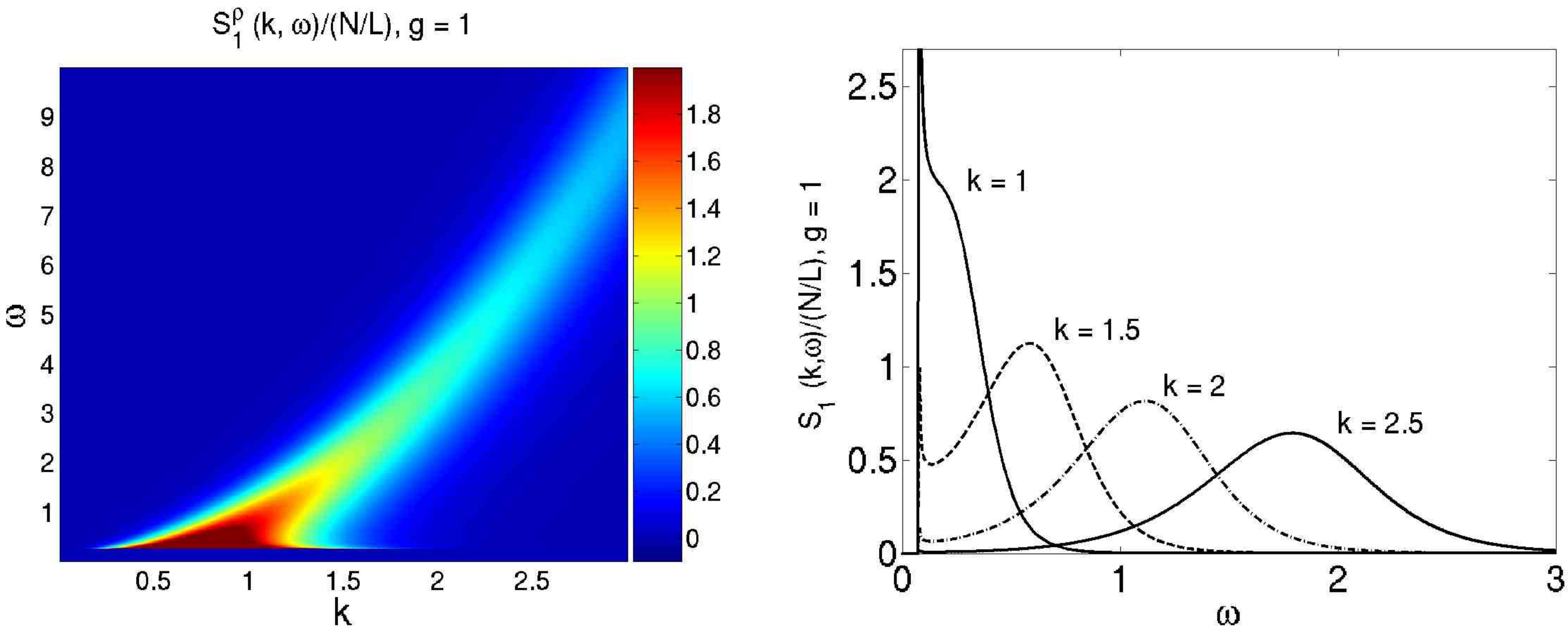

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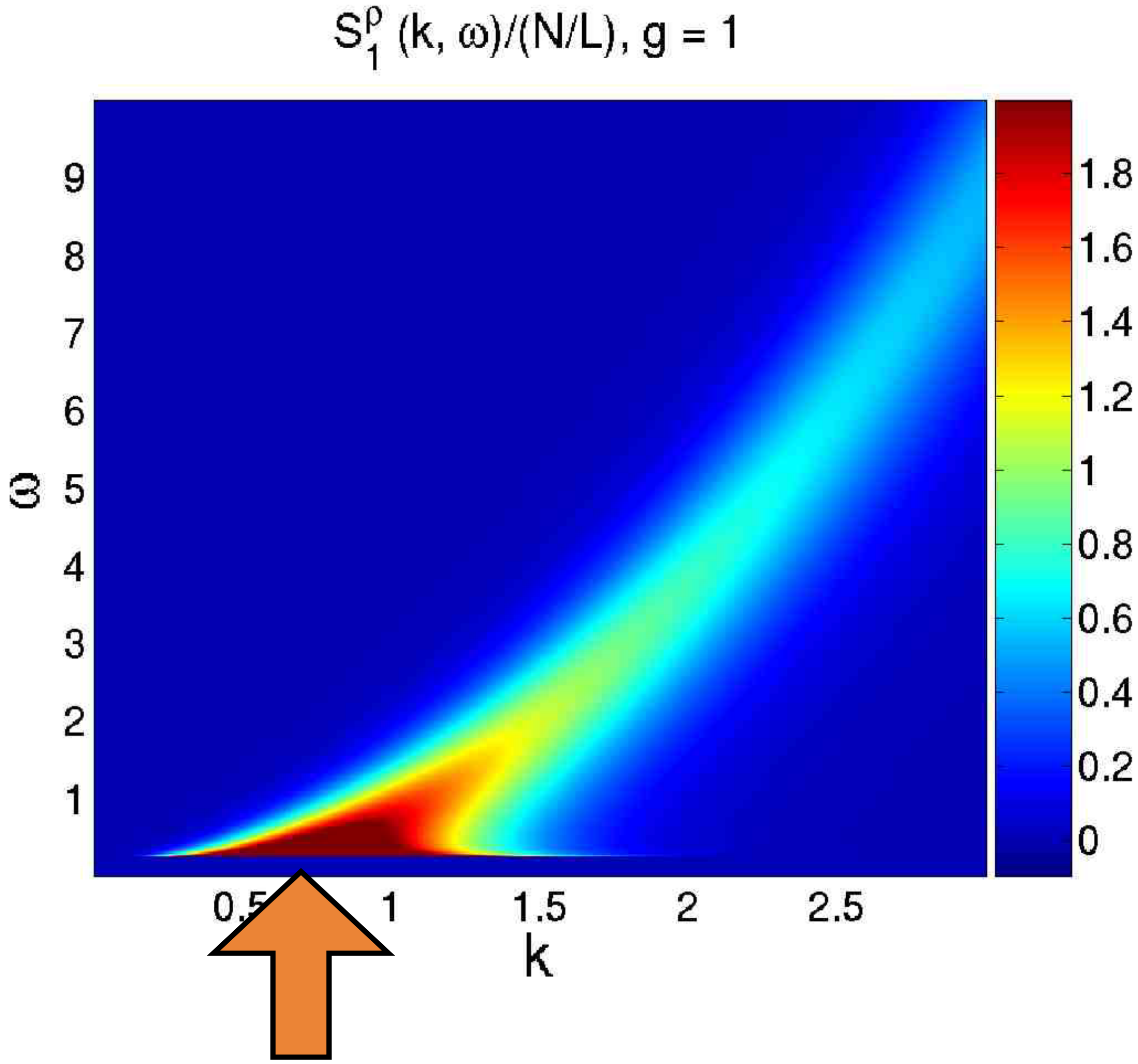

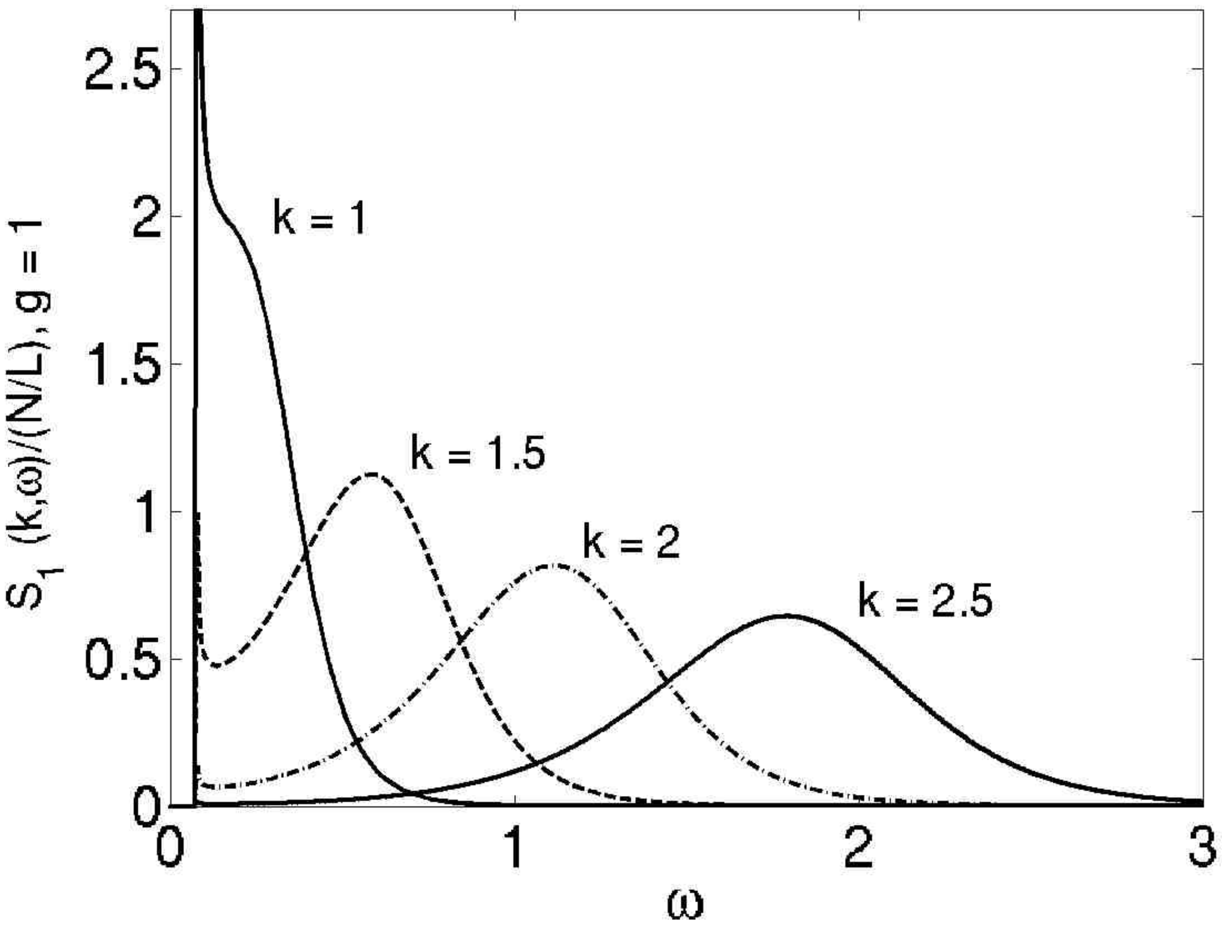

Finite threshold

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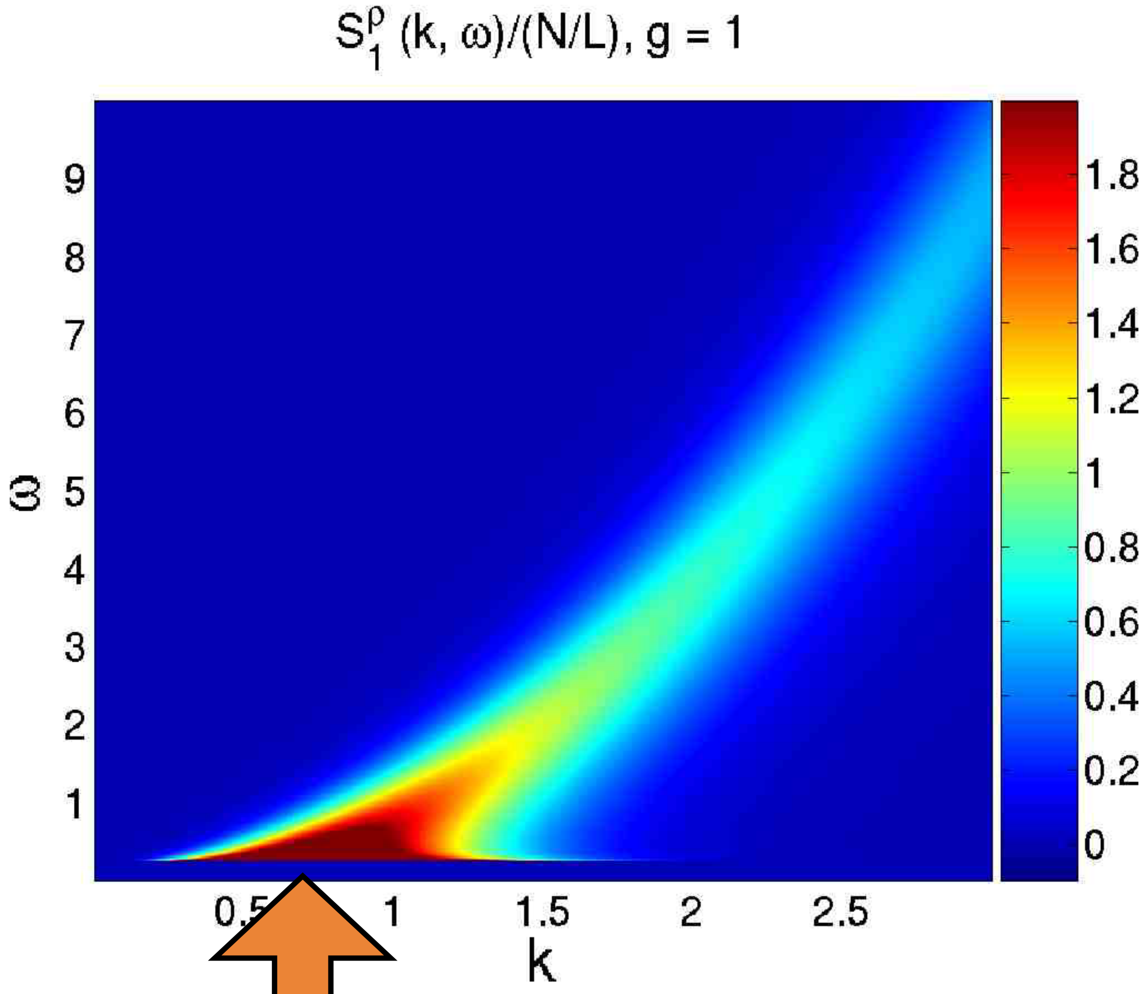

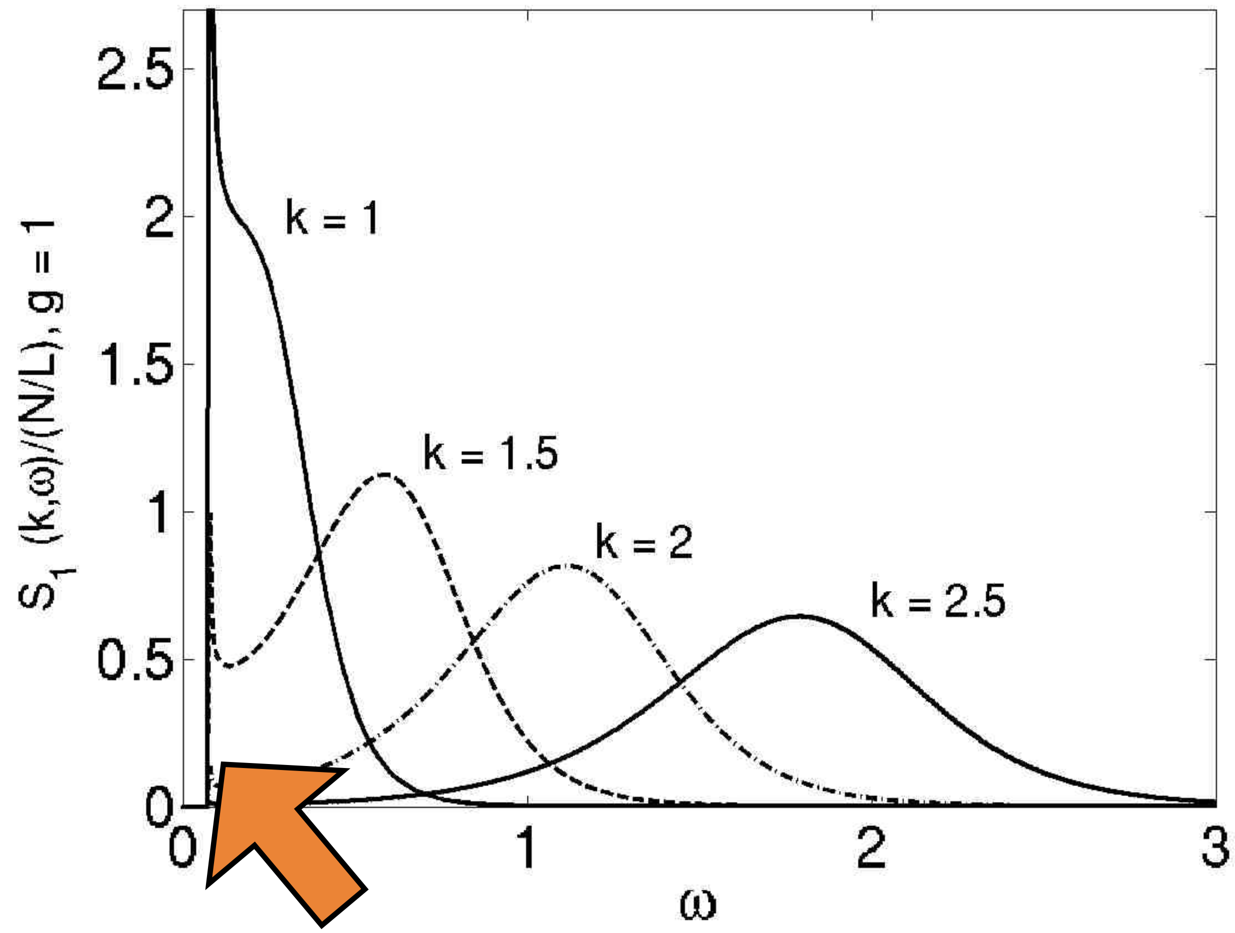

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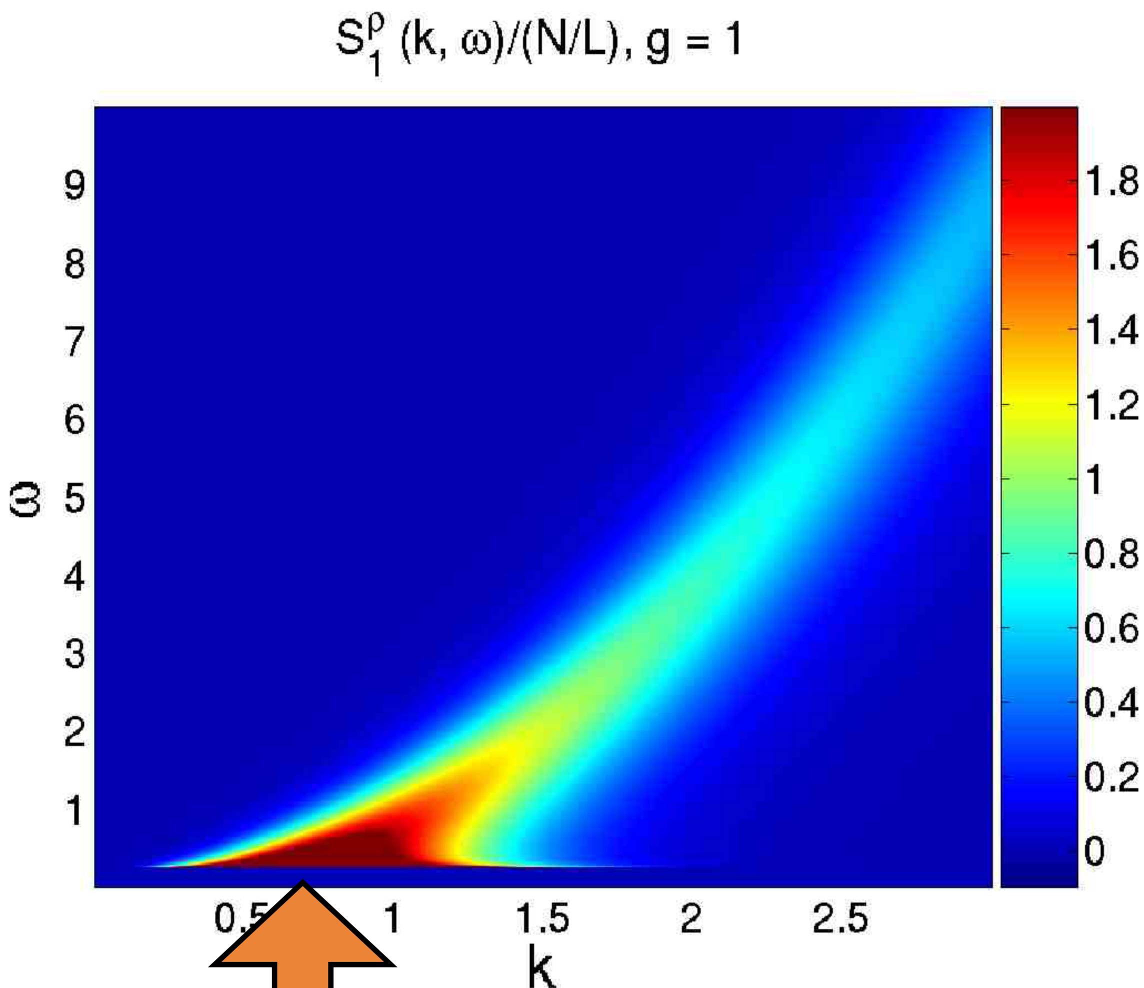

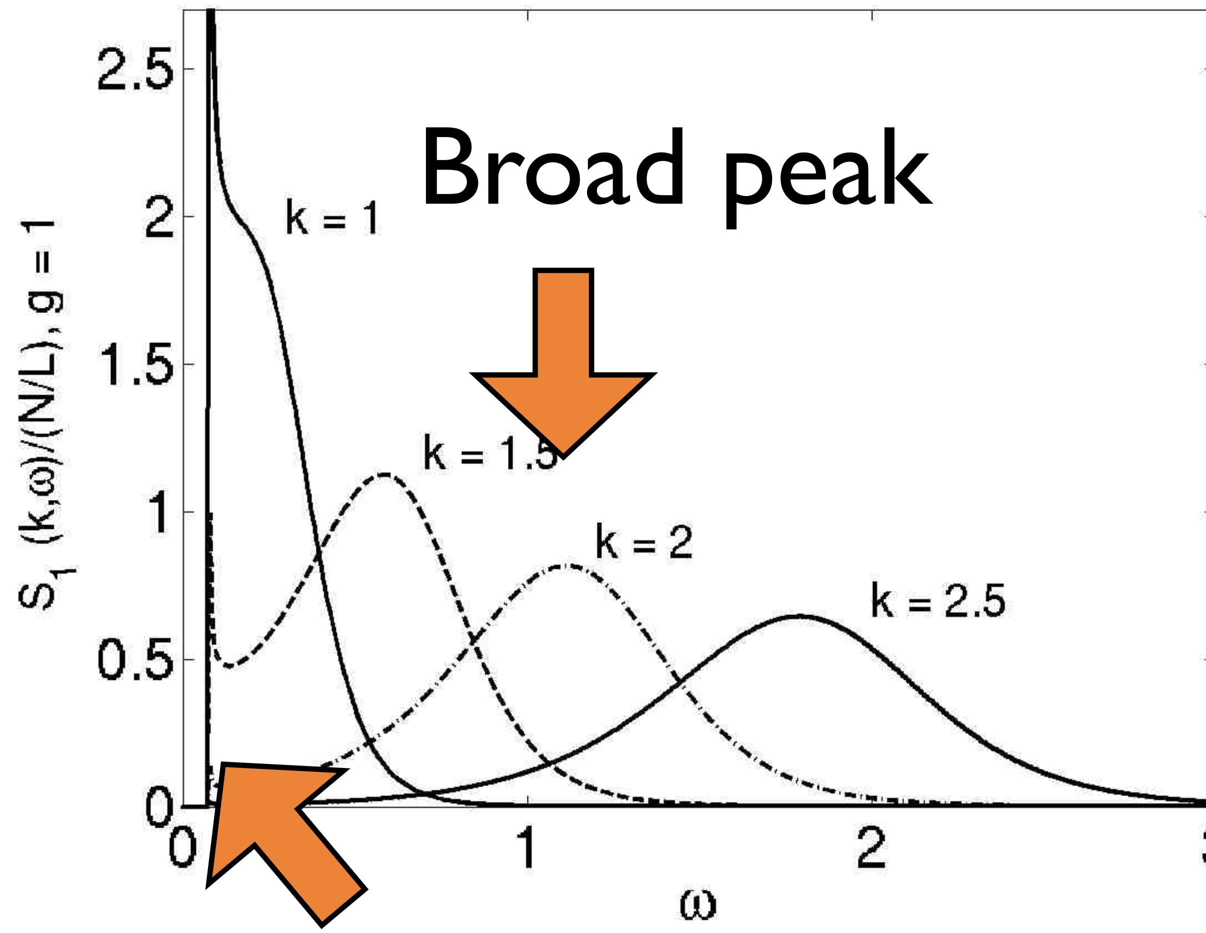

Broad peak

Square-root singularity

Finite threshold

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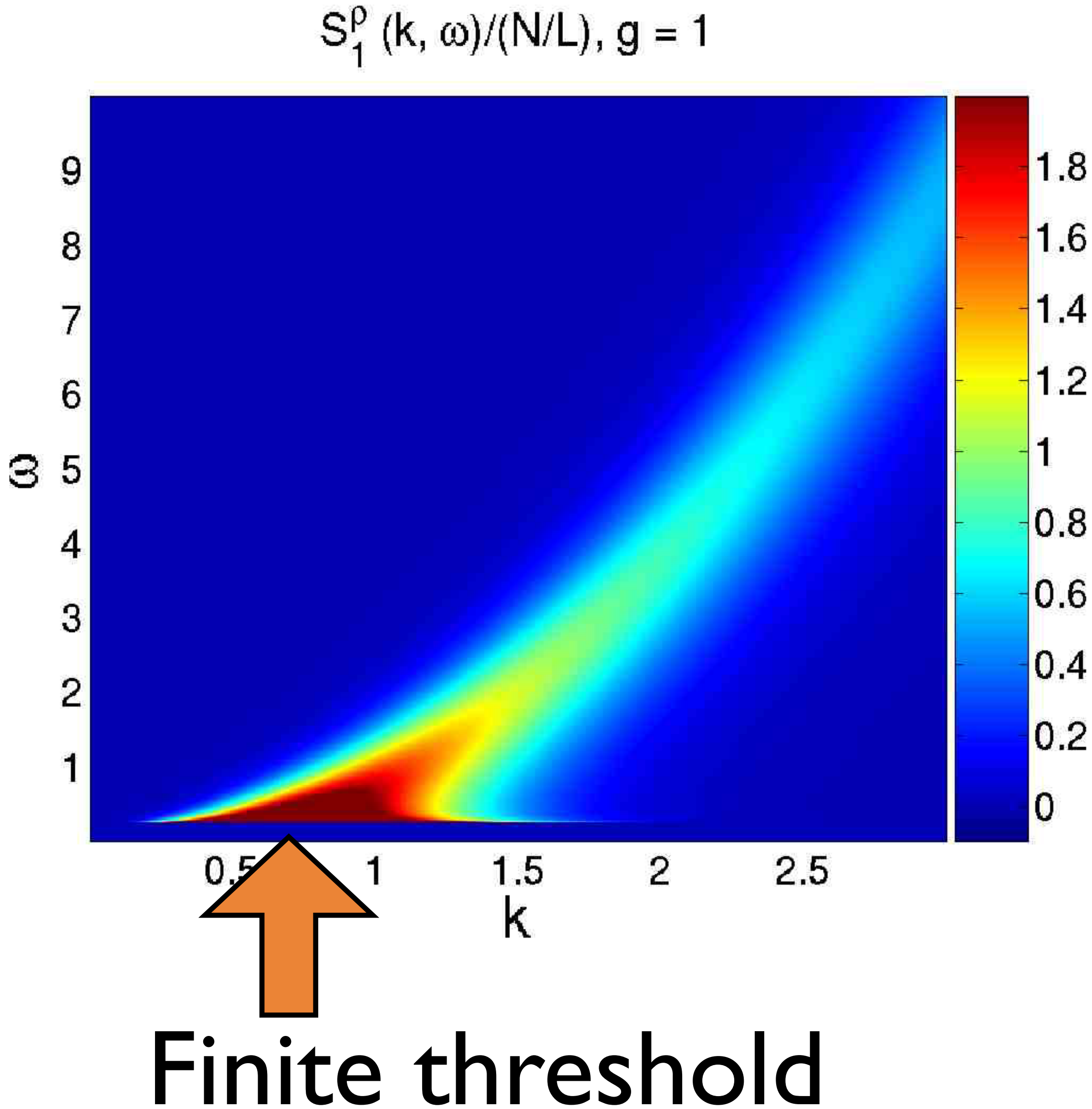
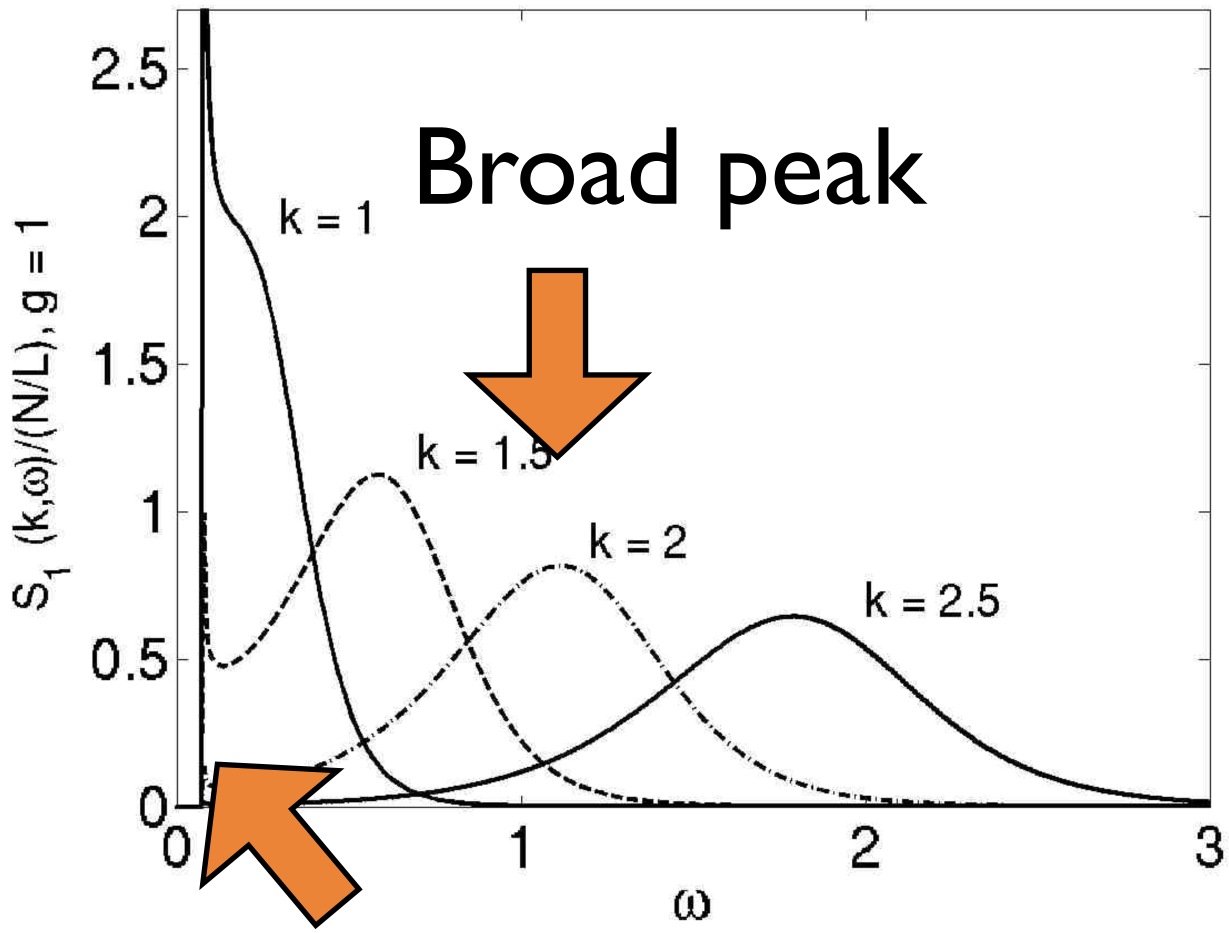
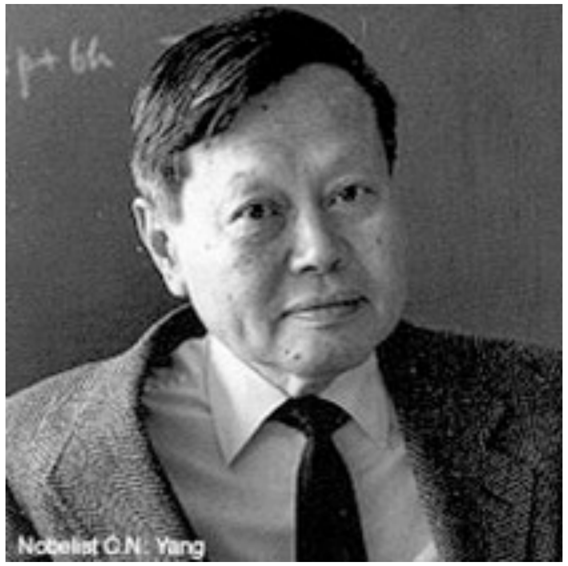




Single-particle part: leads to Mössbauer-like effect
(gas reacts like a single massive particle)

The 2-component Bose gas
(special case of Yang permutation model)



$$H = -\sum_{a=1}^{N_C}\sum_{i=1}^{N_a}\frac{\partial^2}{\partial x_{a,i}^2} + 2c \sum_{(a,i)<(b,j)} \delta(x_{a,i} - x_{b,j})$$

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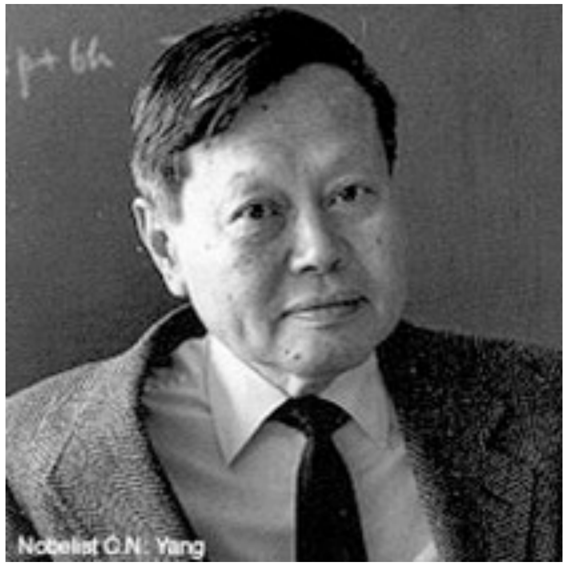

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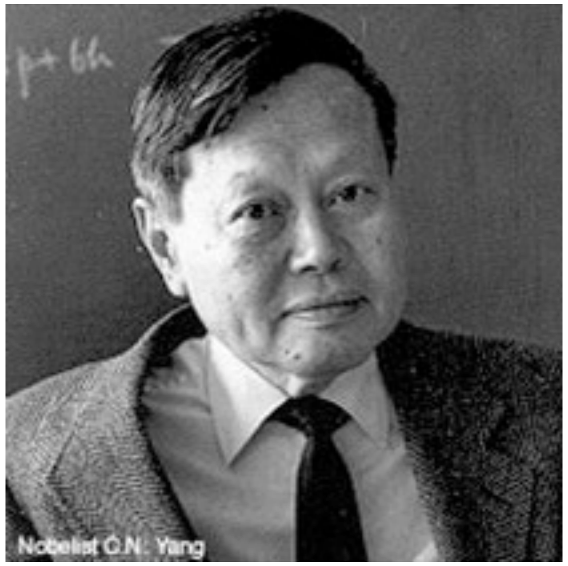

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- Dynamics: hum... nested BA
- Equilibrium thermodynamics: OK !

$$\epsilon(\lambda) = \lambda^2 - \mu - \Omega - a_2 * T \ln(1 + e^{-\epsilon(\lambda)/T}) - \sum_{n=1}^{\infty} a_n * T \ln(1 + e^{-\epsilon_n(\lambda)/T})$$

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$$\lim_{n\to\infty} \frac{\epsilon_n(\lambda)}{n} = 2\Omega$$

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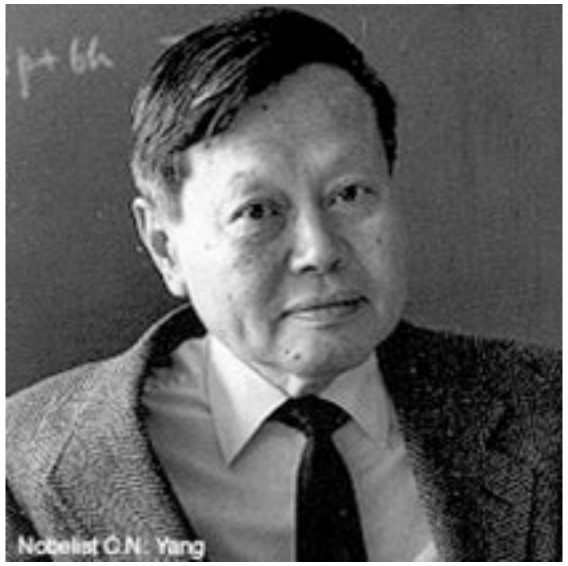

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→ Numerical solution

J.-S. C., A. Klauser & J. van den Brink, PRA 2009

The 2-component Bose gas

Ferromagnetism using interacting bosons

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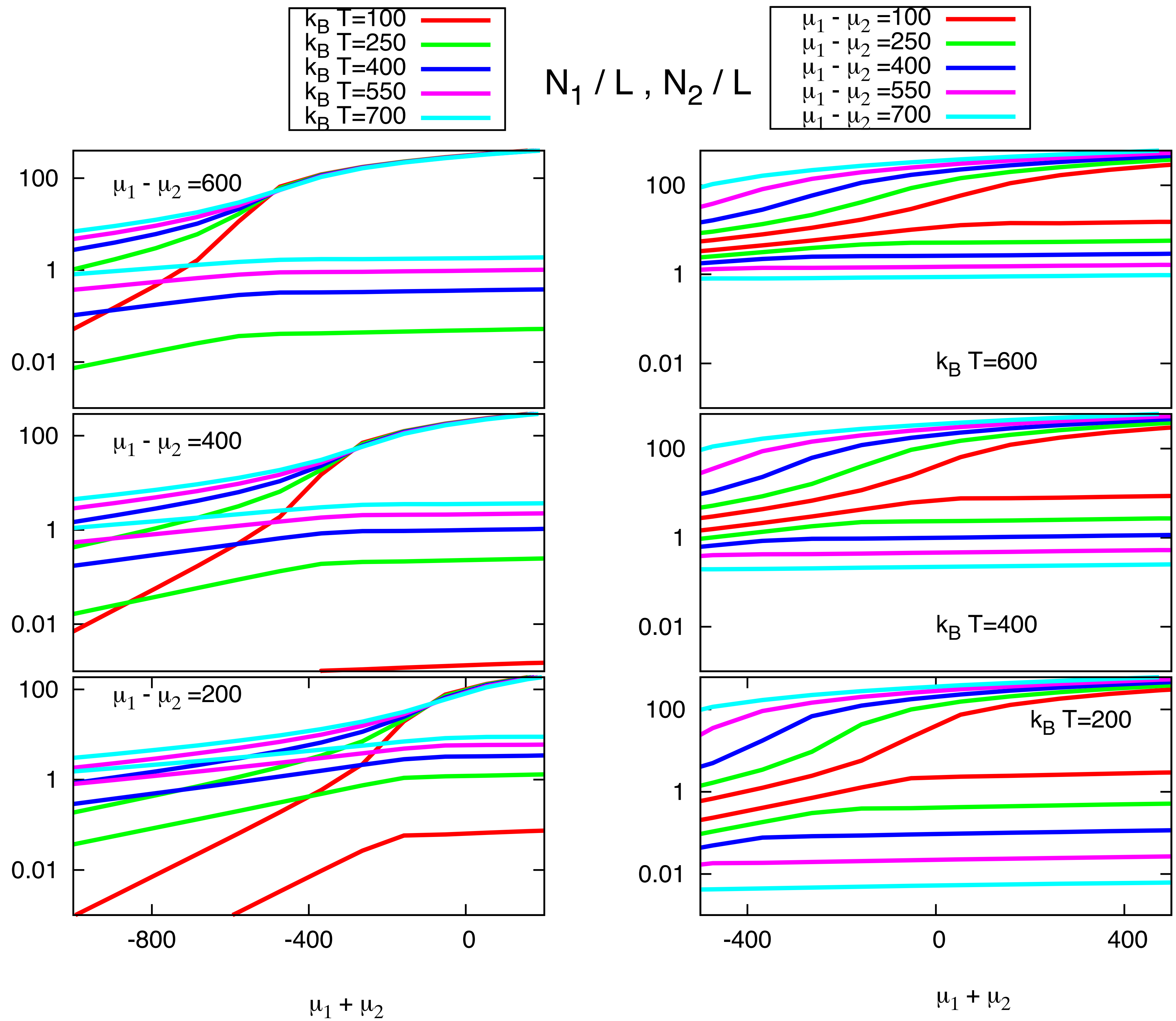
Populations as a function of total chemical potential



The 2-component Bose gas

Ferromagnetism using interacting bosons

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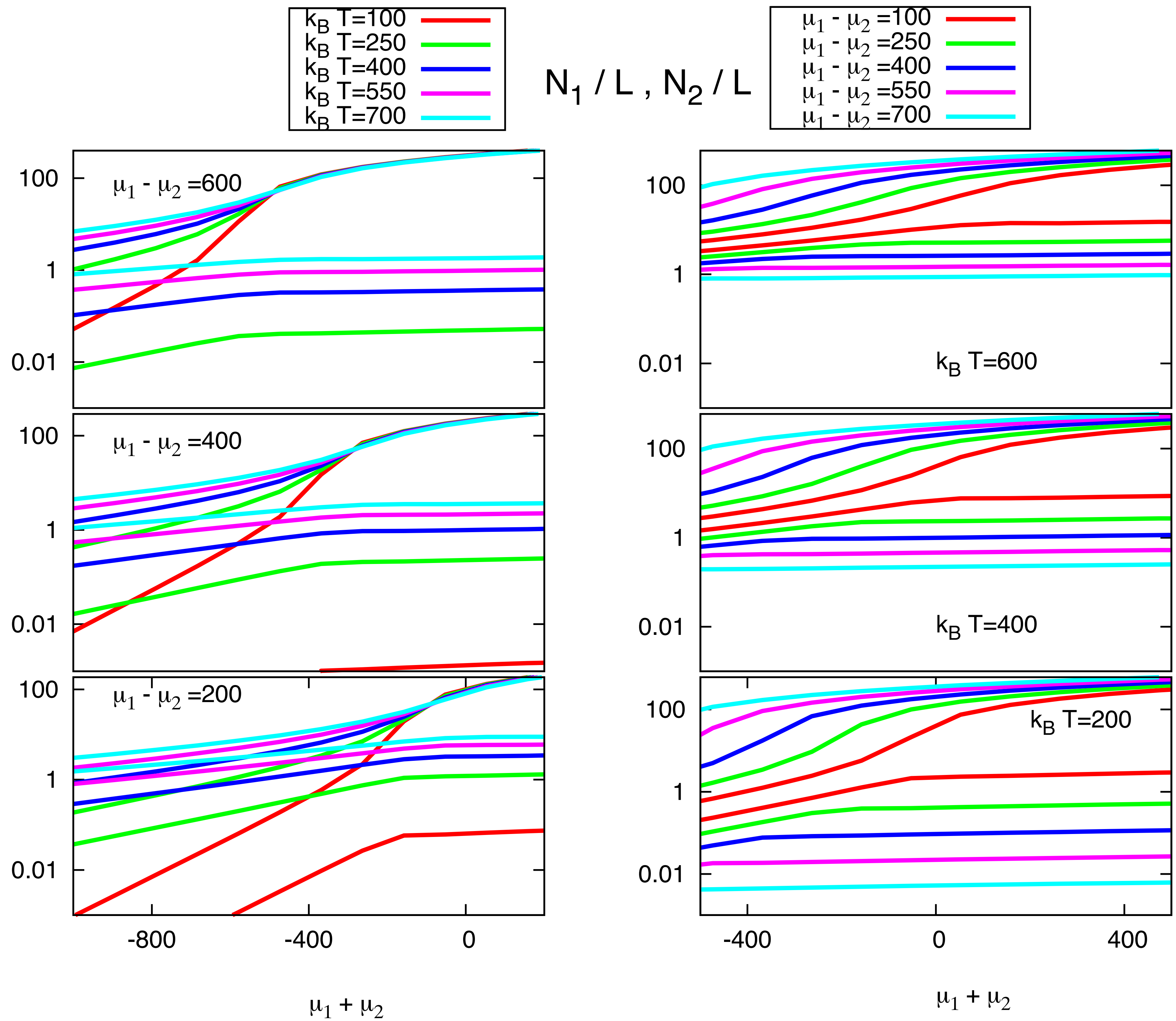
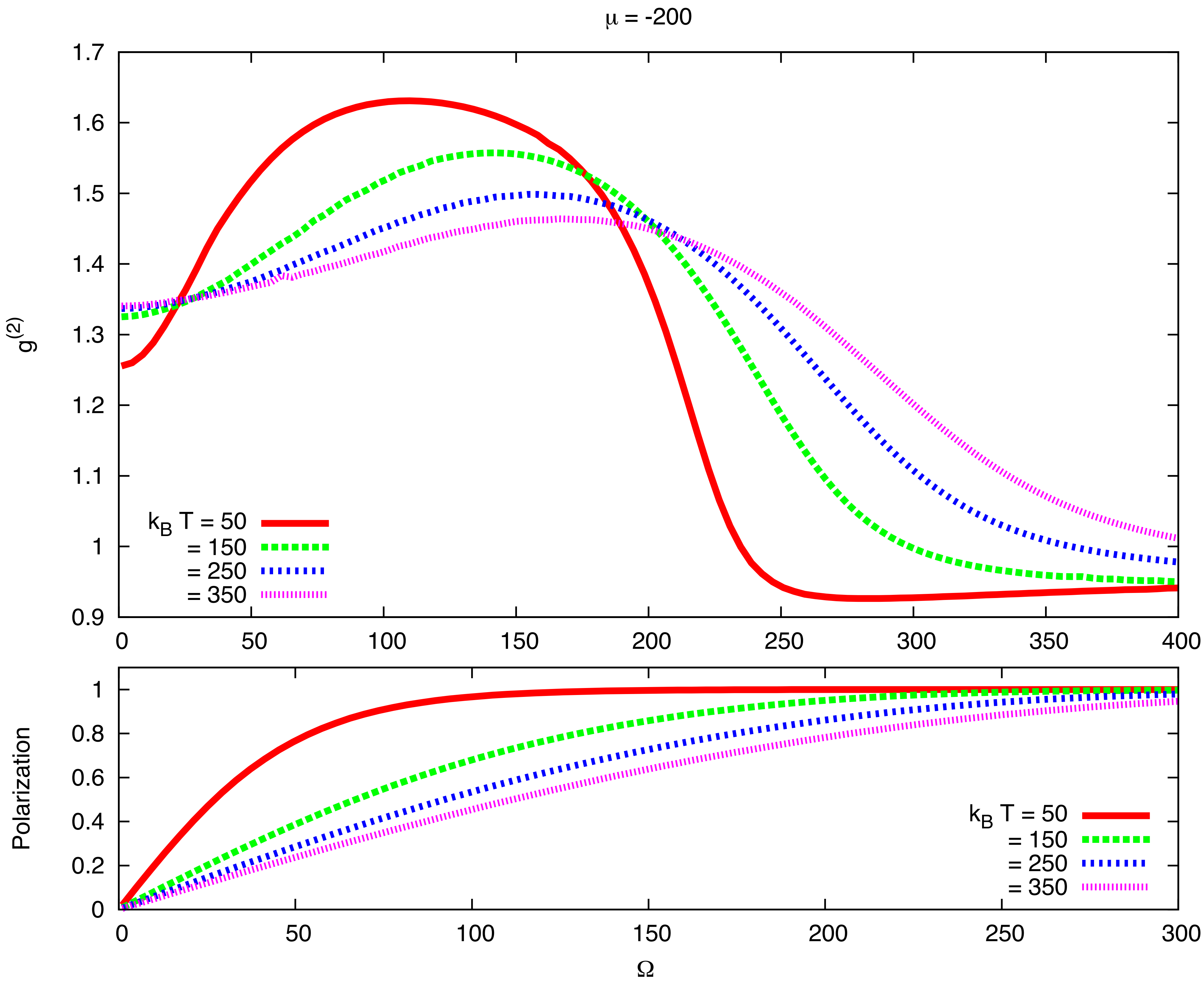
This + LDA: predictions for density profile in a trap

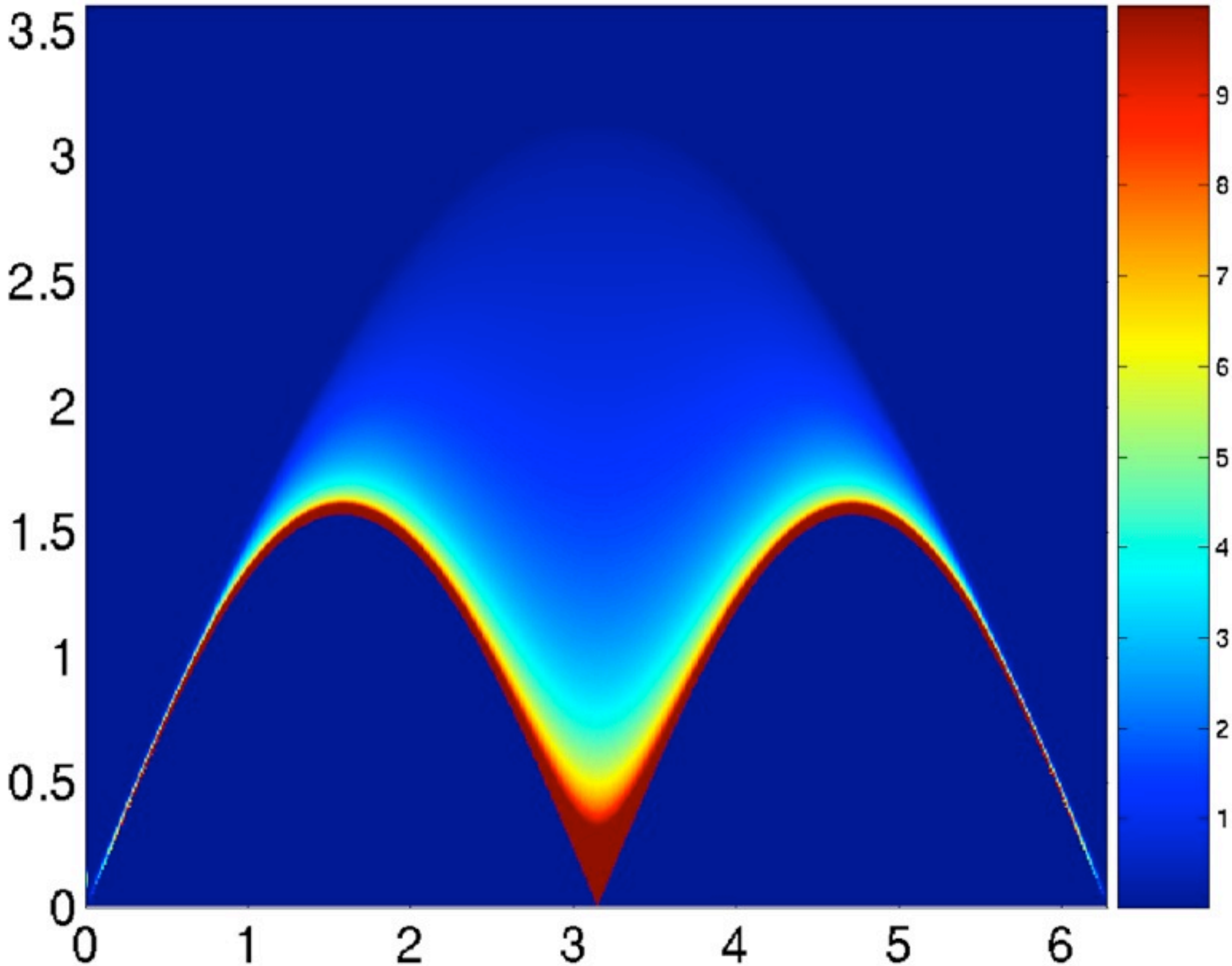


2CBG: nonmonotonic g(2)

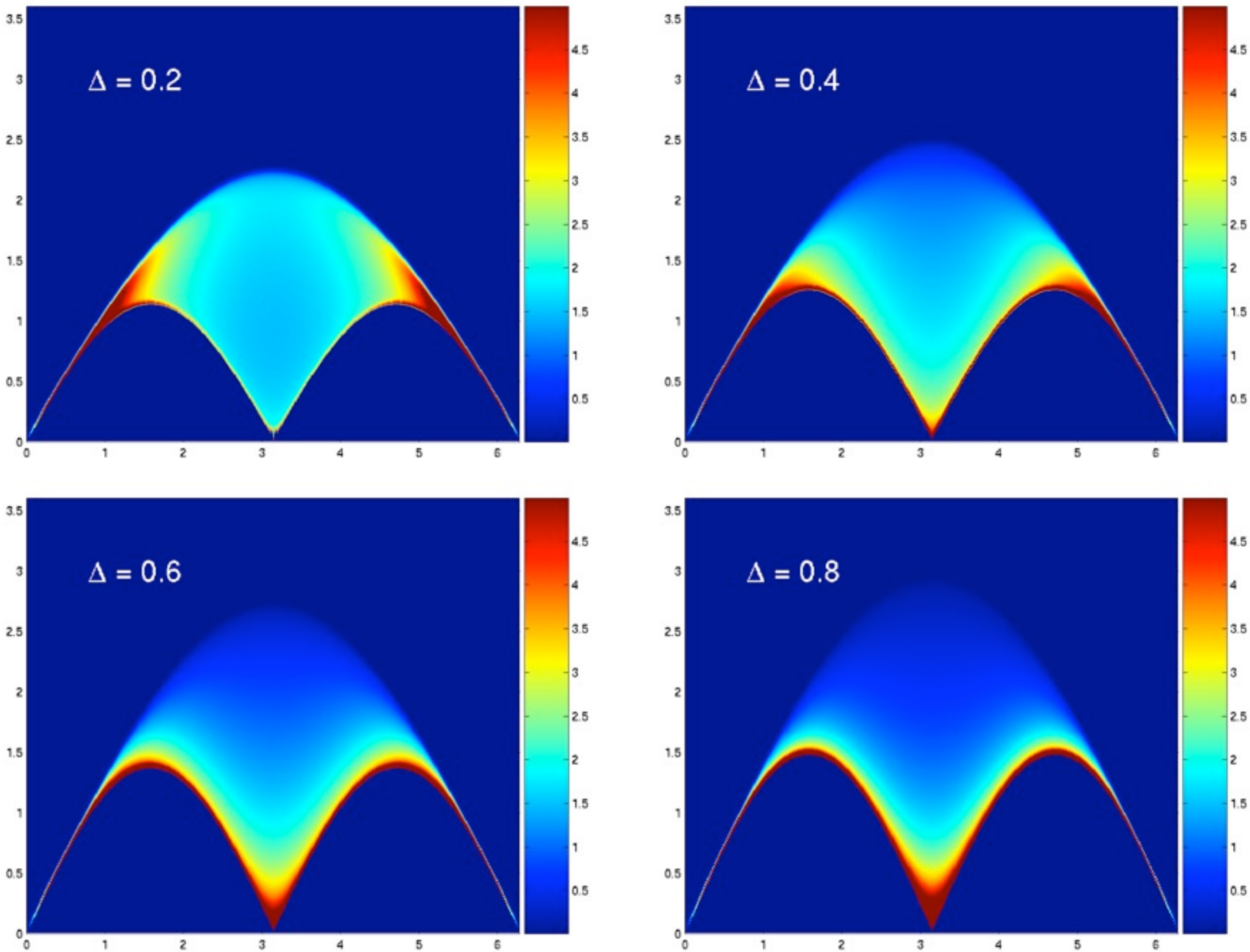


Heisenberg chains

$$S(k,\omega), \quad \Delta = 1, \quad h = 0$$



Zero field chain: longitudinal SF



Method 2: analytics (XXX, $h = 0$)

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Missing part: higher spinon numbers

Four spinon part of zero-field structure factor in the thermodynamic limit

(Abada, Bougourzi, Si-Lakhal 1997, revised in JSC & R. Hagemans JSTAT 2006)

At each point, 4 spinon SF is two-fold integral:

$$S_4(k,\omega) = C_4 \int_{\mathcal{D}_K} dK \int_{\Omega_l(k,\omega,K)}^{\Omega_u(k,\omega,K)} d\Omega \, \frac{J(k,\omega,K,\Omega)}{\left\{ \left[\omega_{2,u}^2(K) - \Omega^2\right] \left[\omega_{2,u}^2(k-K) - (\omega-\Omega)^2\right] \right\}^{1/2}}$$

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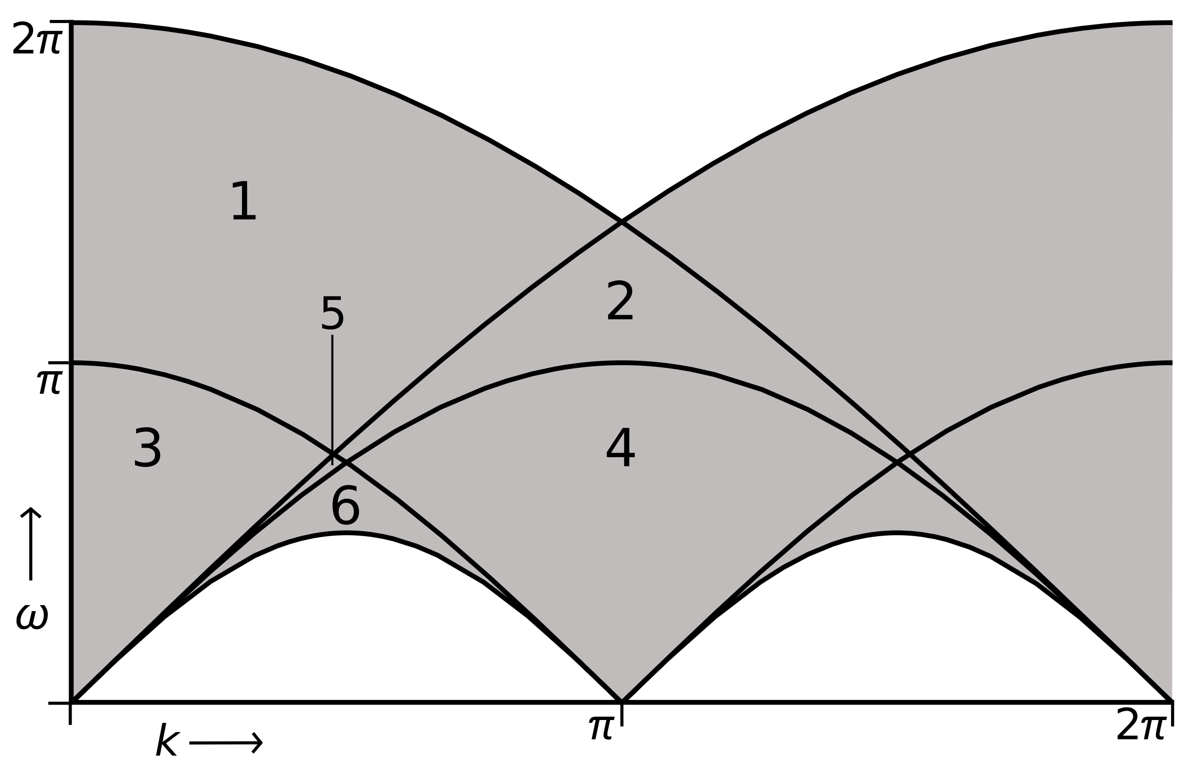

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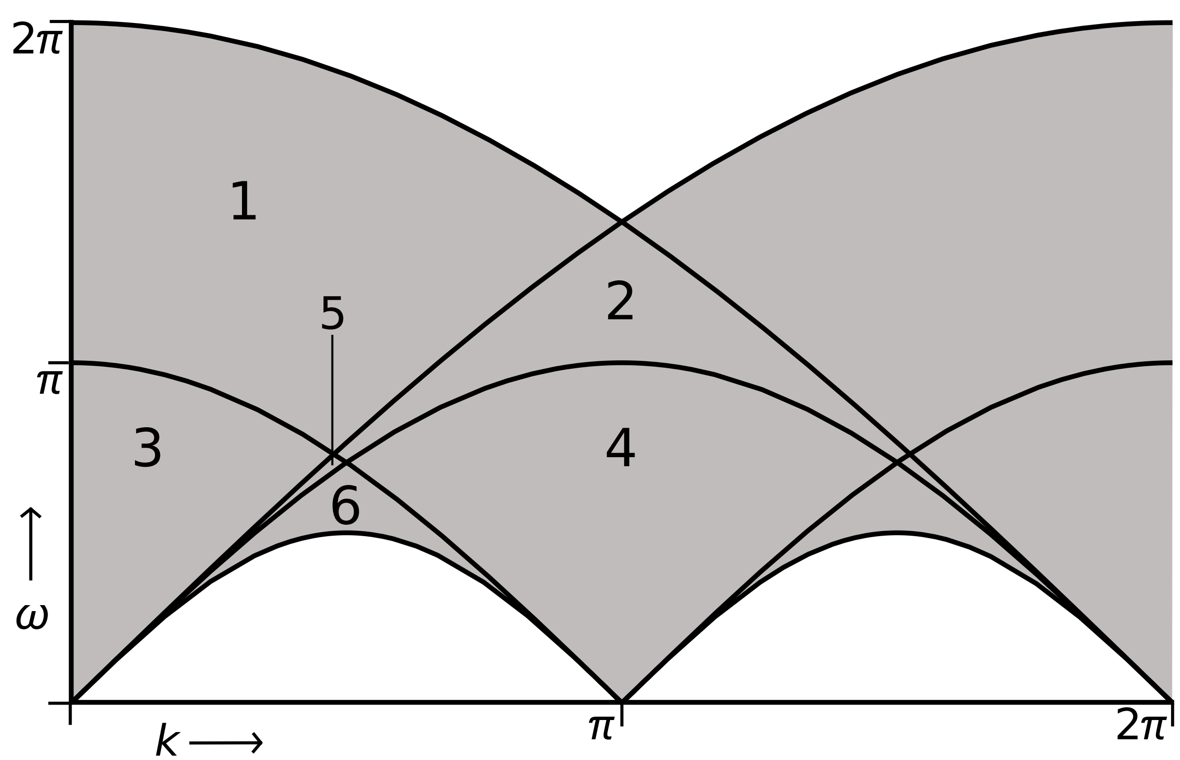
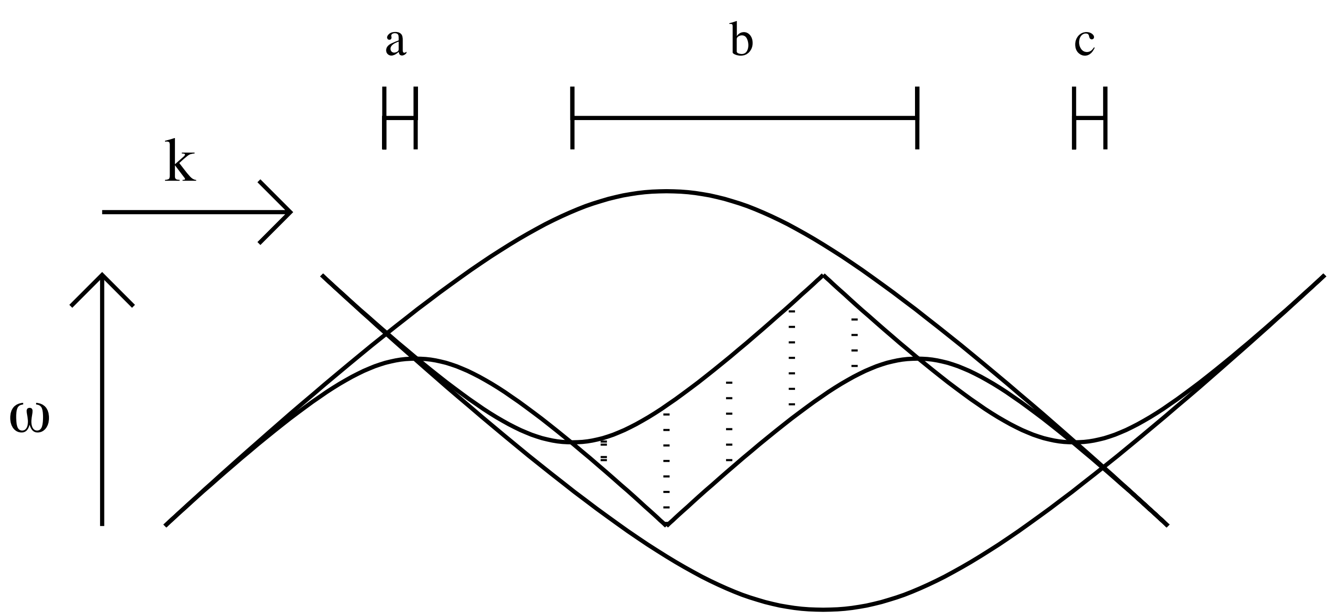
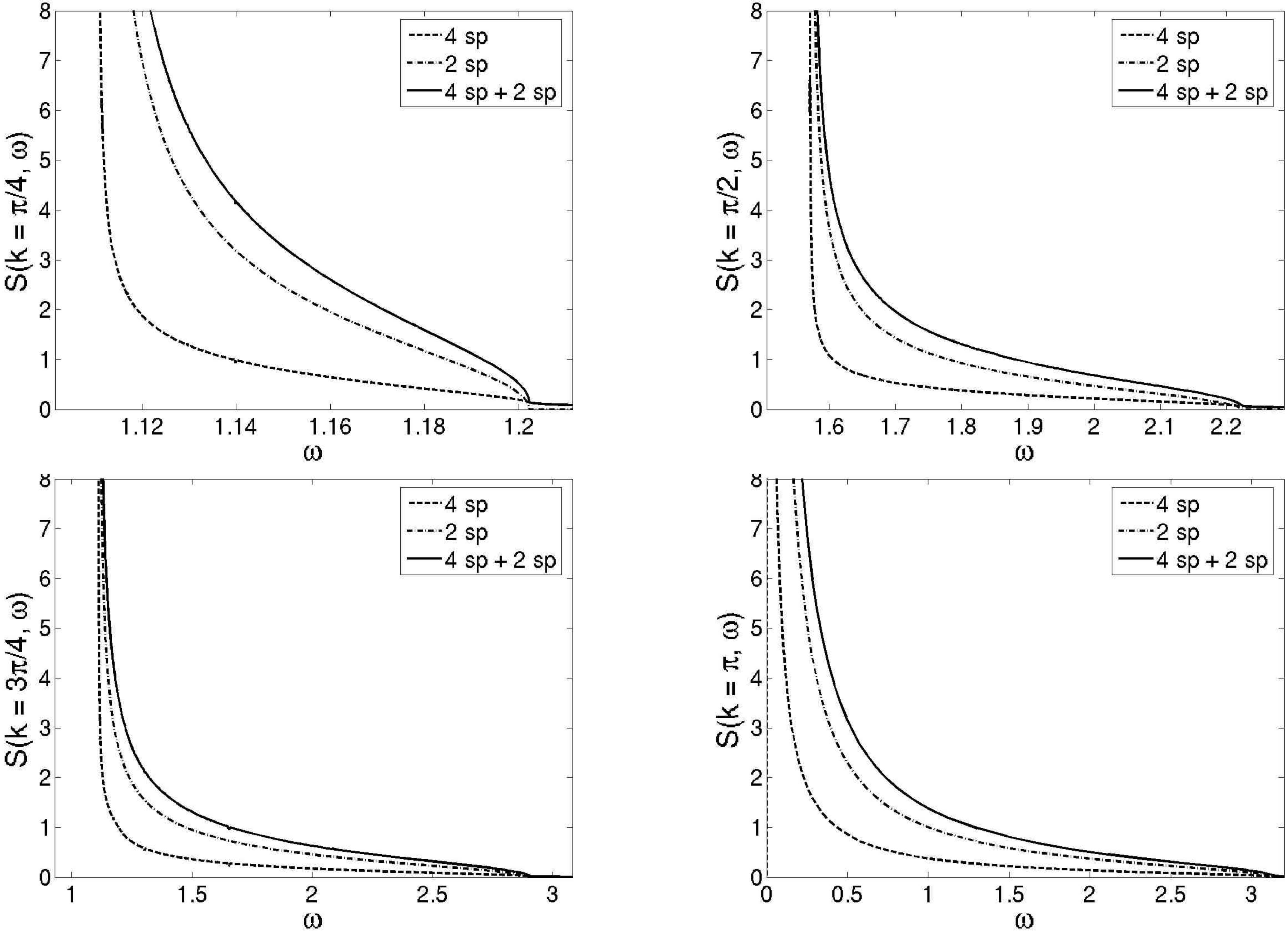
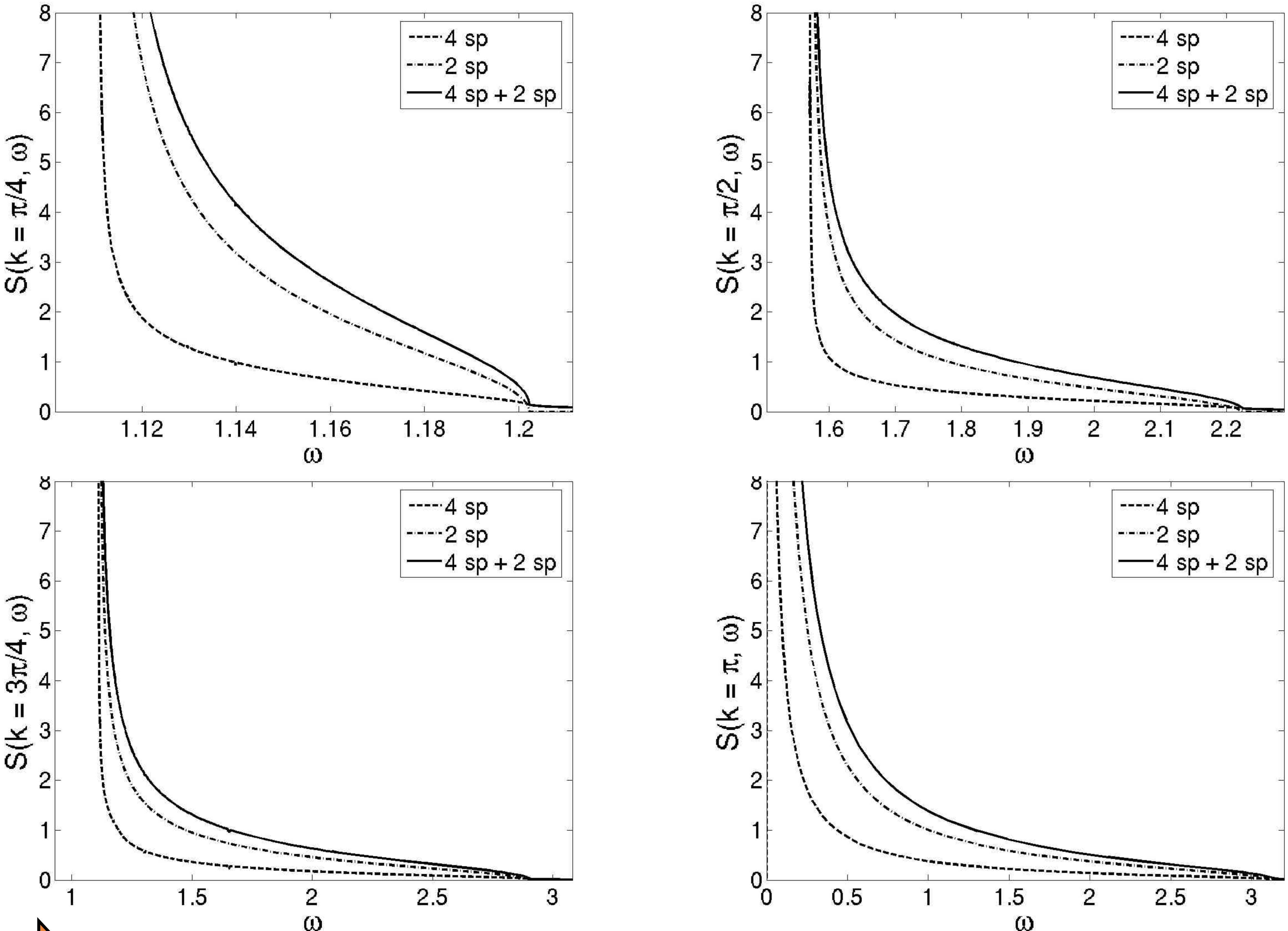


Integration regions: intersection of two 2-spinon continua

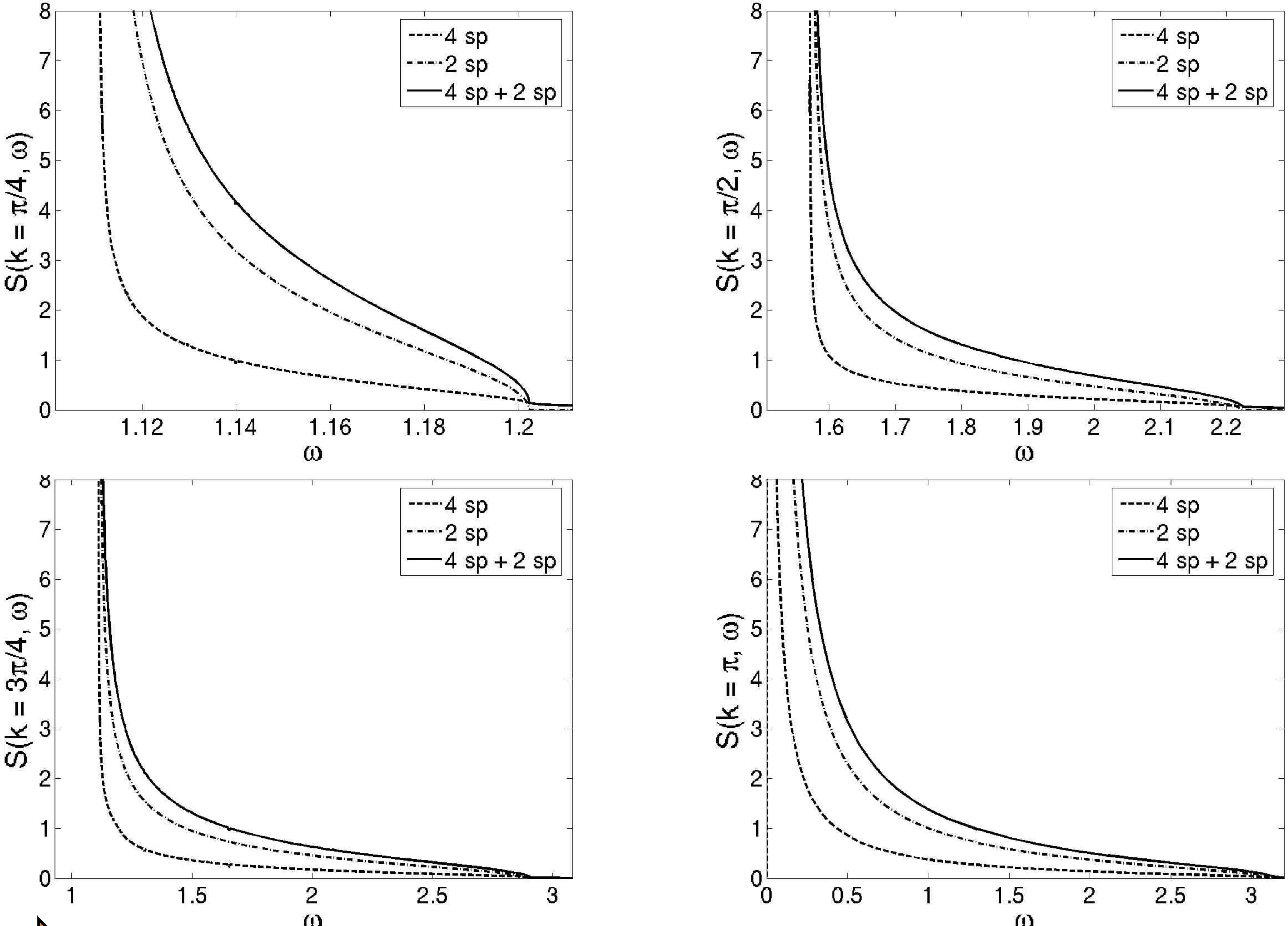




S(k = π/4, ω)
S(k = π/2, ω)
S(k = 3π/4, ω)
S(k = π, ω)
ω
4 sp
2 sp
4 sp + 2 sp



4-spinon states carry about 27% of full intensity



4-spinon states carry about 27% of full intensity

2 + 4 spinons: approx 98% of correlations !

Analytics (II): gapped XXZ, h = 0

(Bougourzi, Karbach, Müller 1998, revisited in JSC, Mossel & Pérez Castillo, JSTAT 2008)

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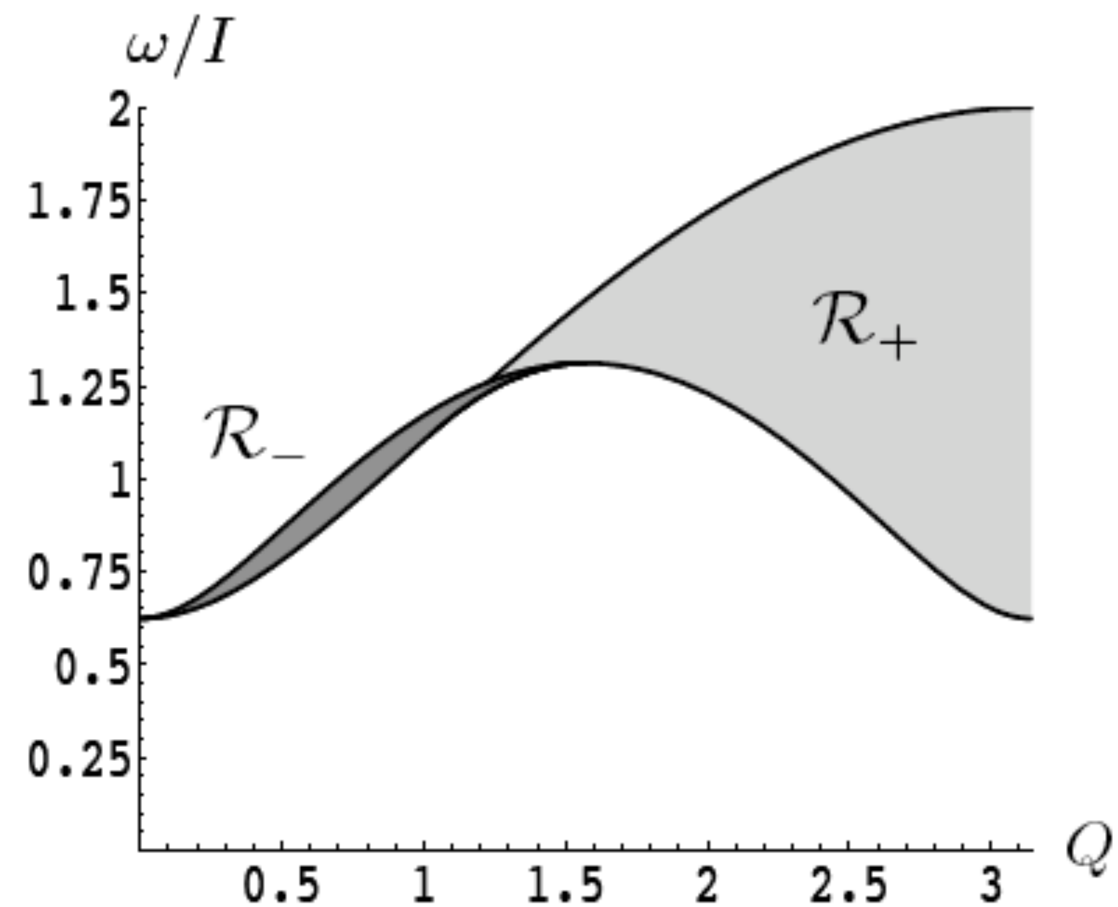

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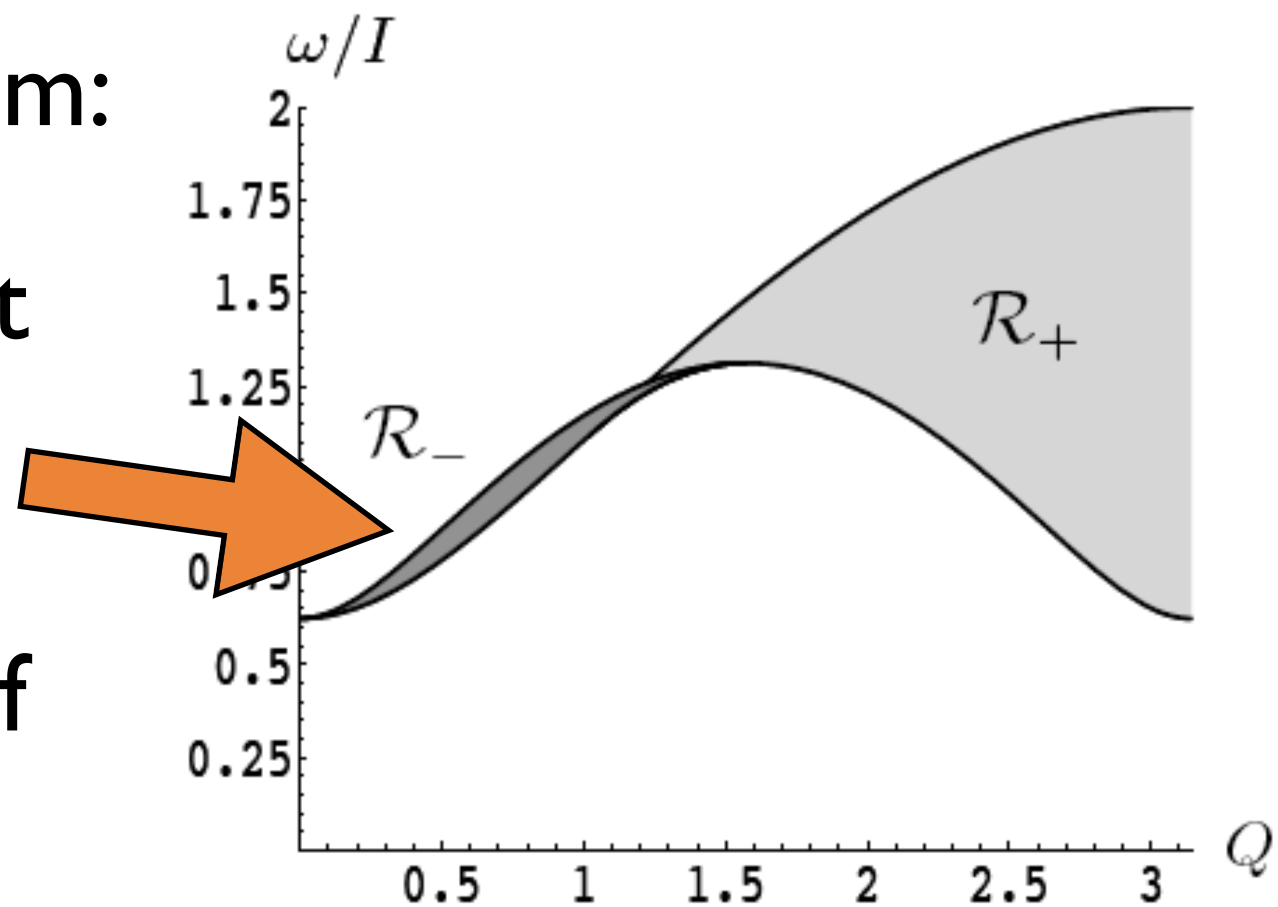
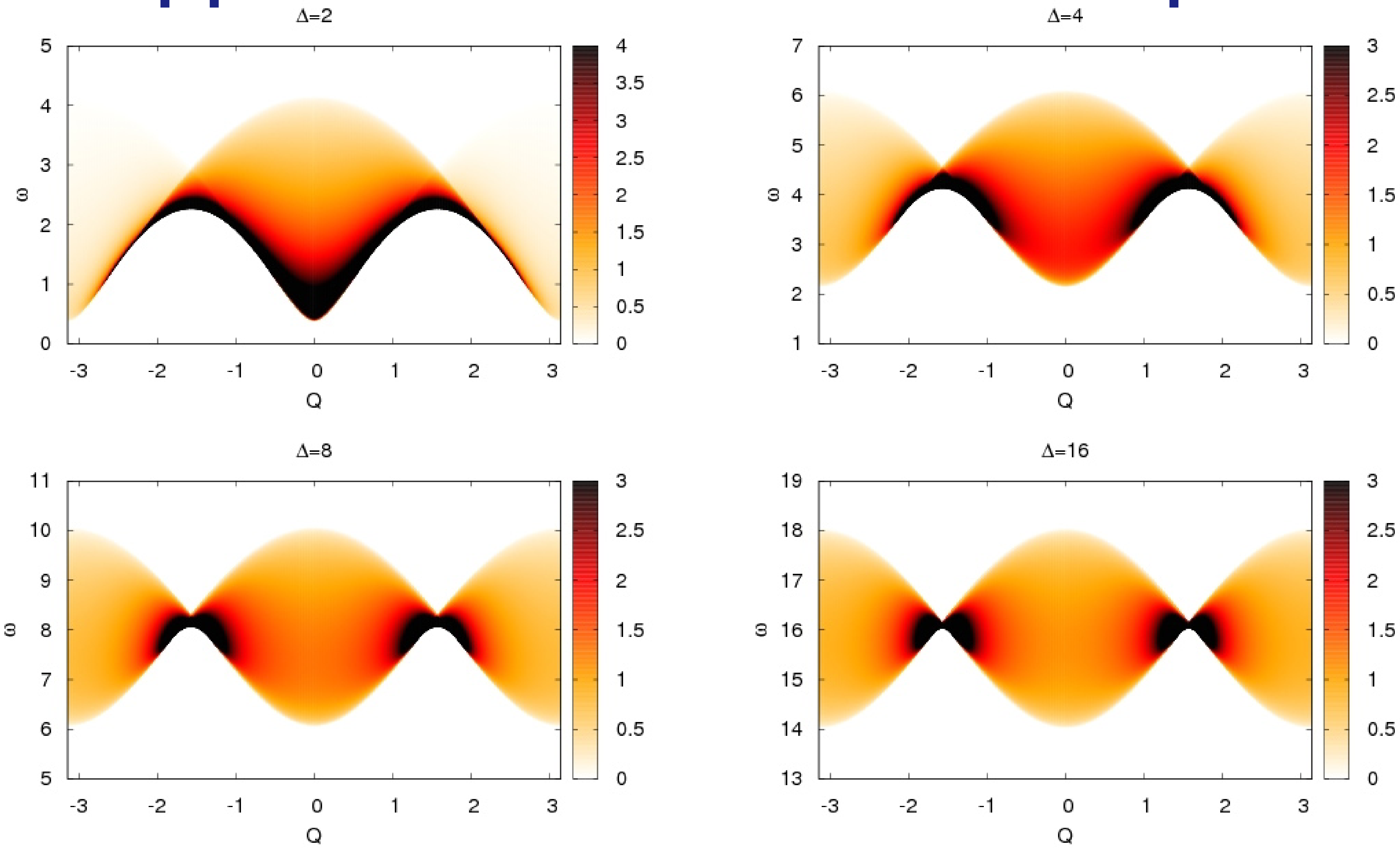
Nontrivial 2-spinon continuum:

‘Folding up’ of continuum at small momentum transfer (curvature of dispersion relation changes sign as fn of momentum)



Gapped XXZ AFM, h = 0, 2spinons



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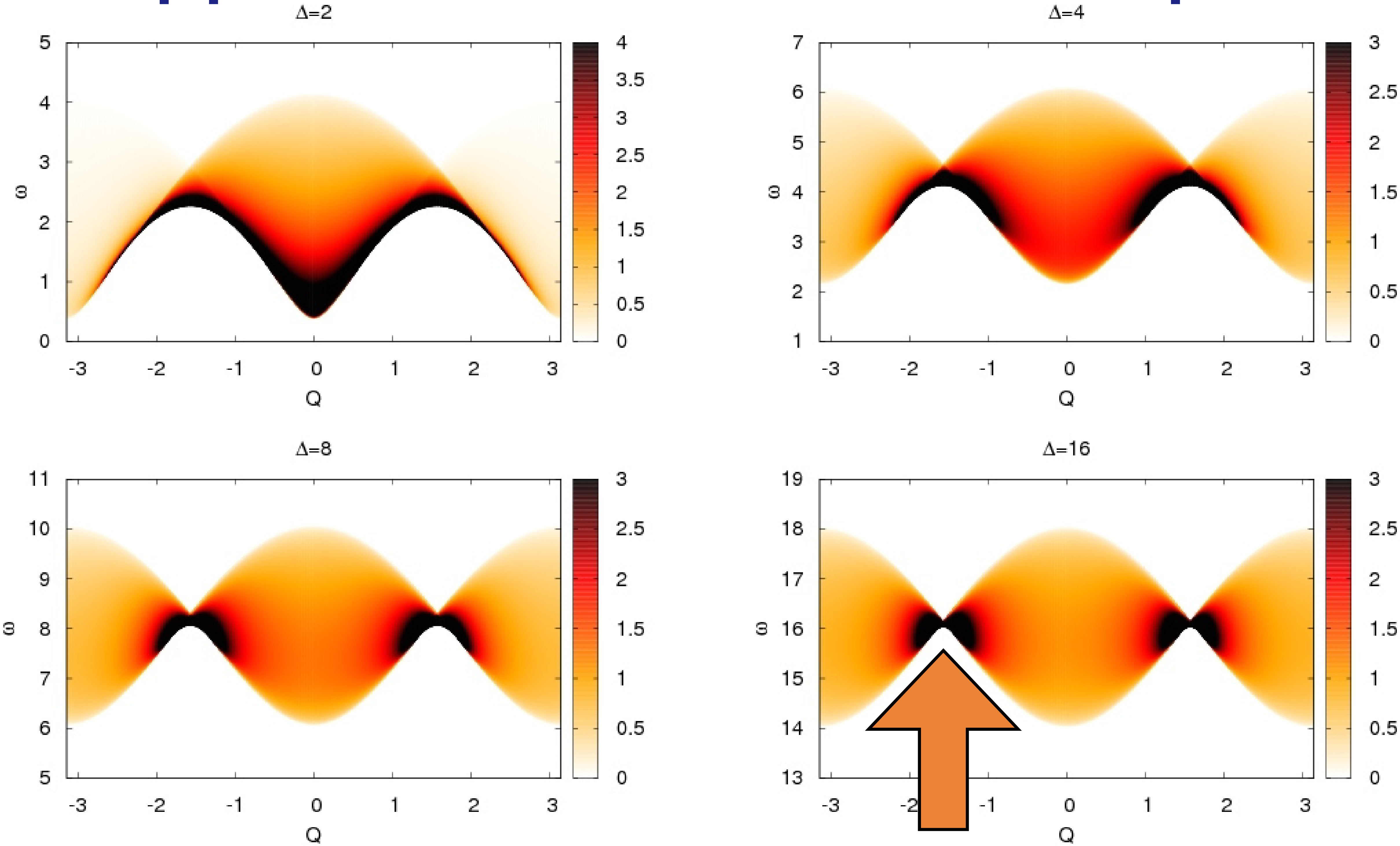

π periodicity only recovered in true Ising limit

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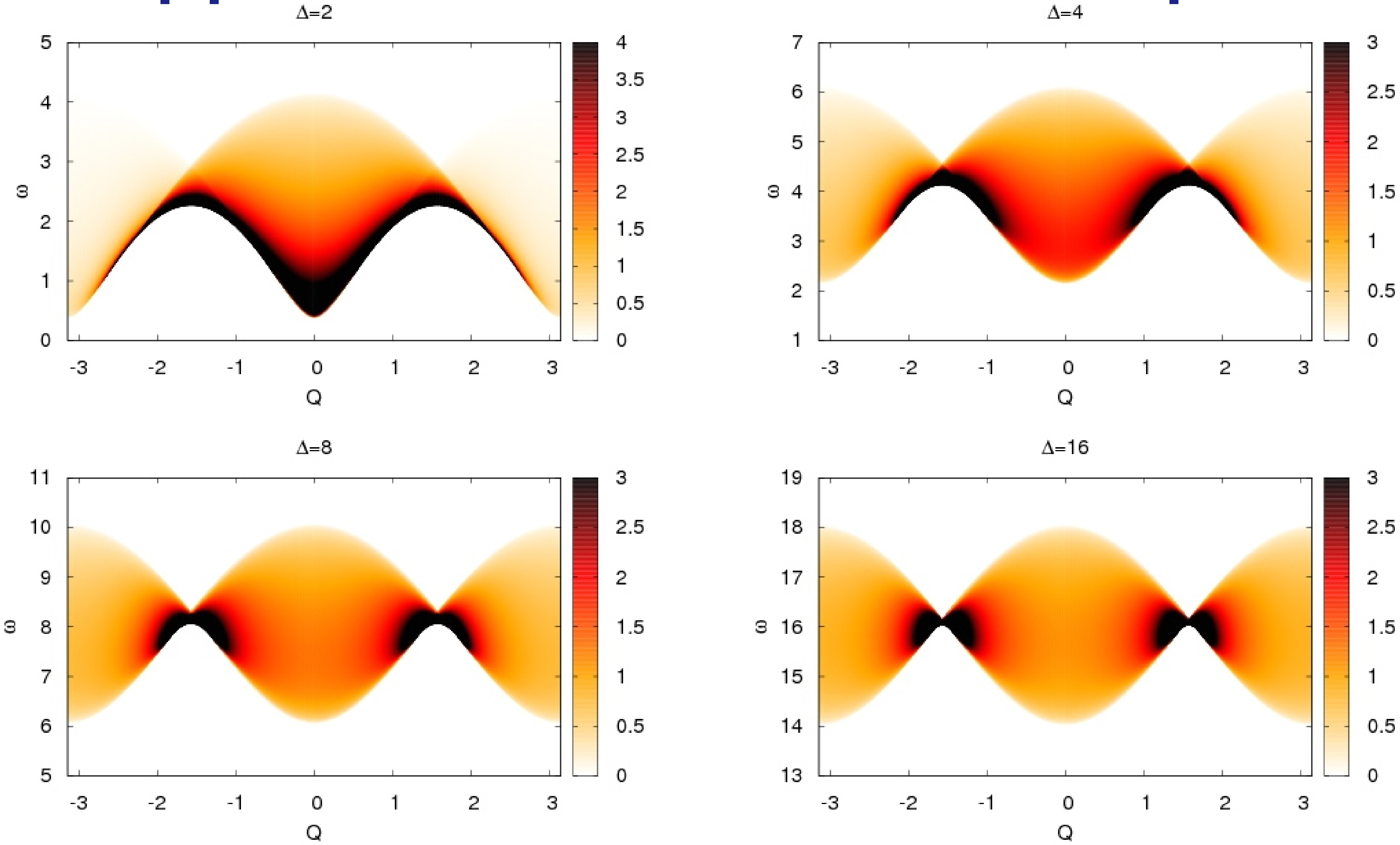

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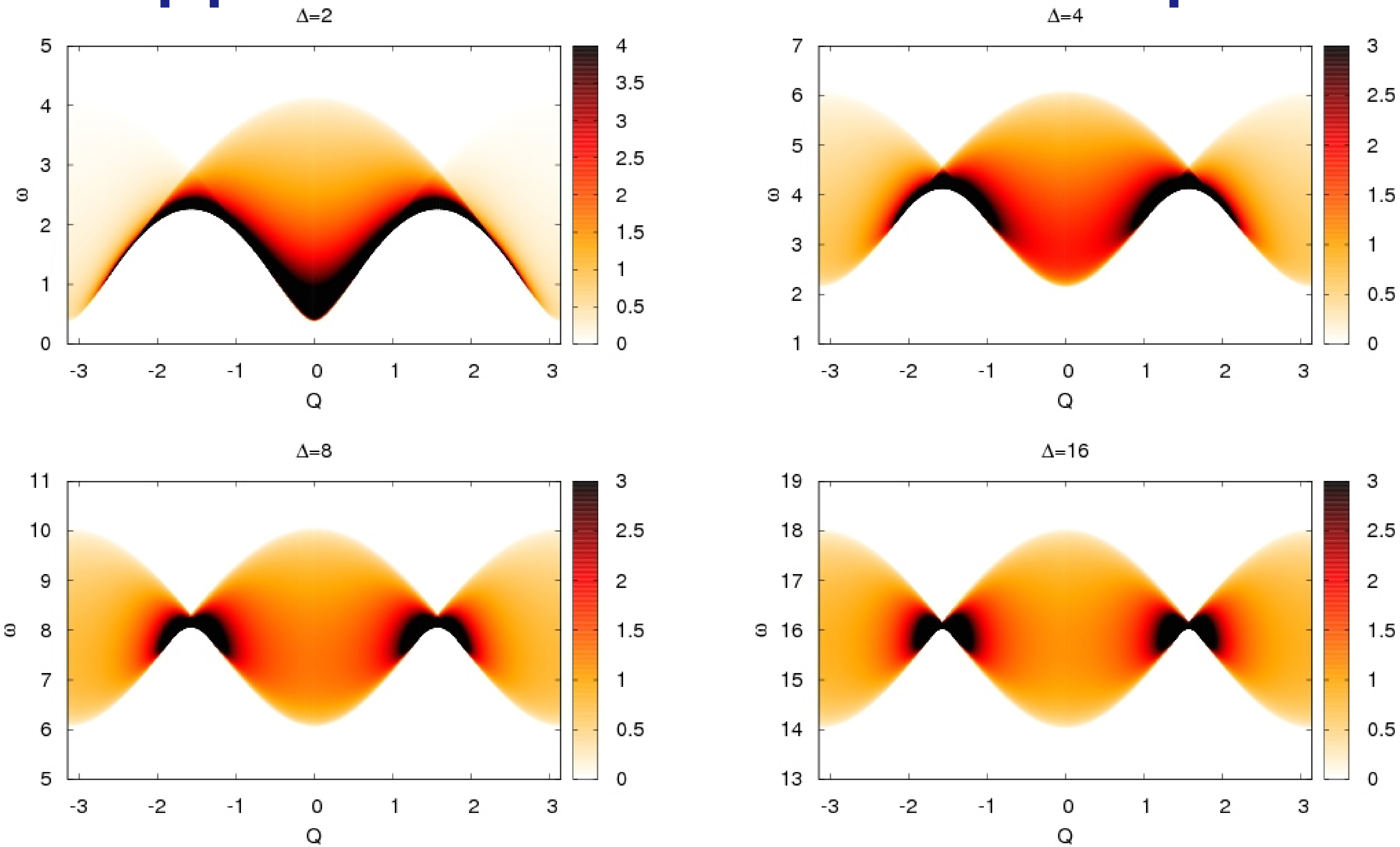
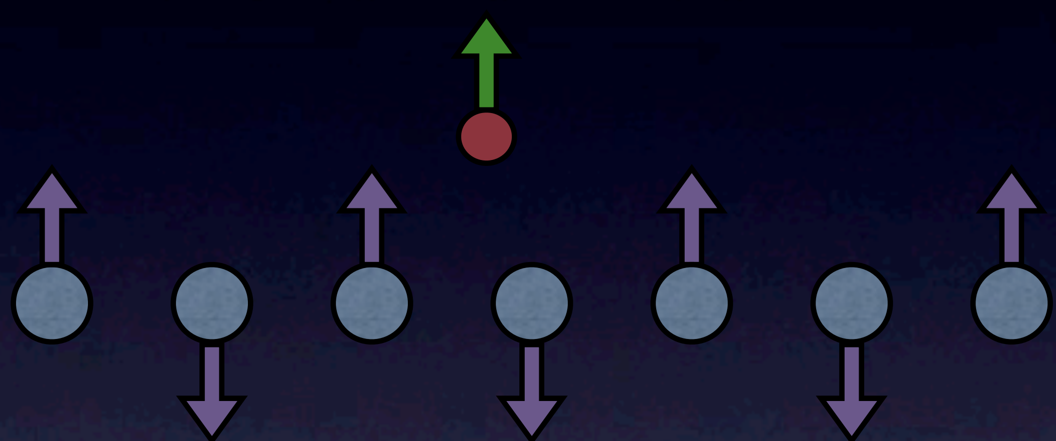


EXACT correlation function in thermodynamic limit for energies below twice the gap

Neutron scattering

Neutron scattering



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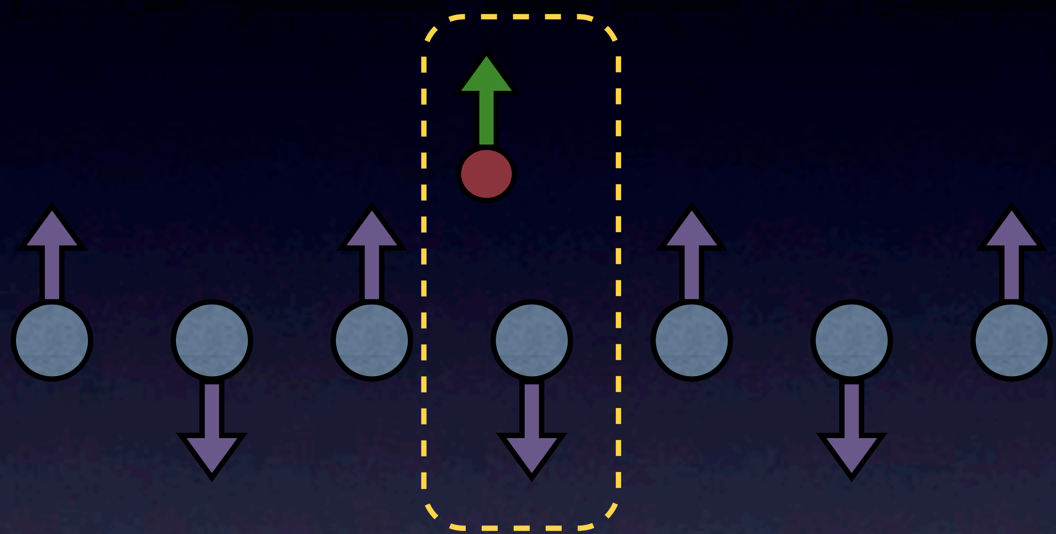

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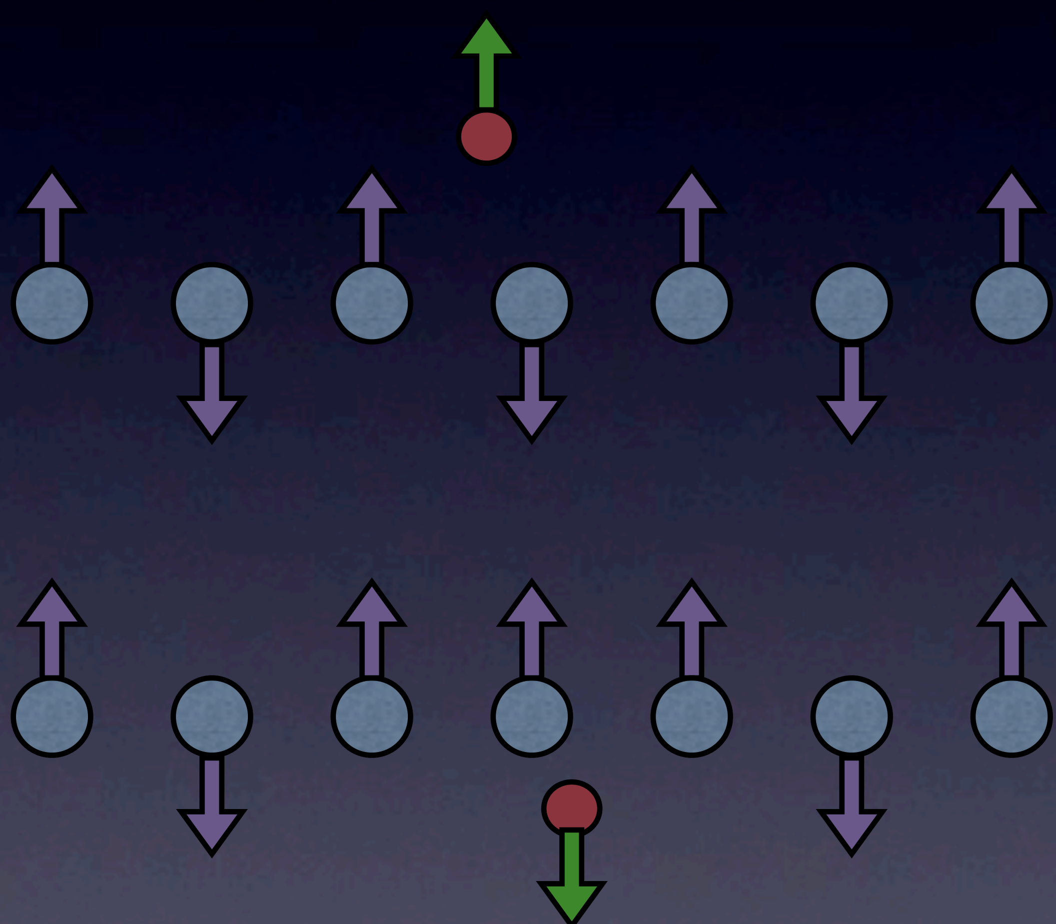

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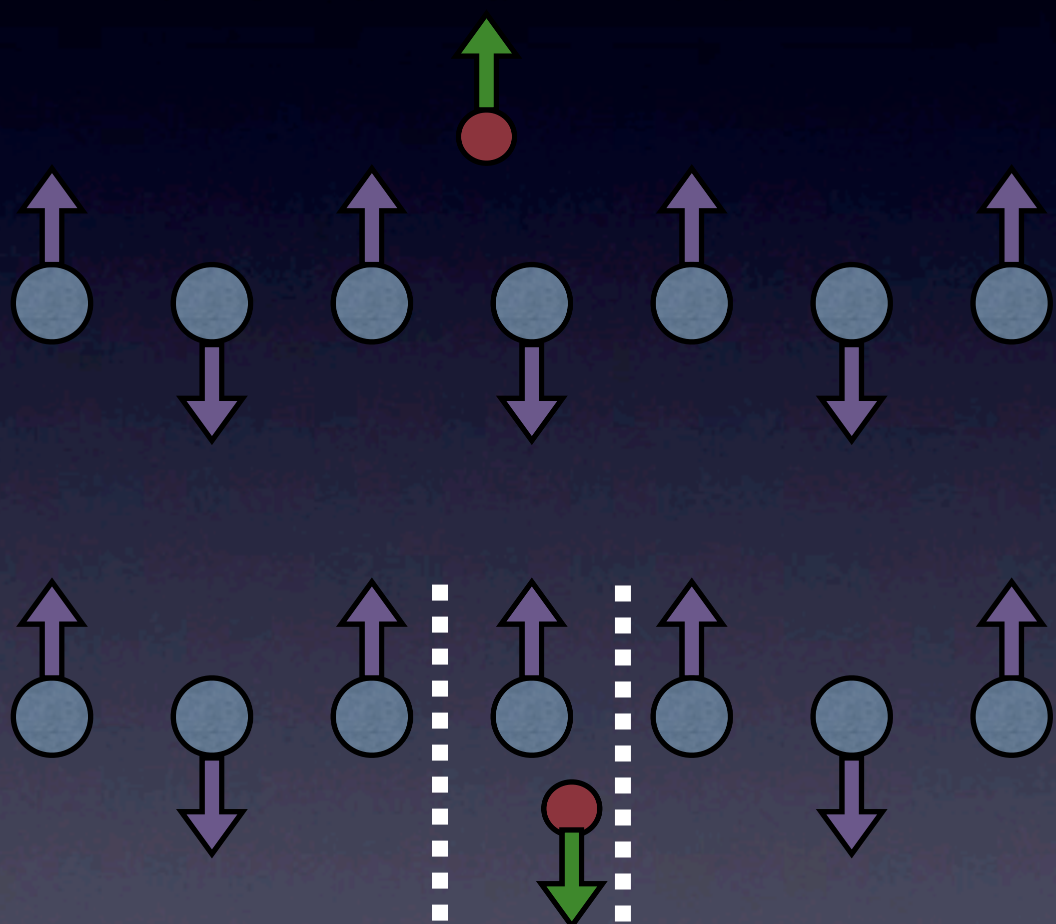

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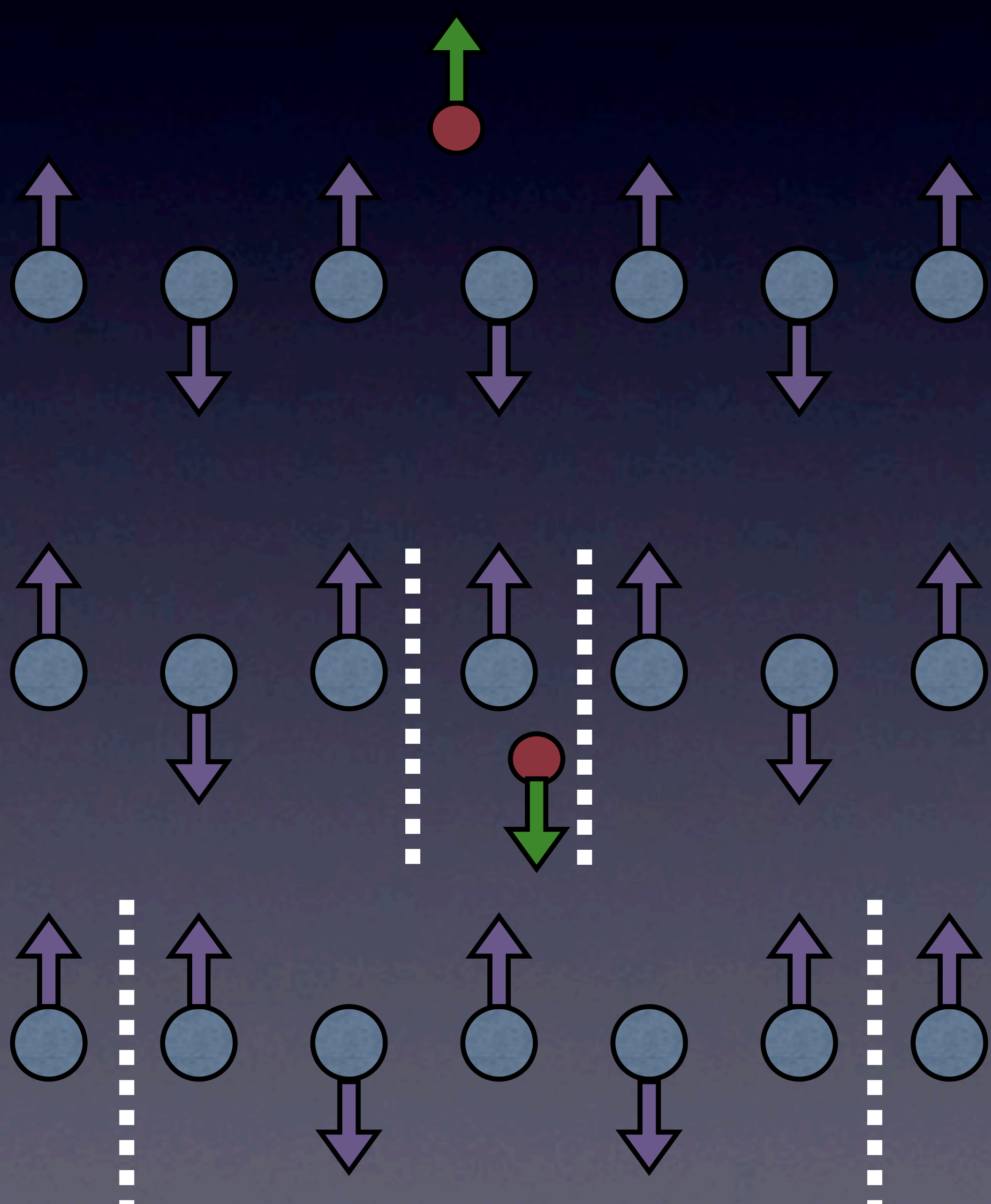

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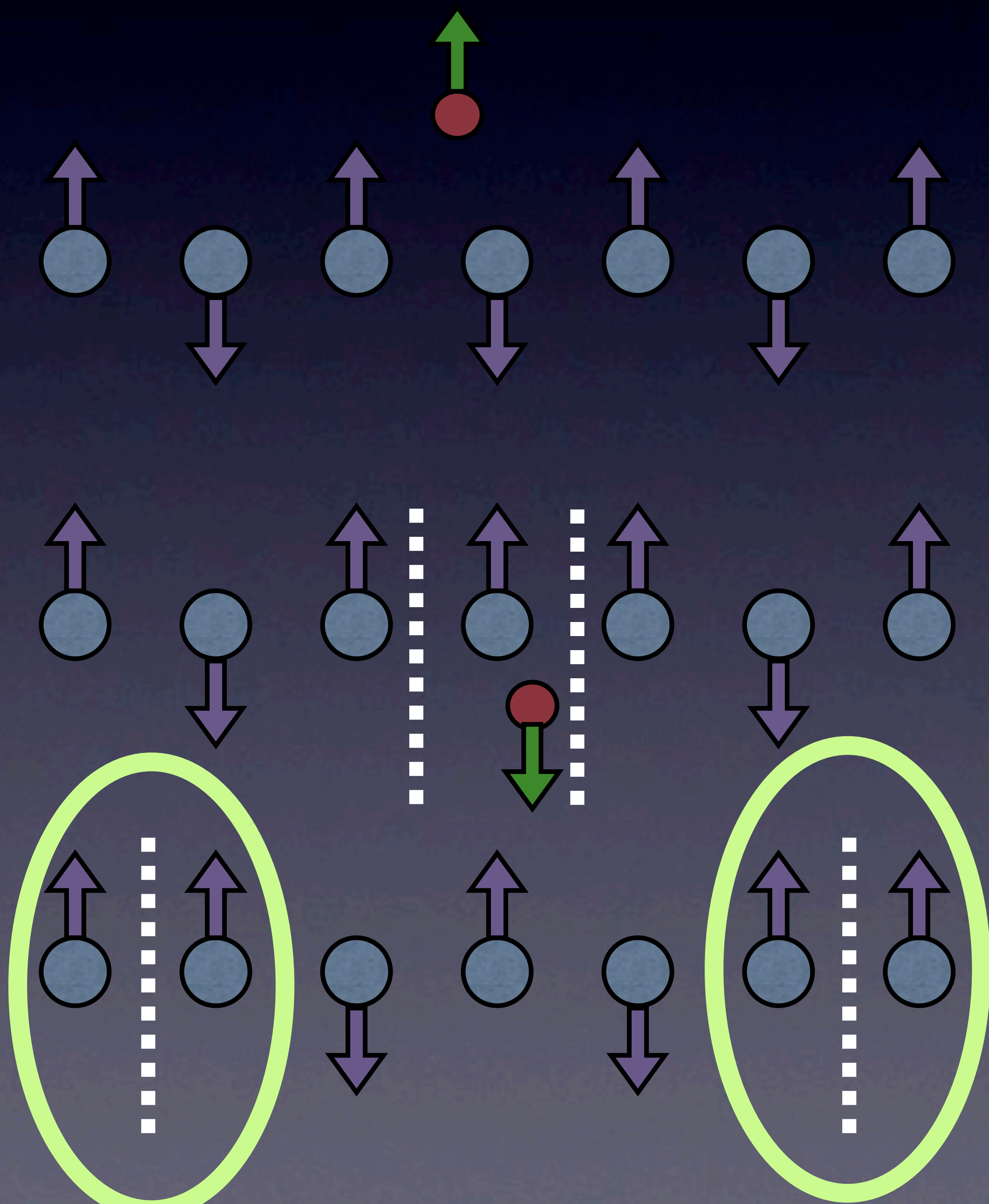
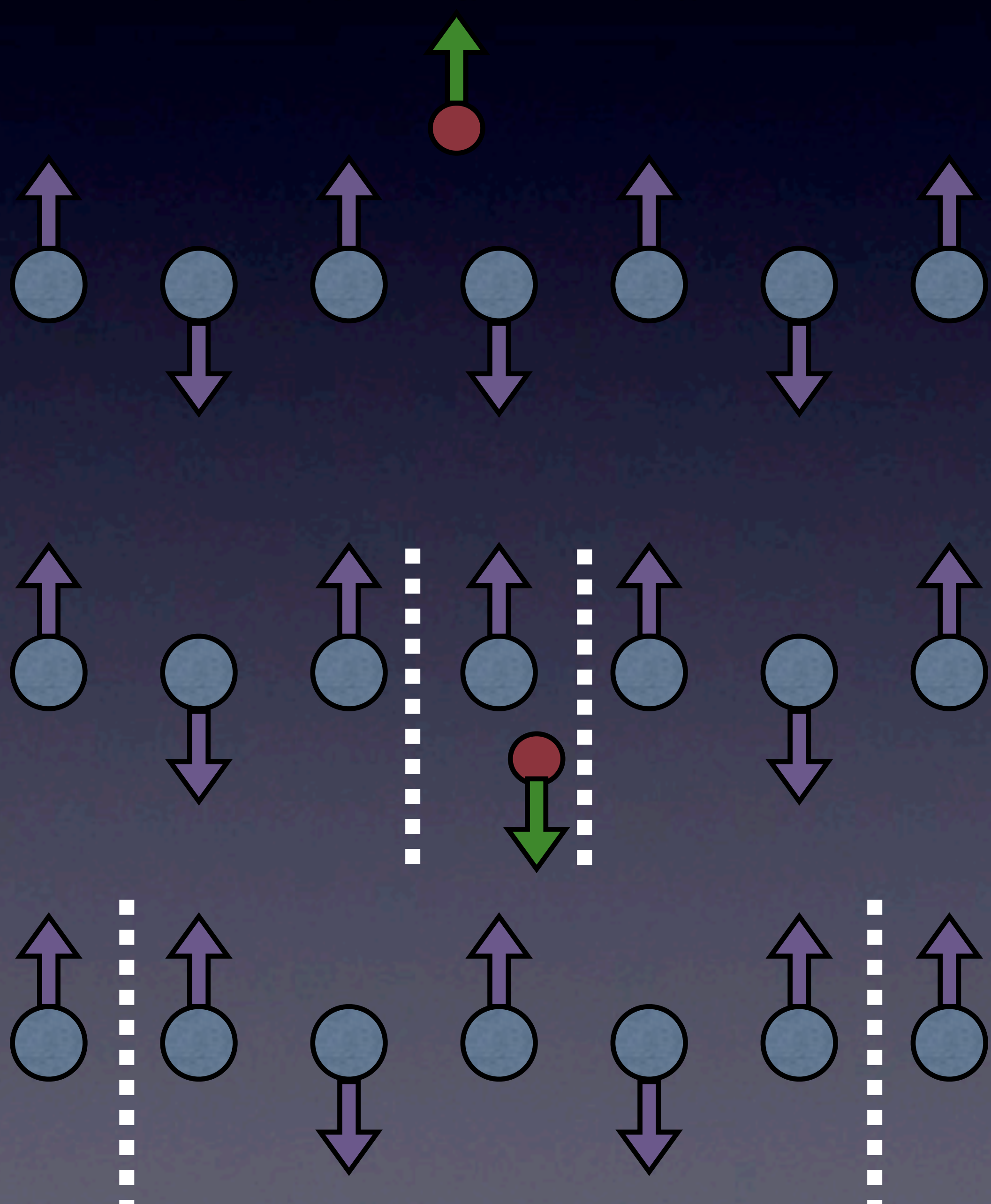
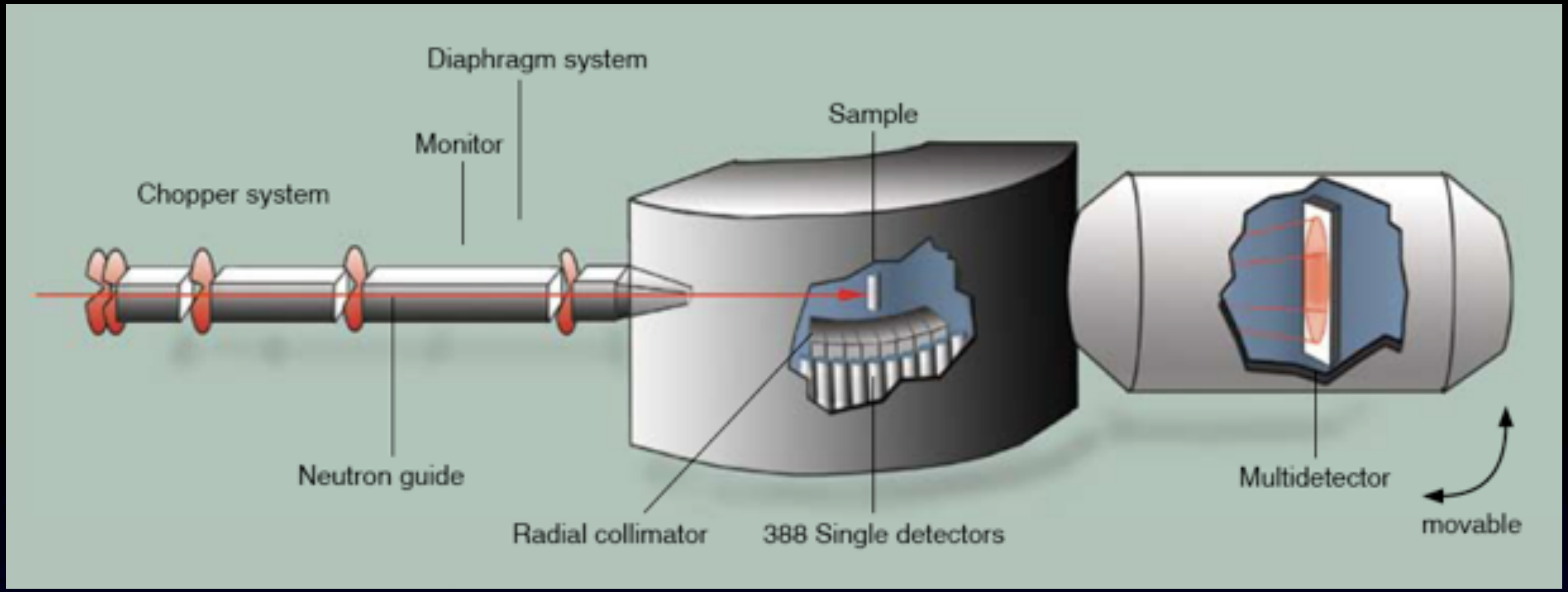


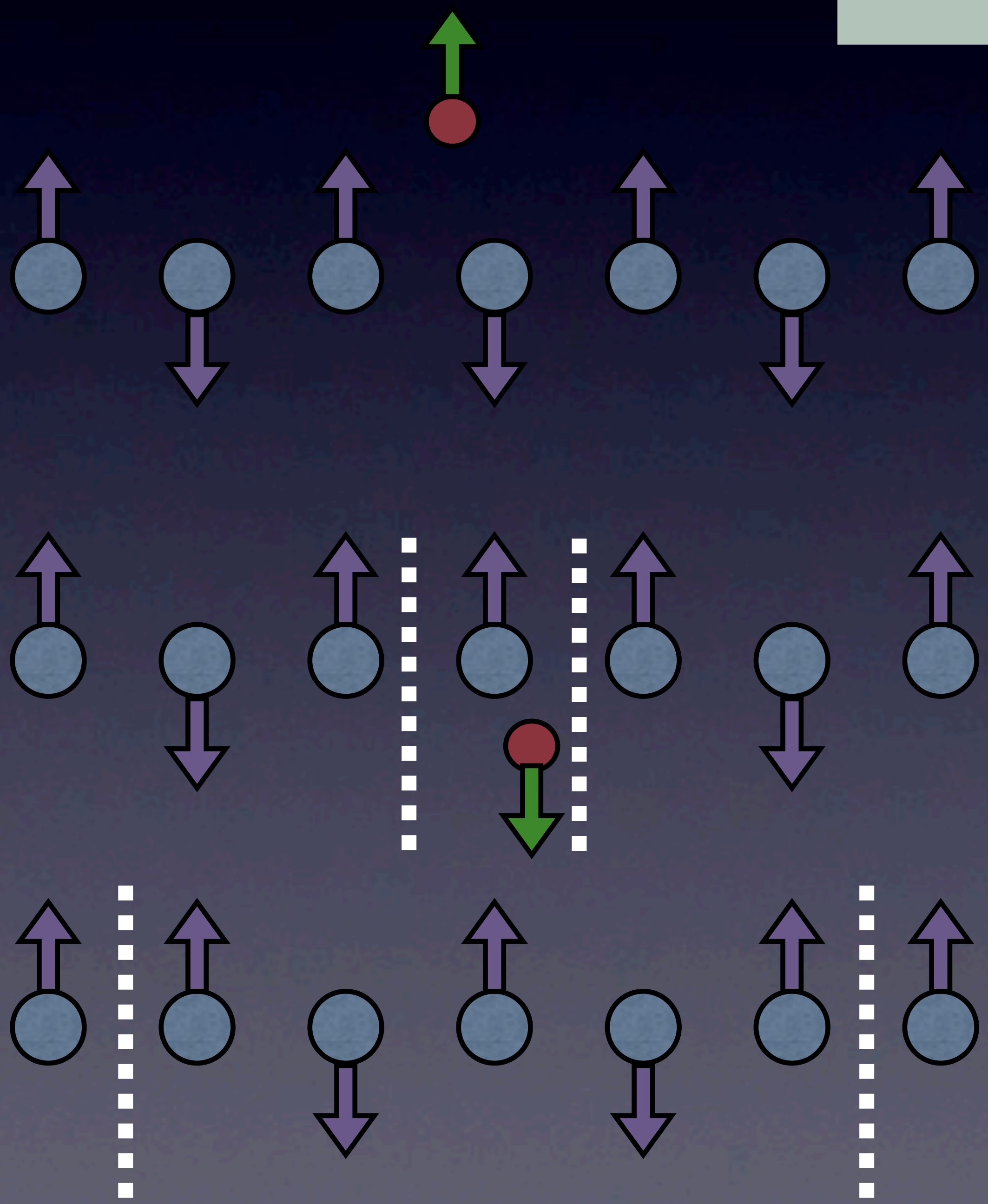
'new' particles:
spinons (quantum solitons)

Neutron scattering



Neutron scattering





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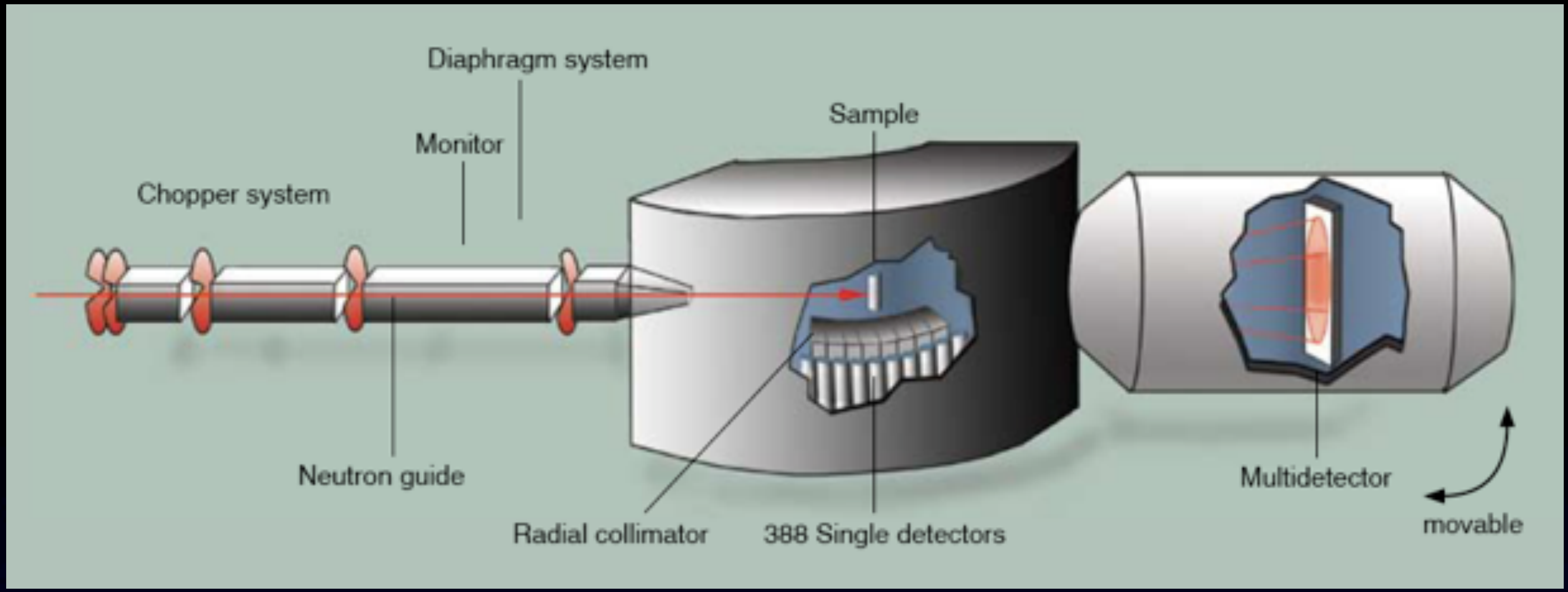

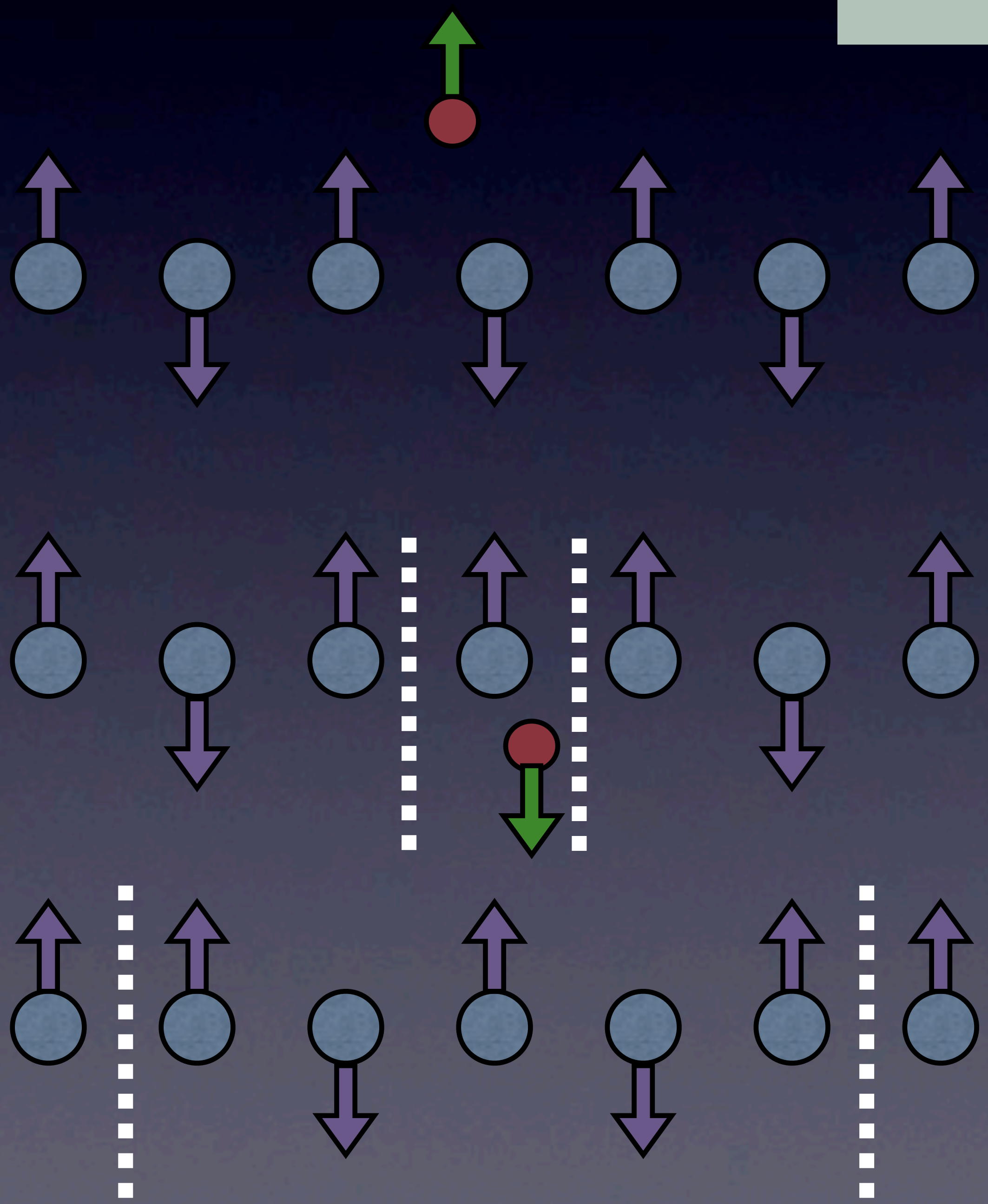

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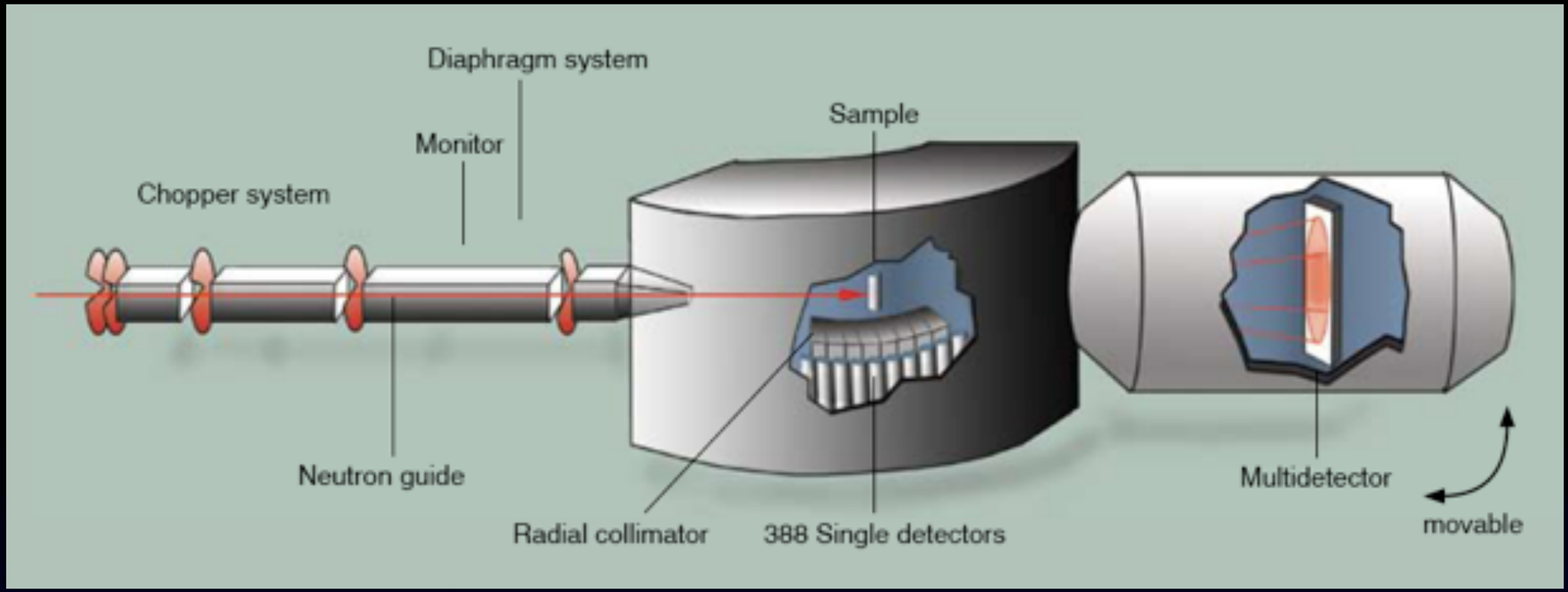

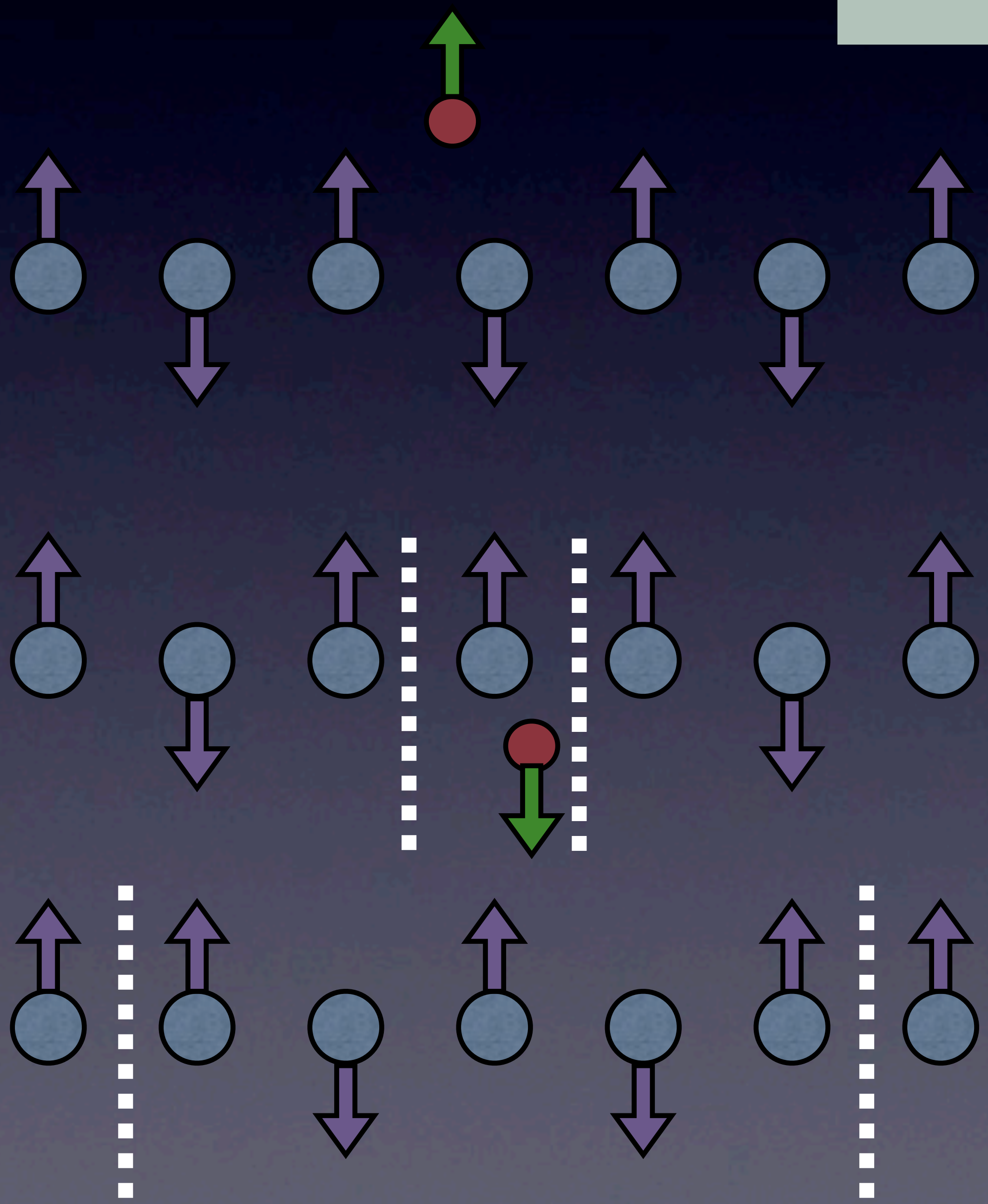

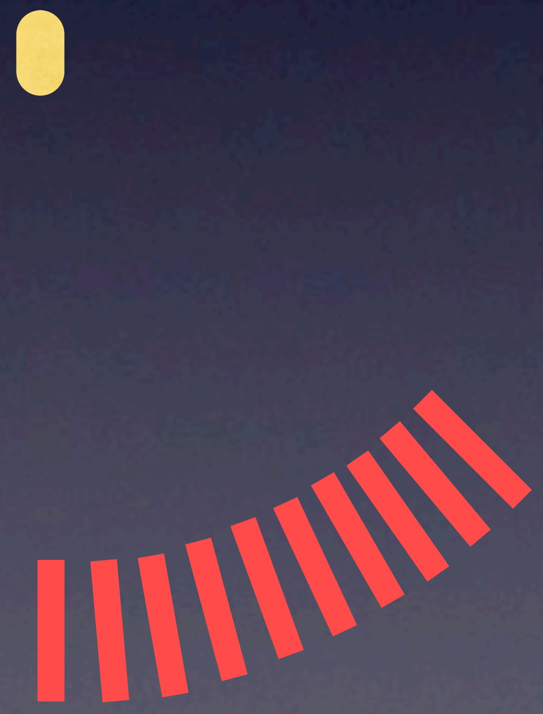

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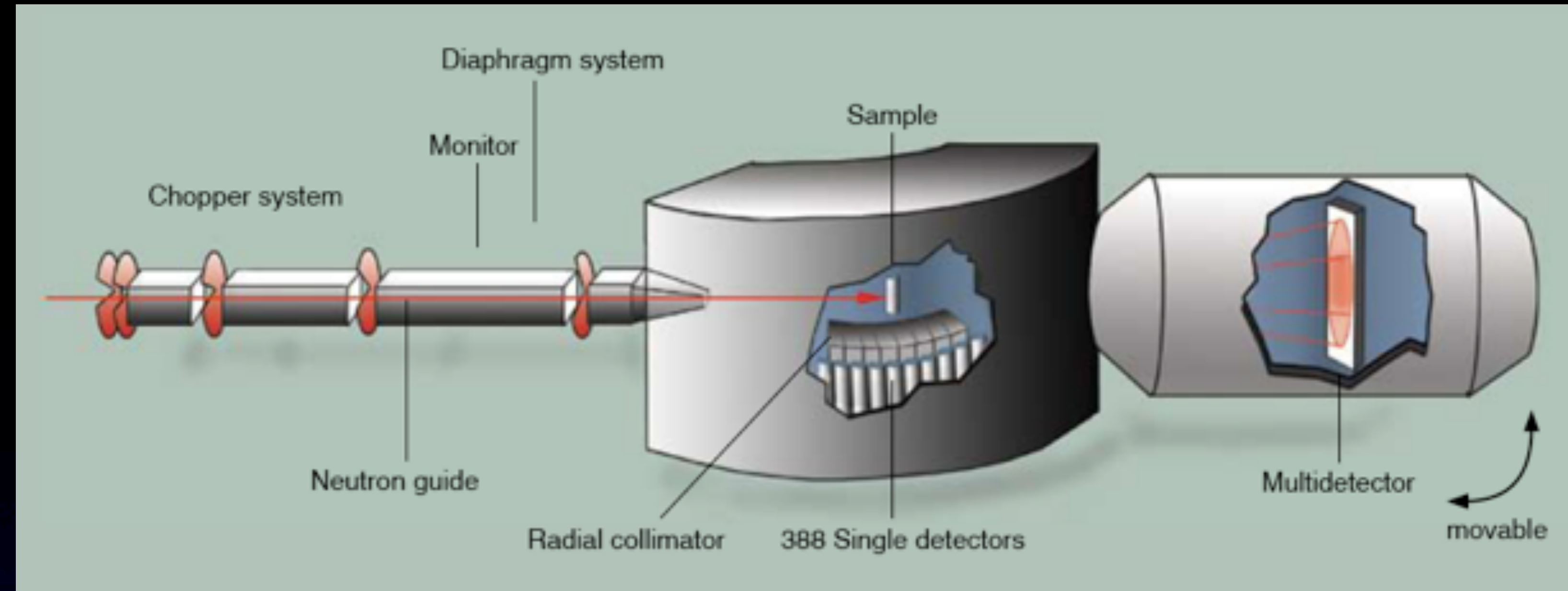

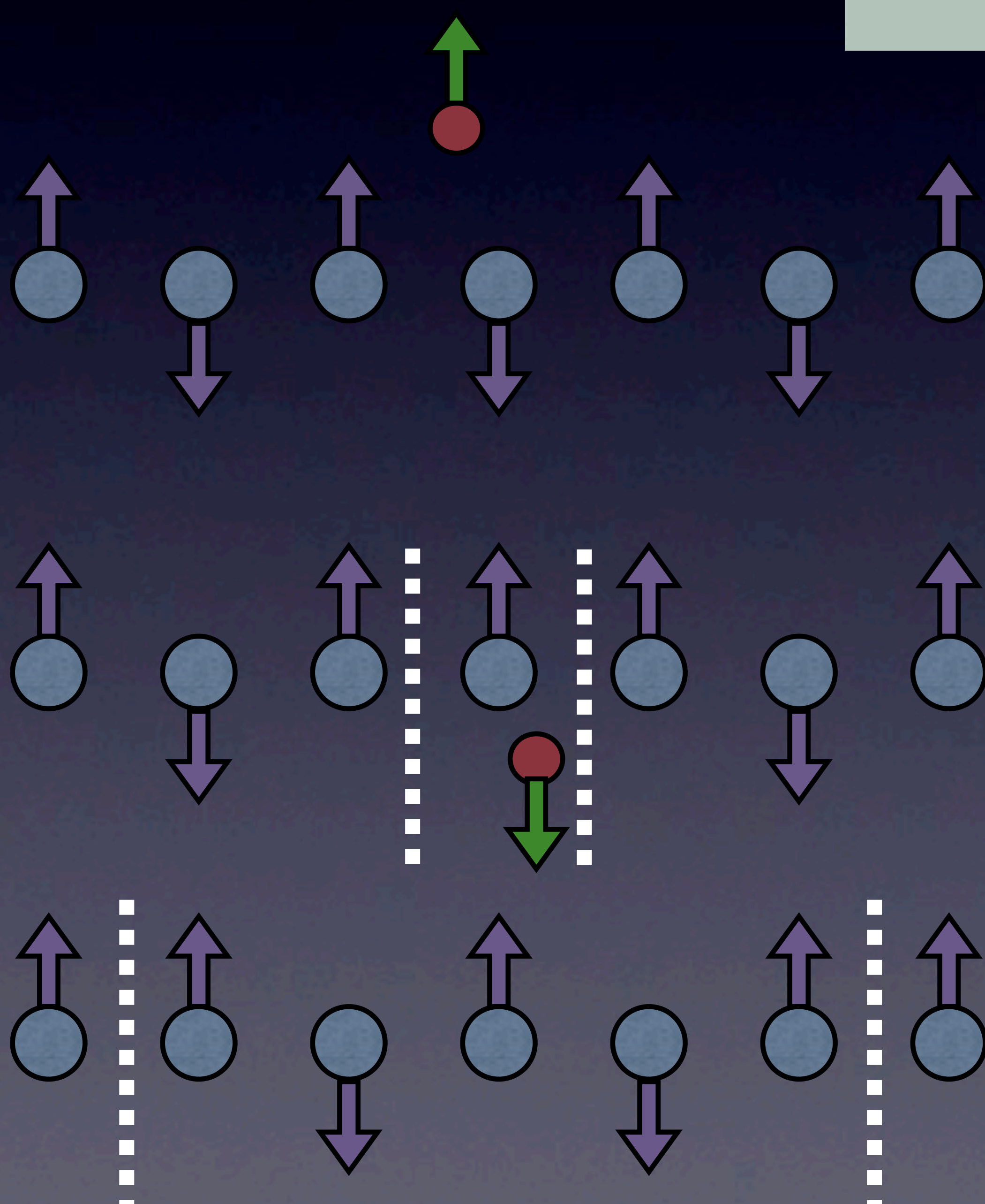

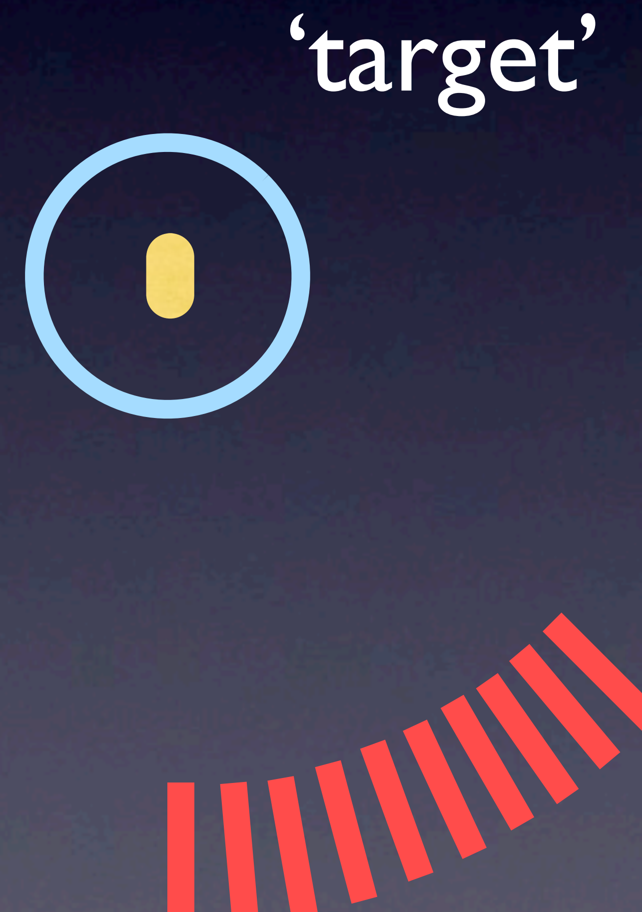

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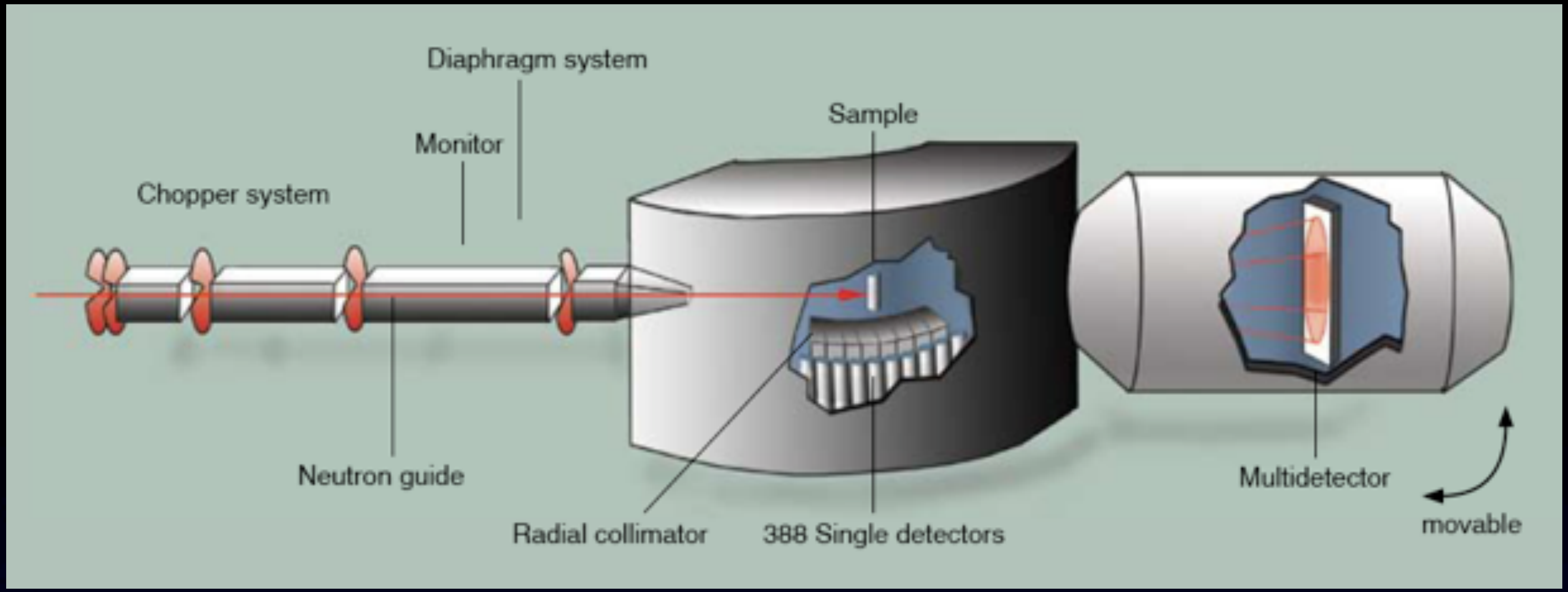

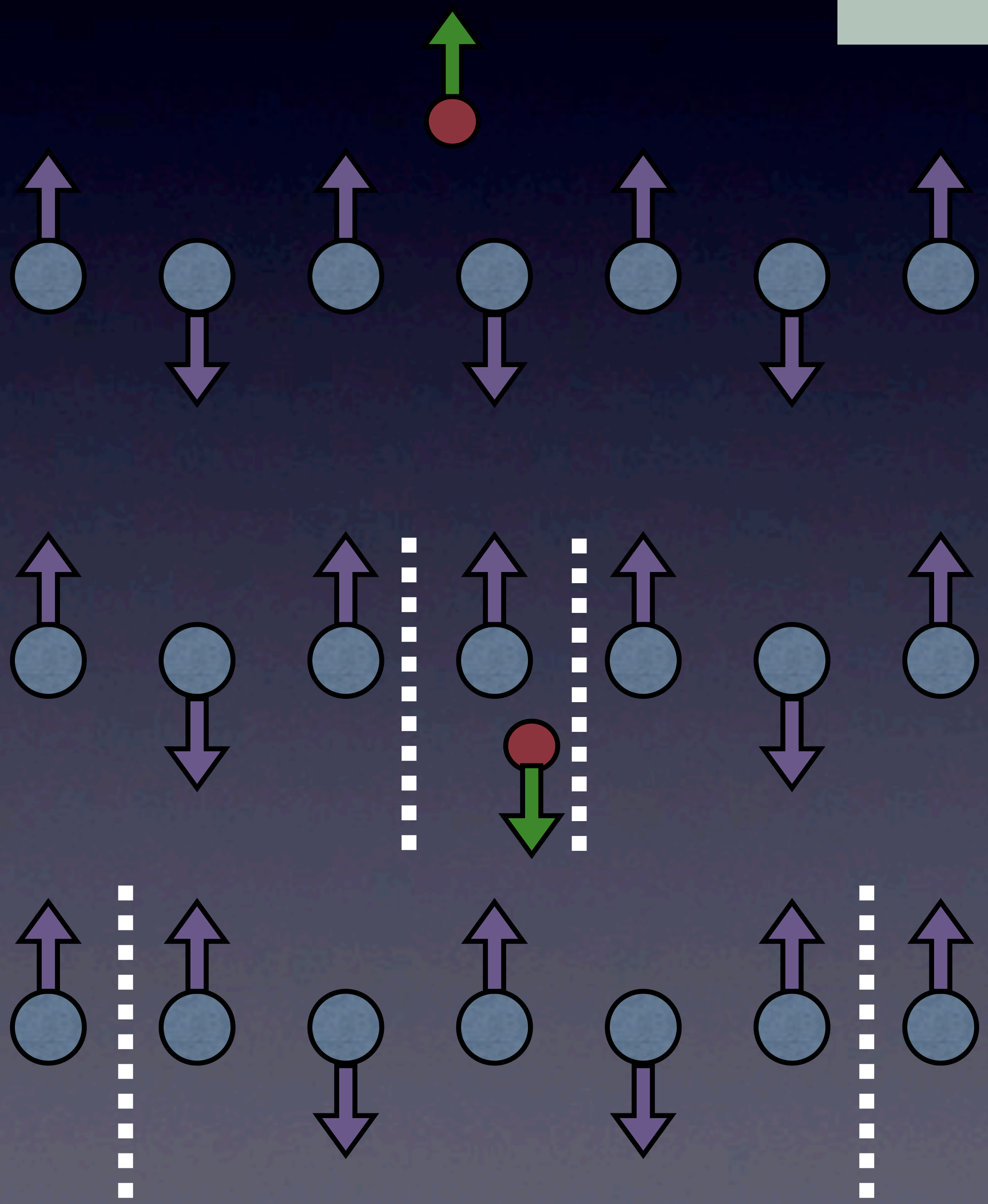

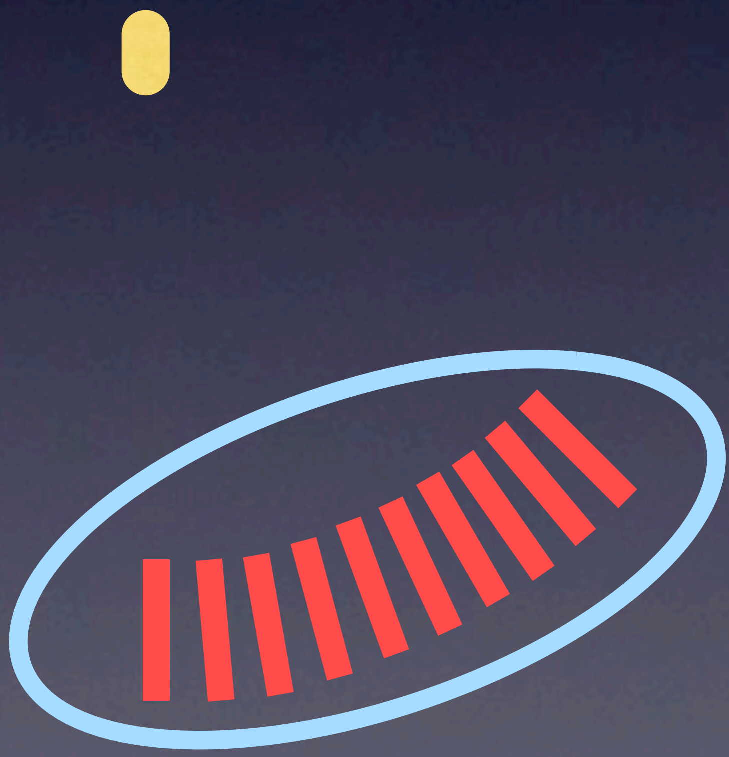

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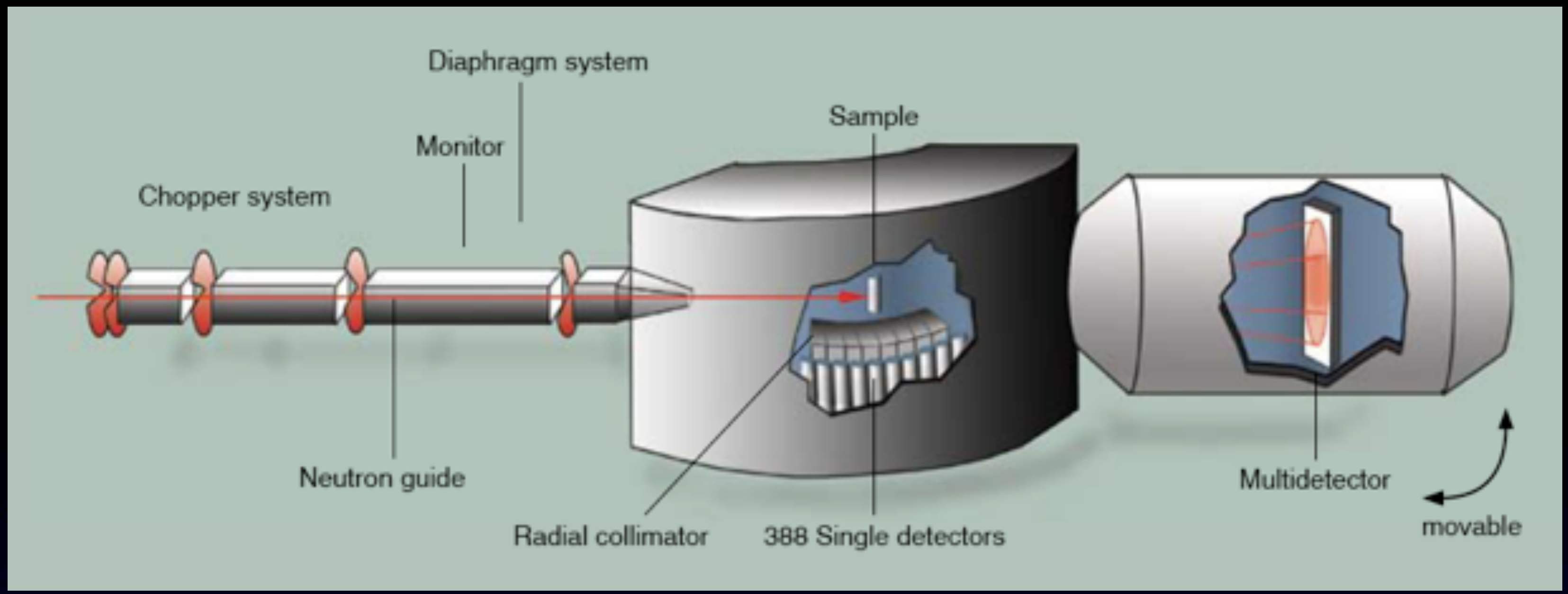

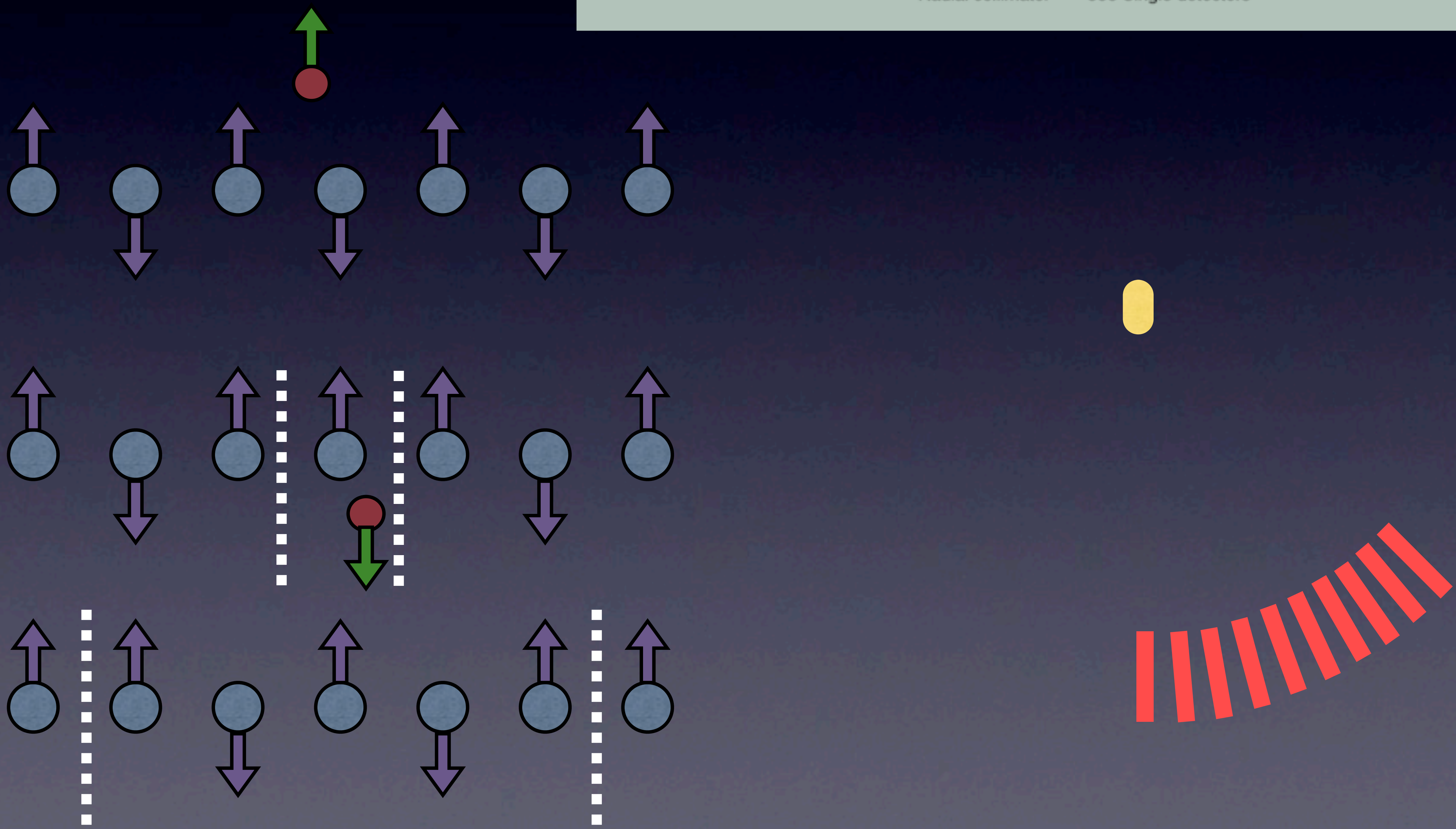

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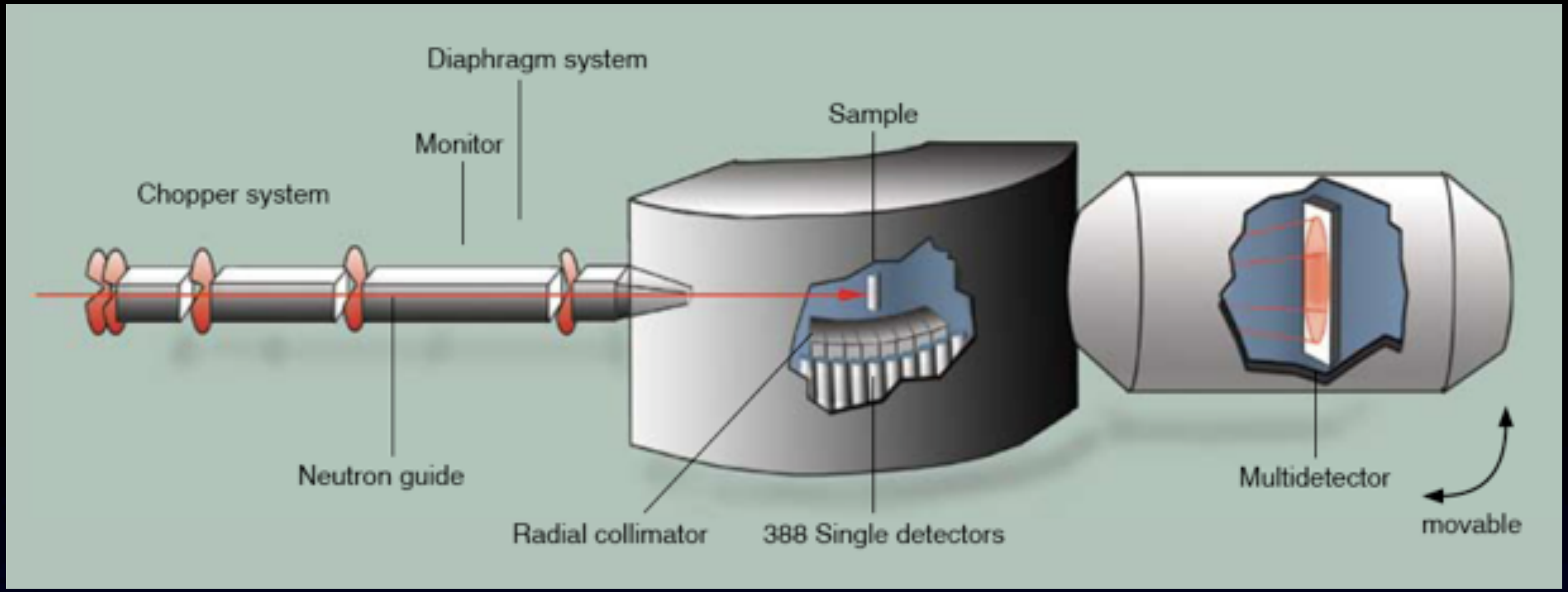

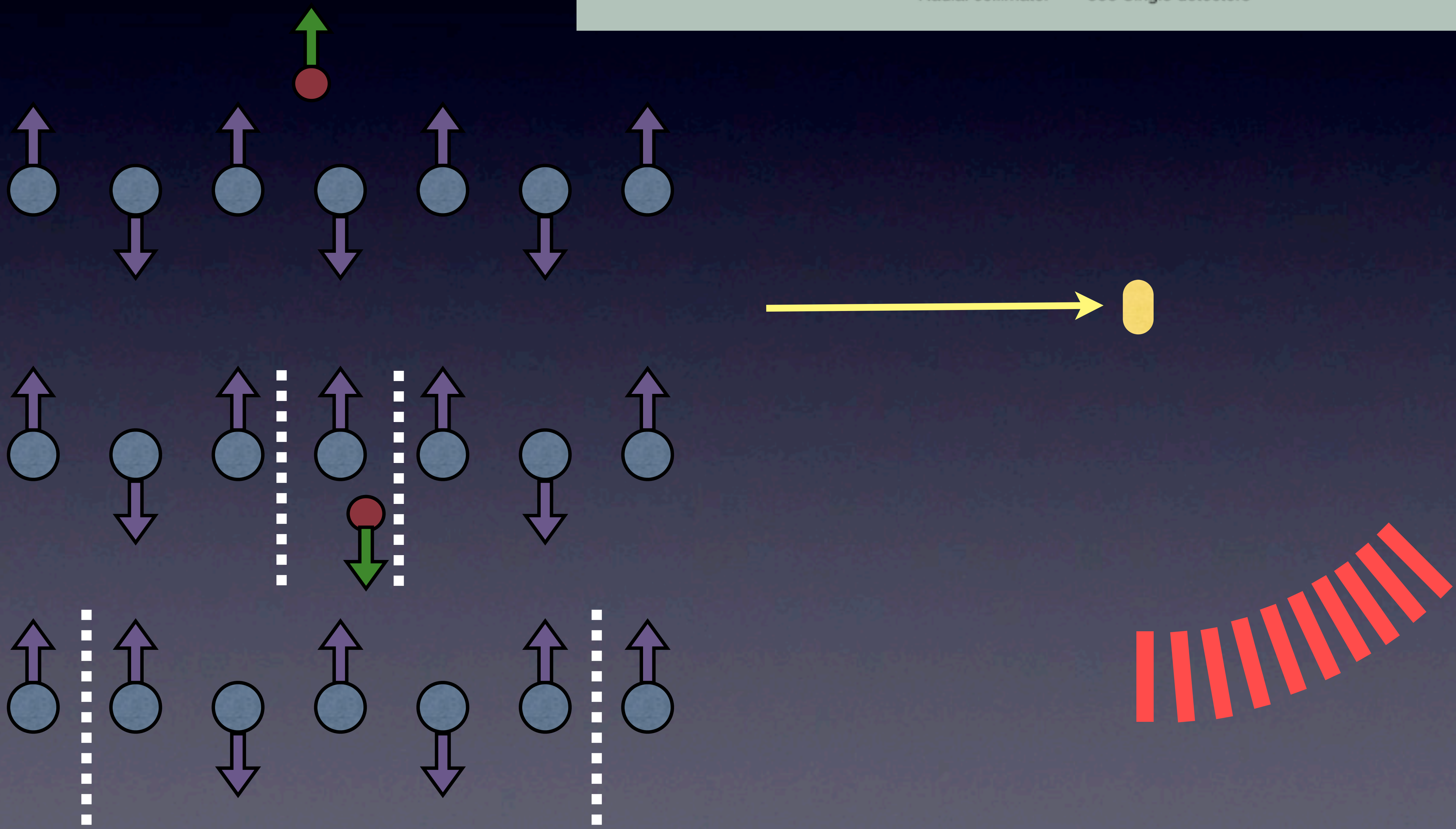

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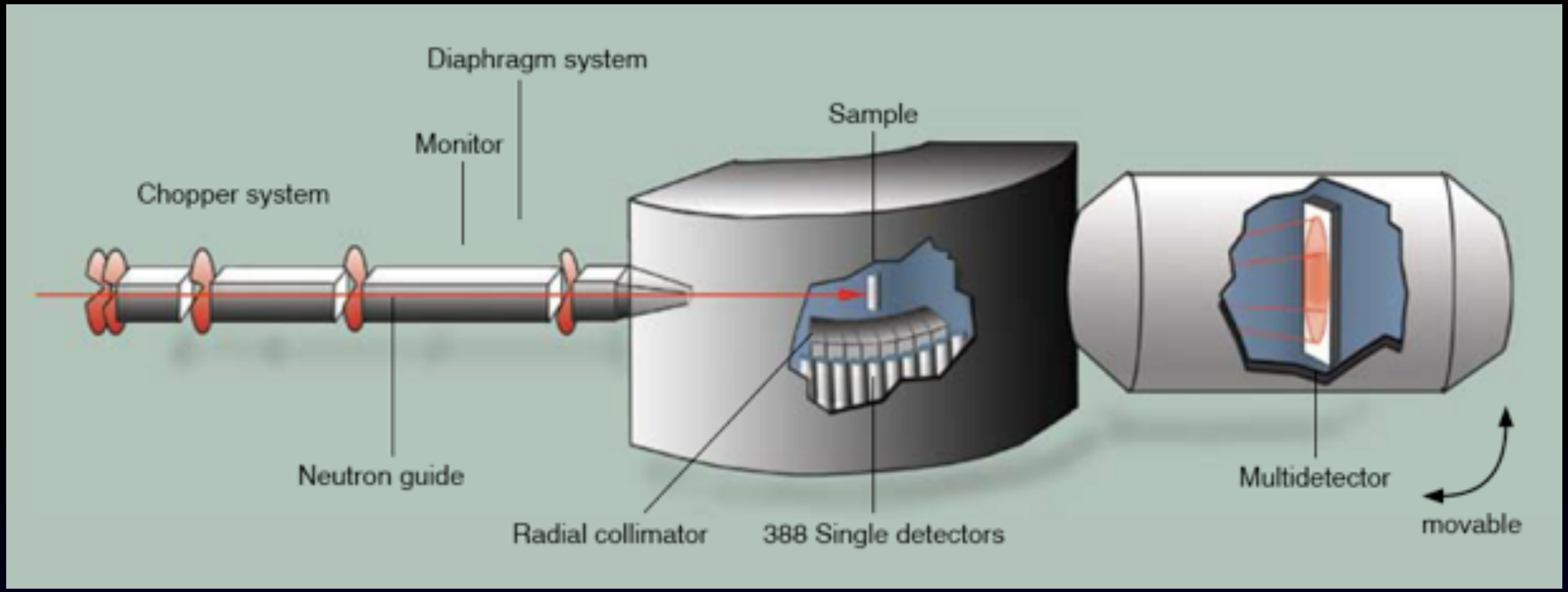
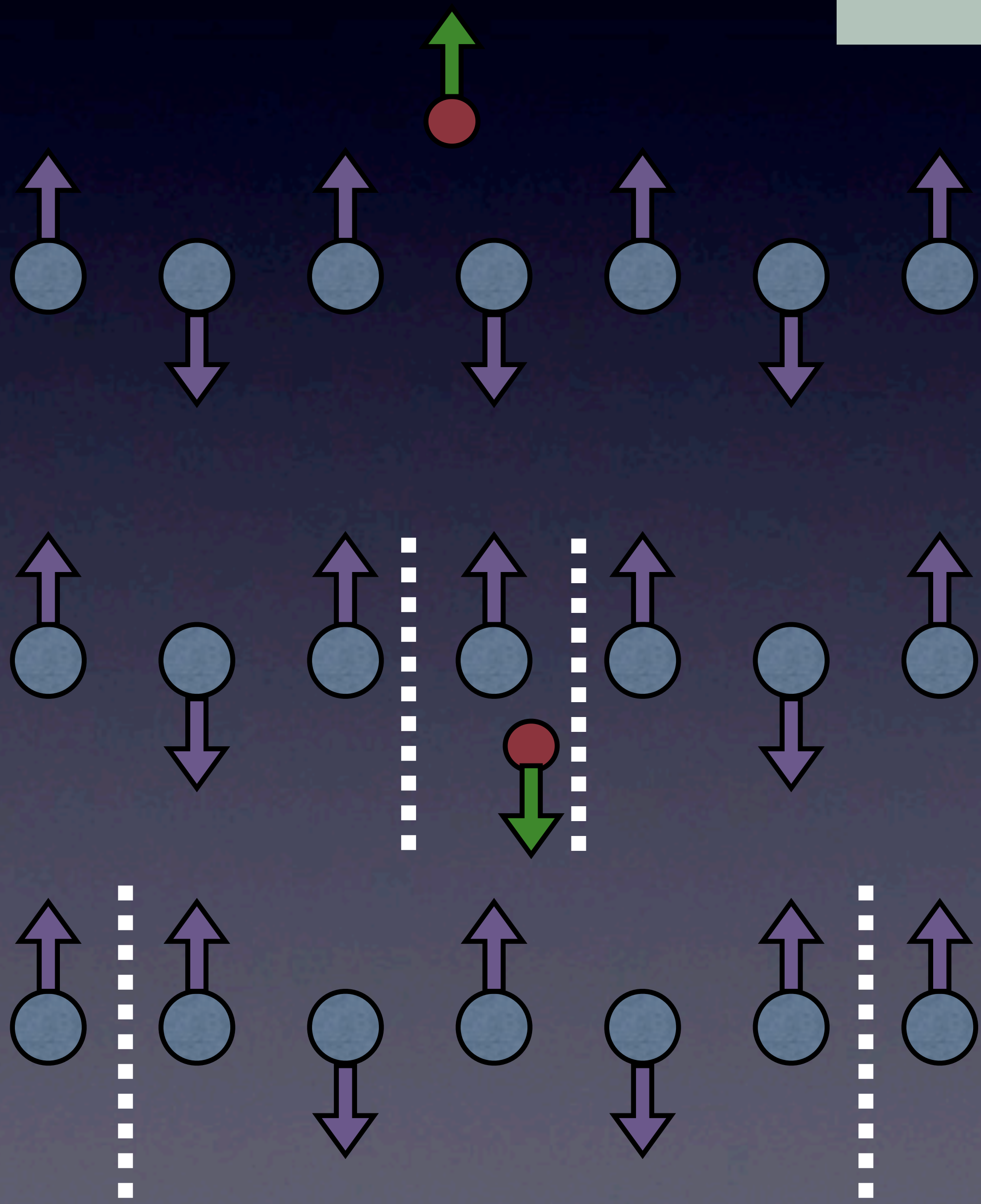
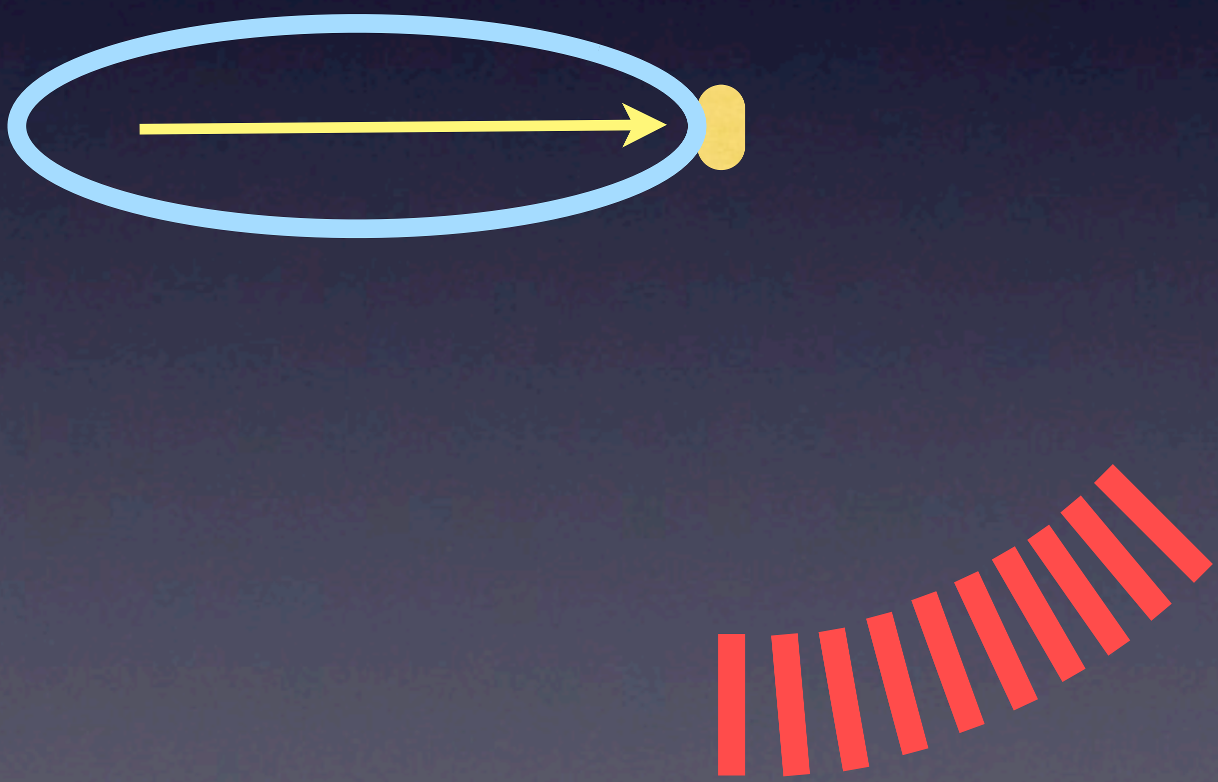


neutrons
from reactor





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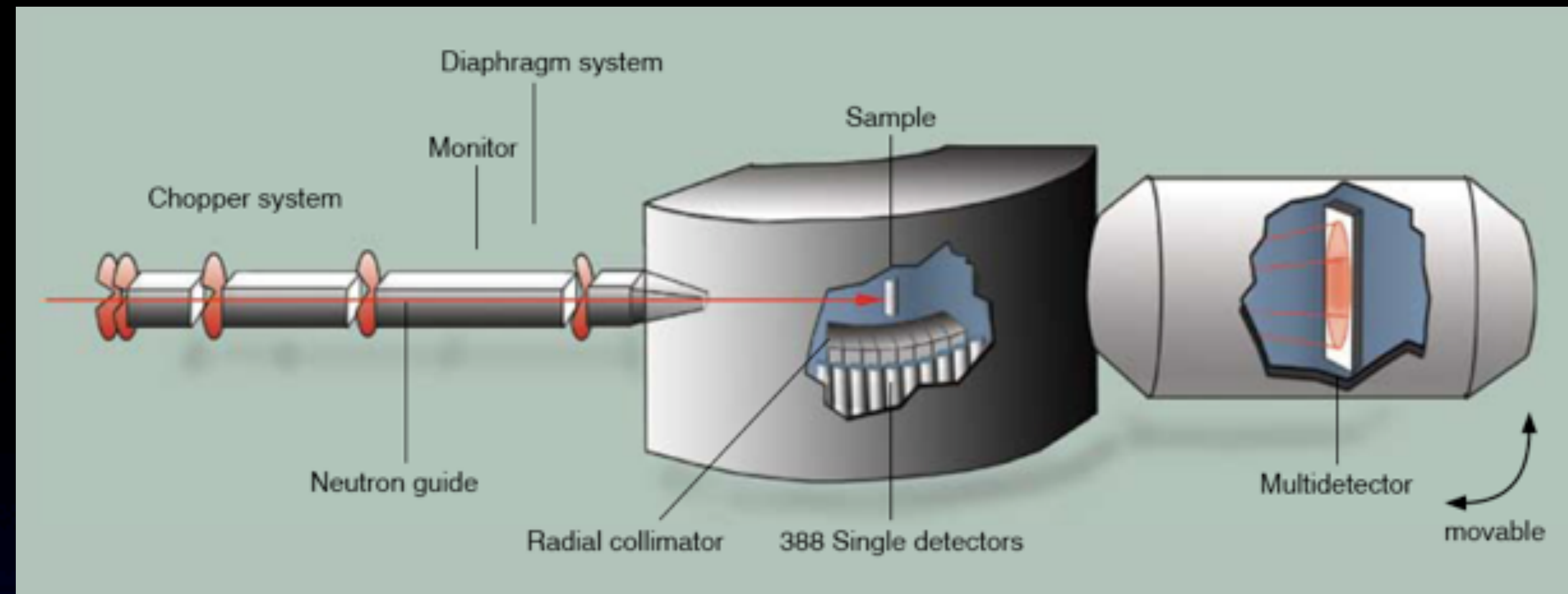

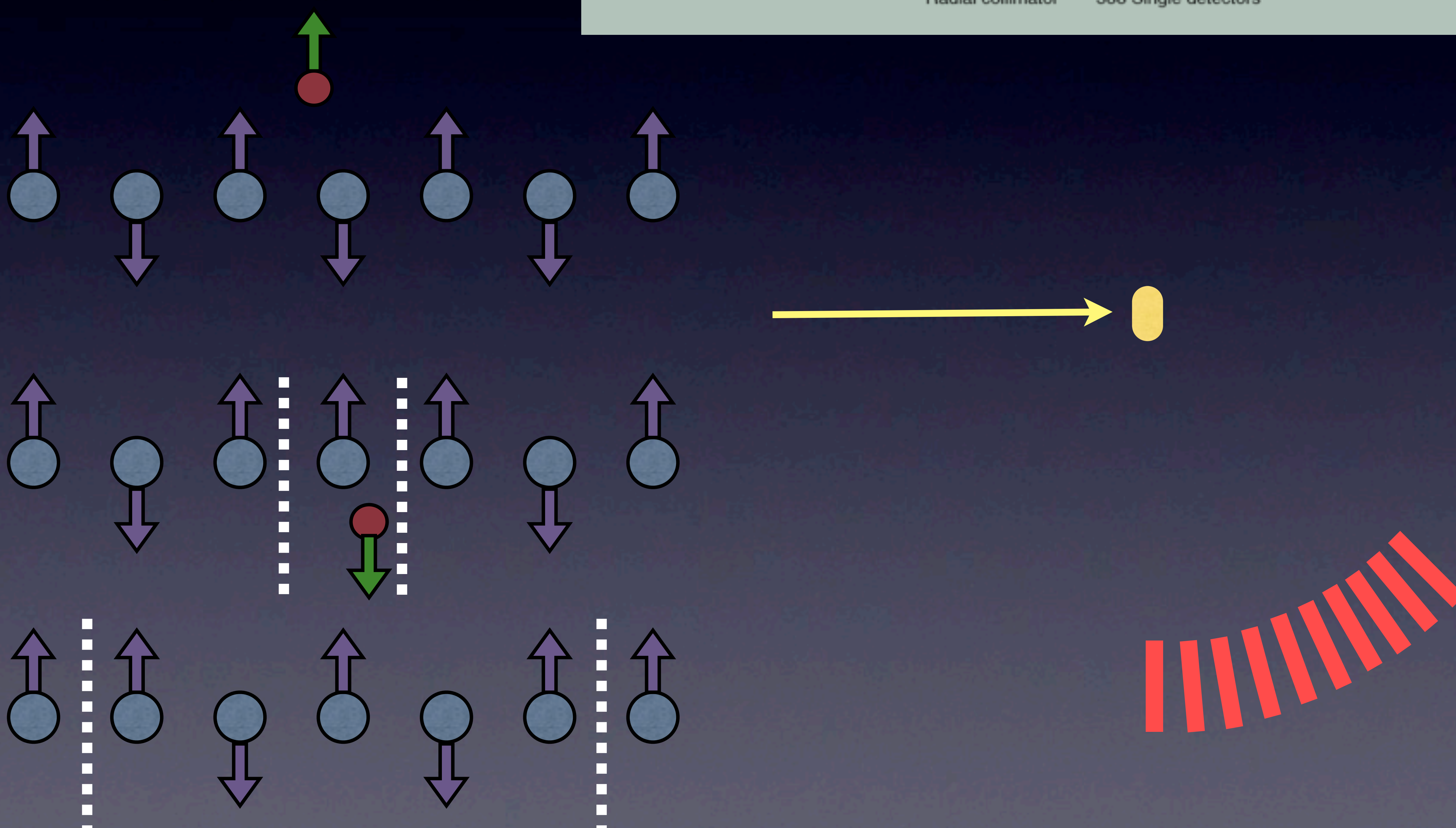

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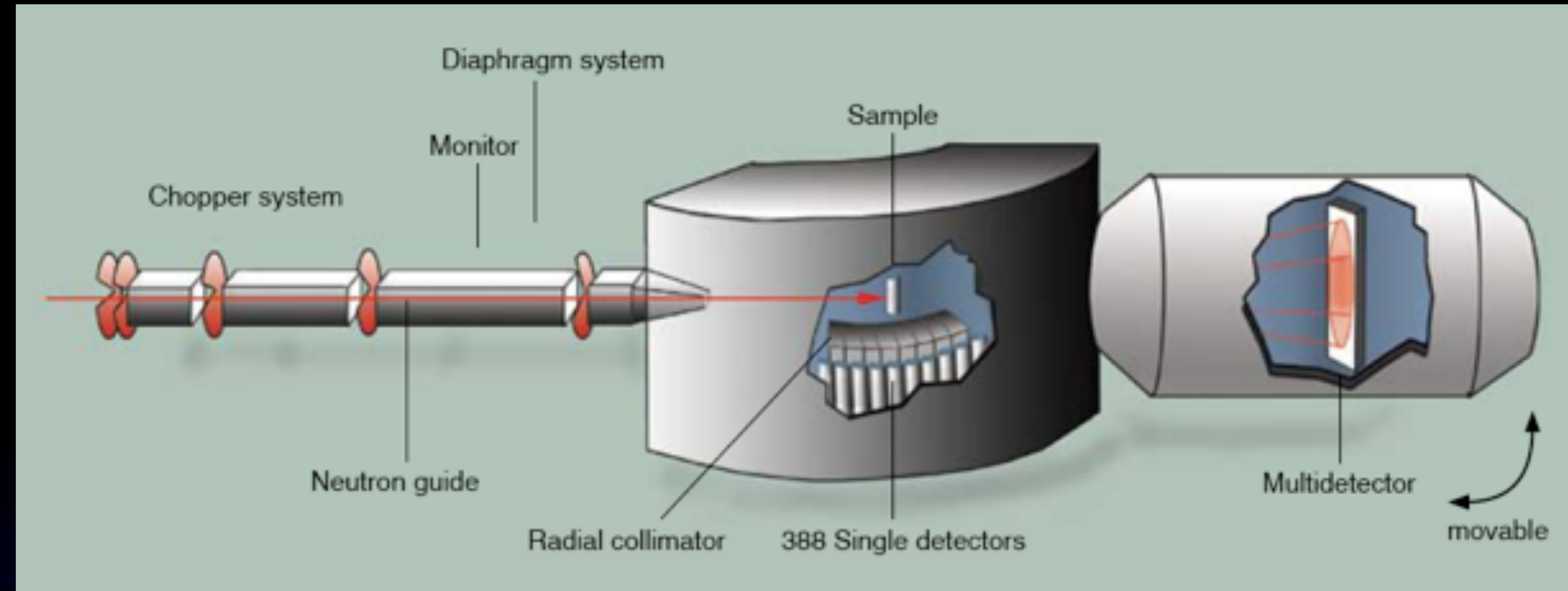
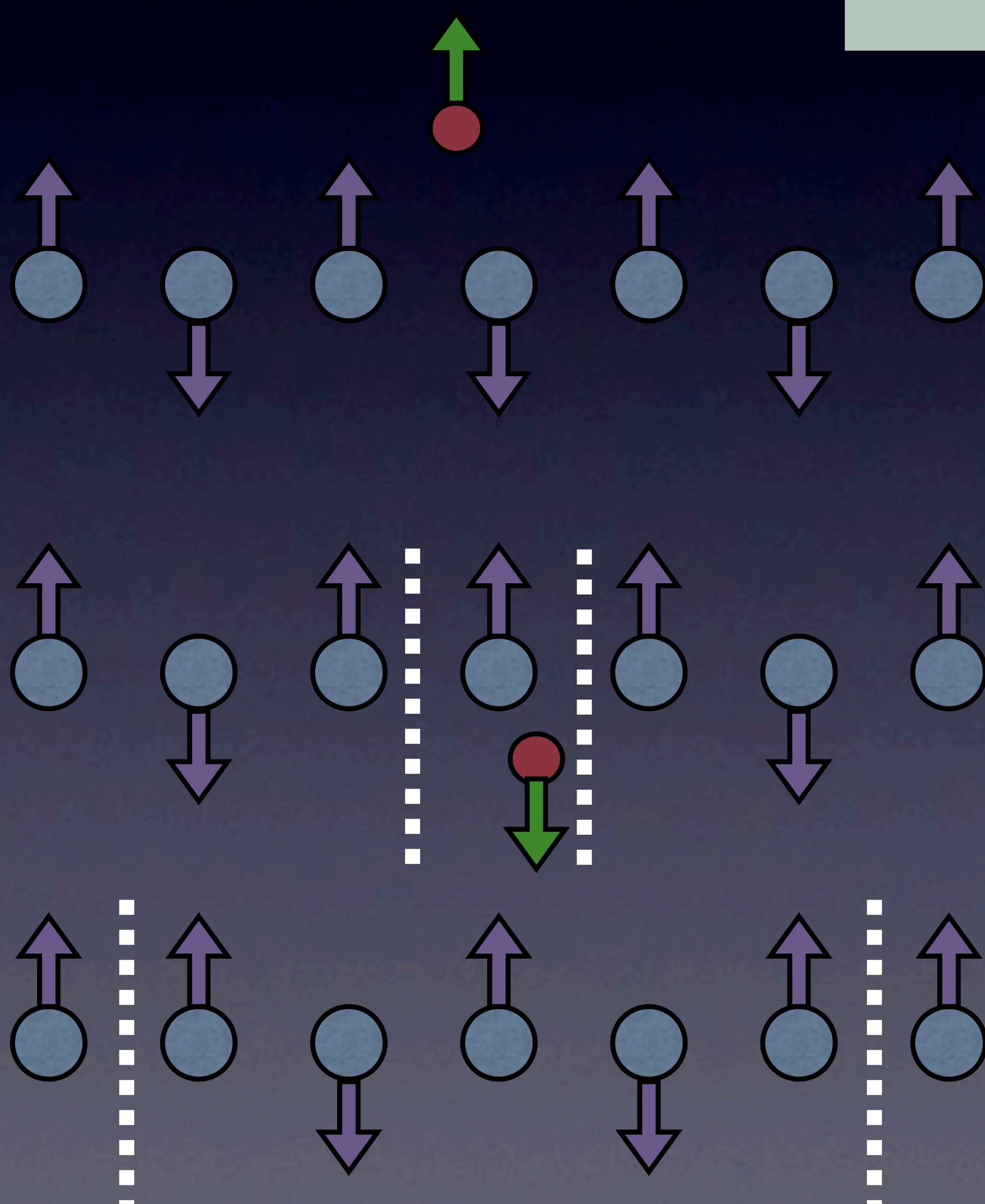
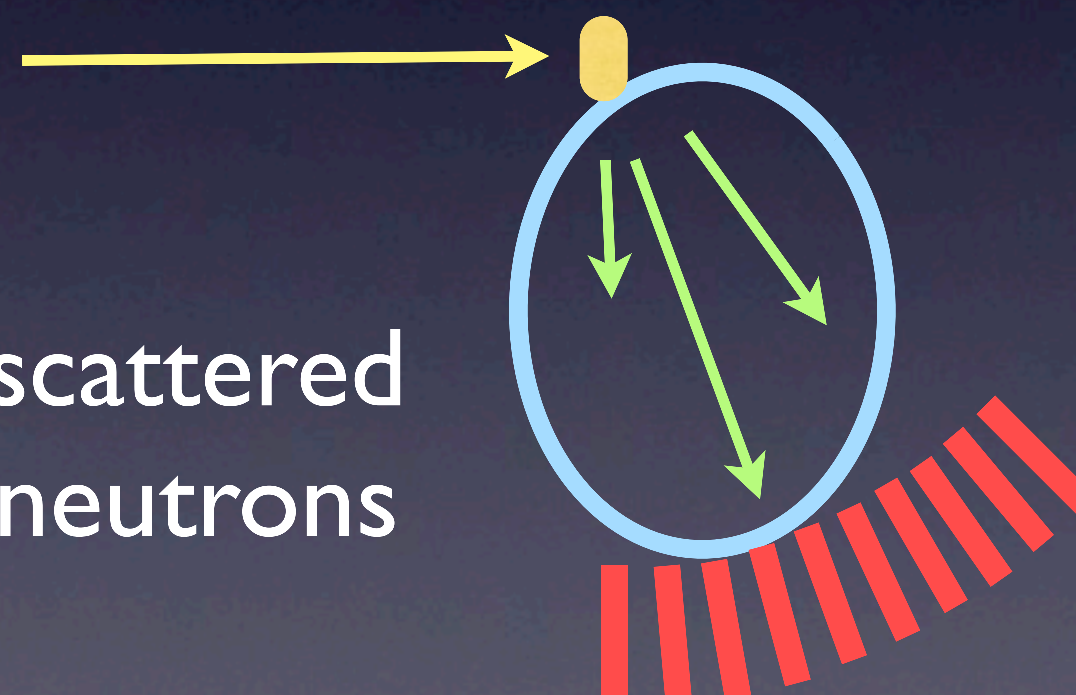




scattered neutrons



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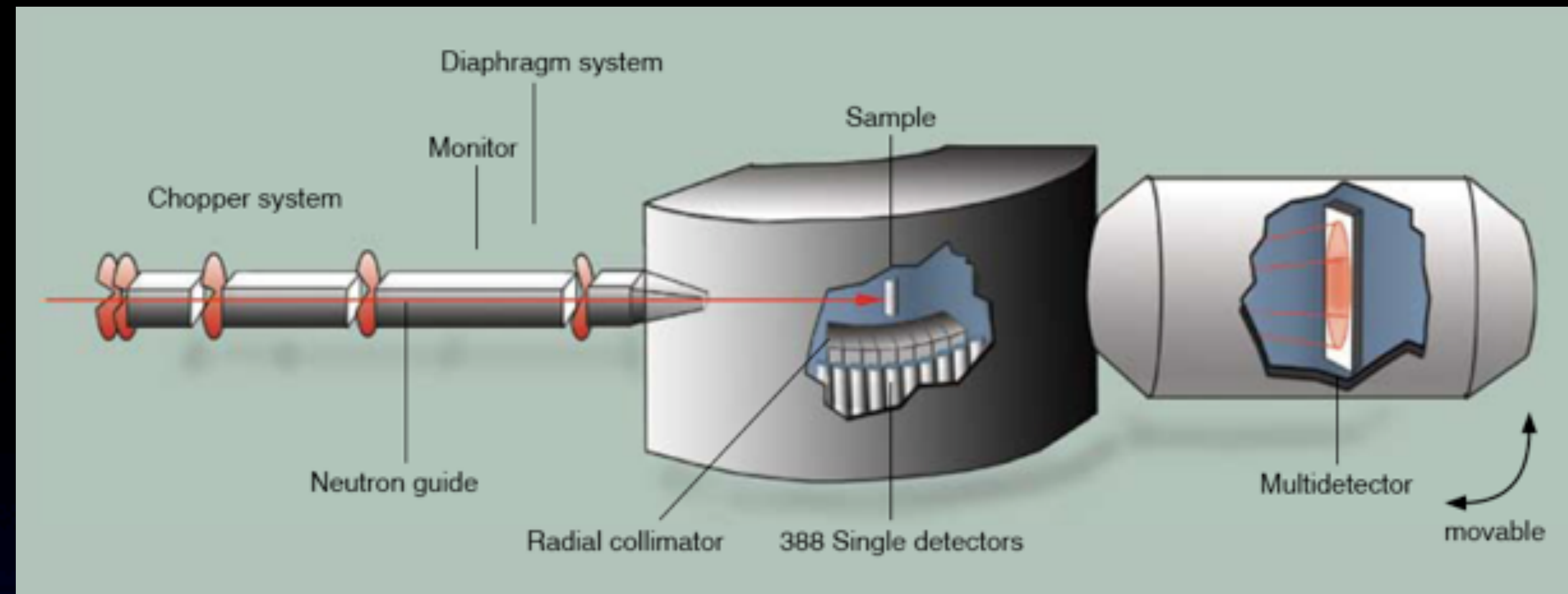

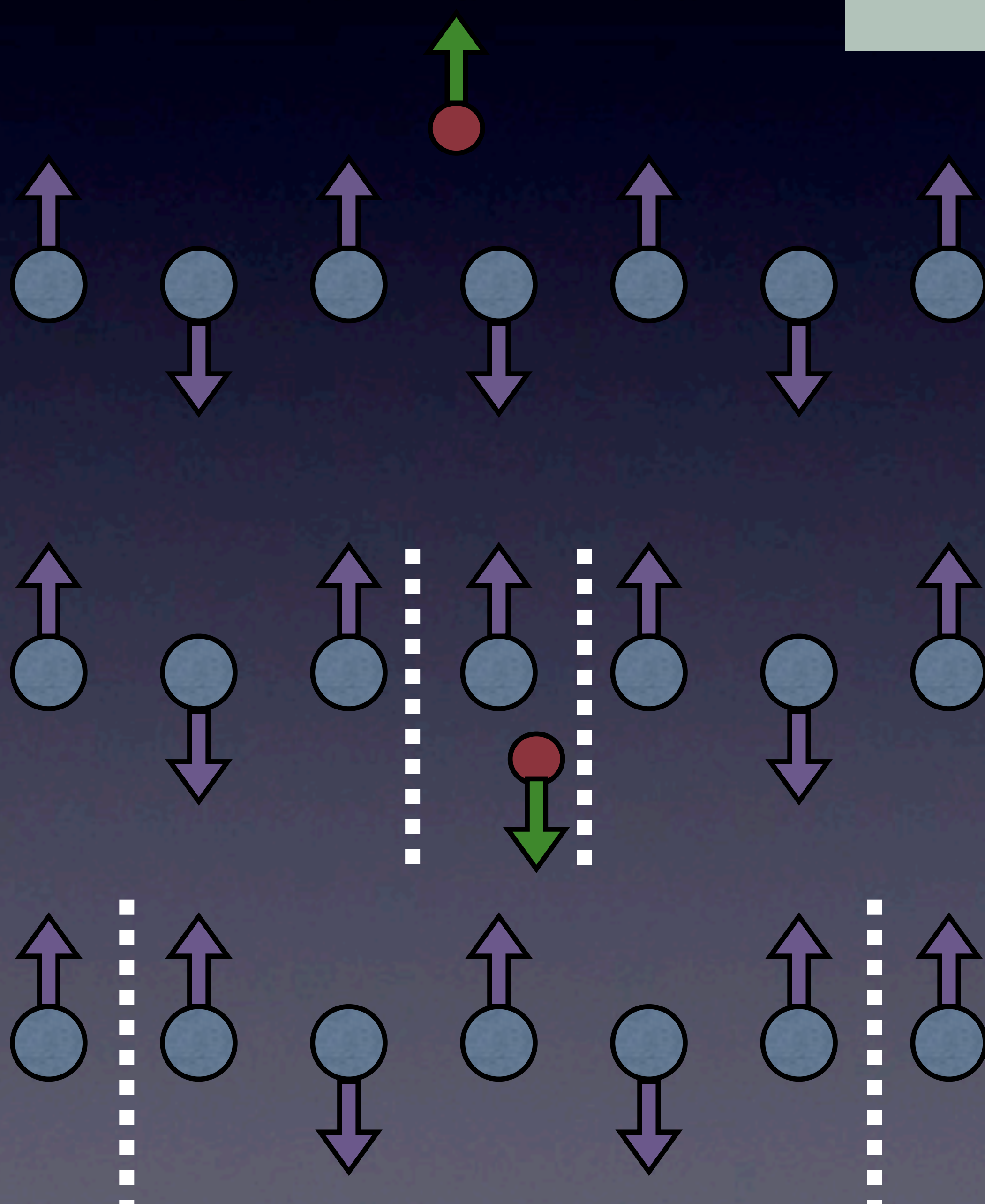

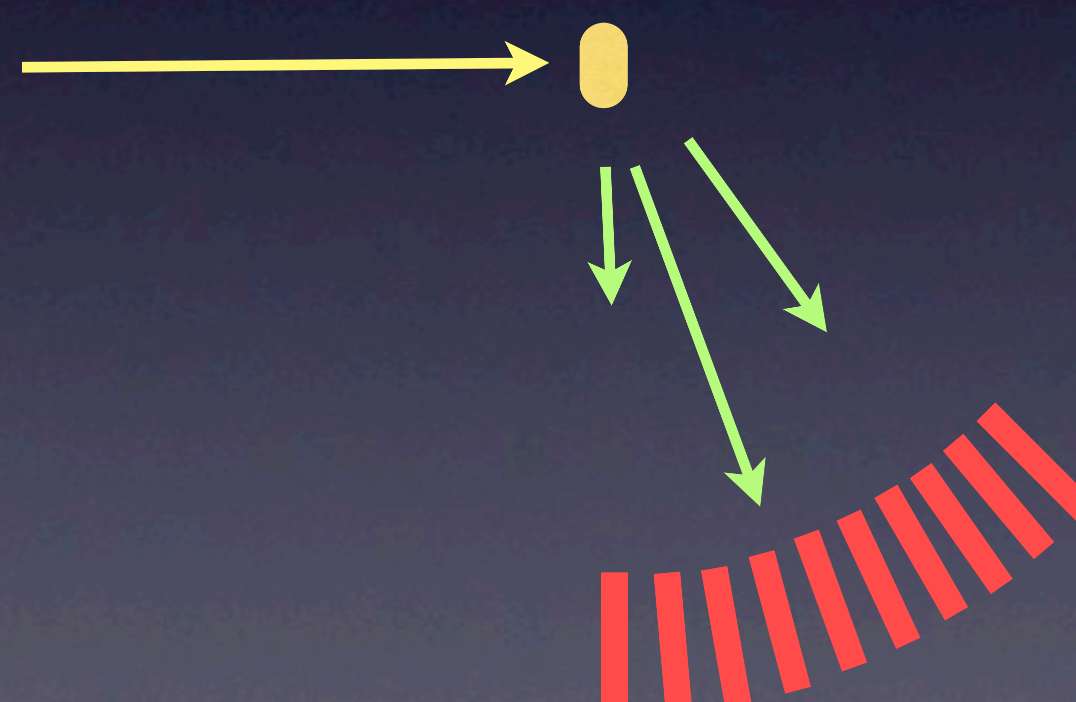

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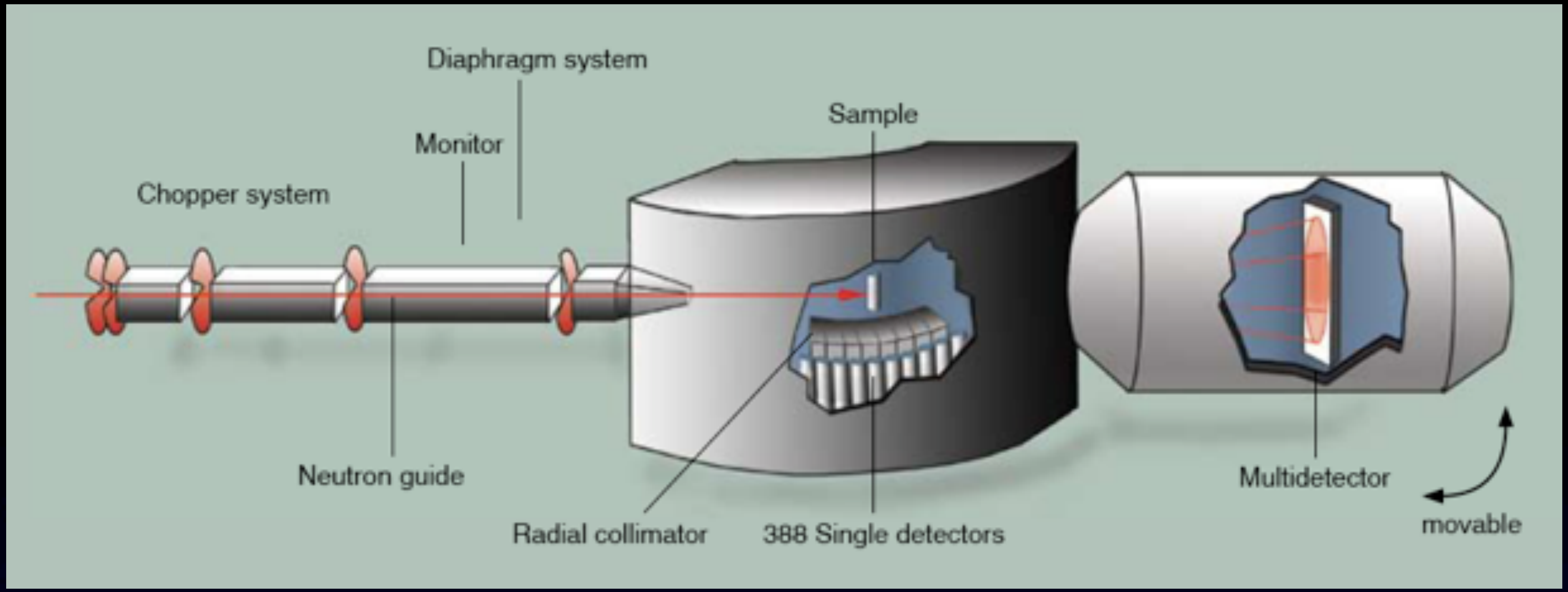

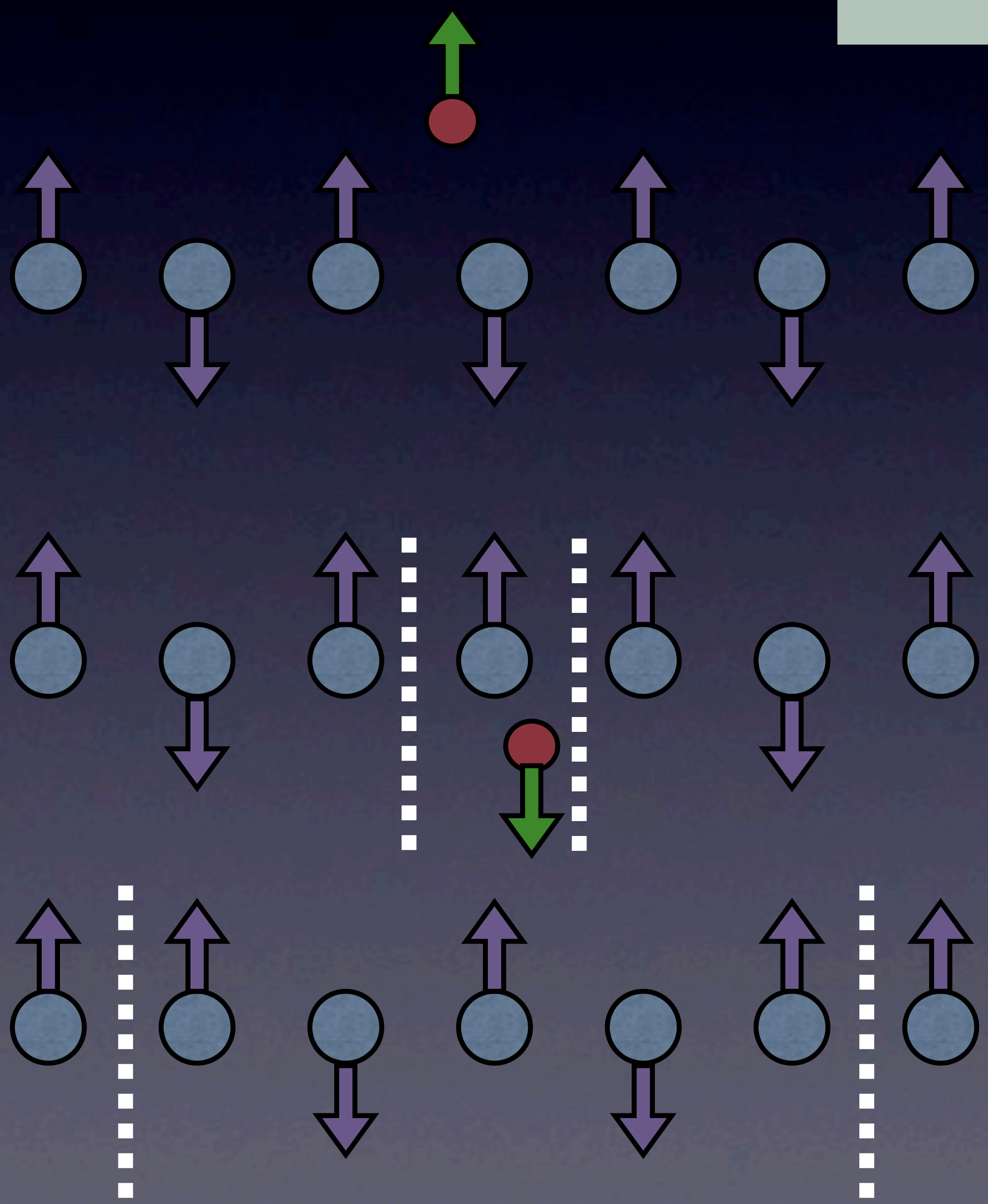

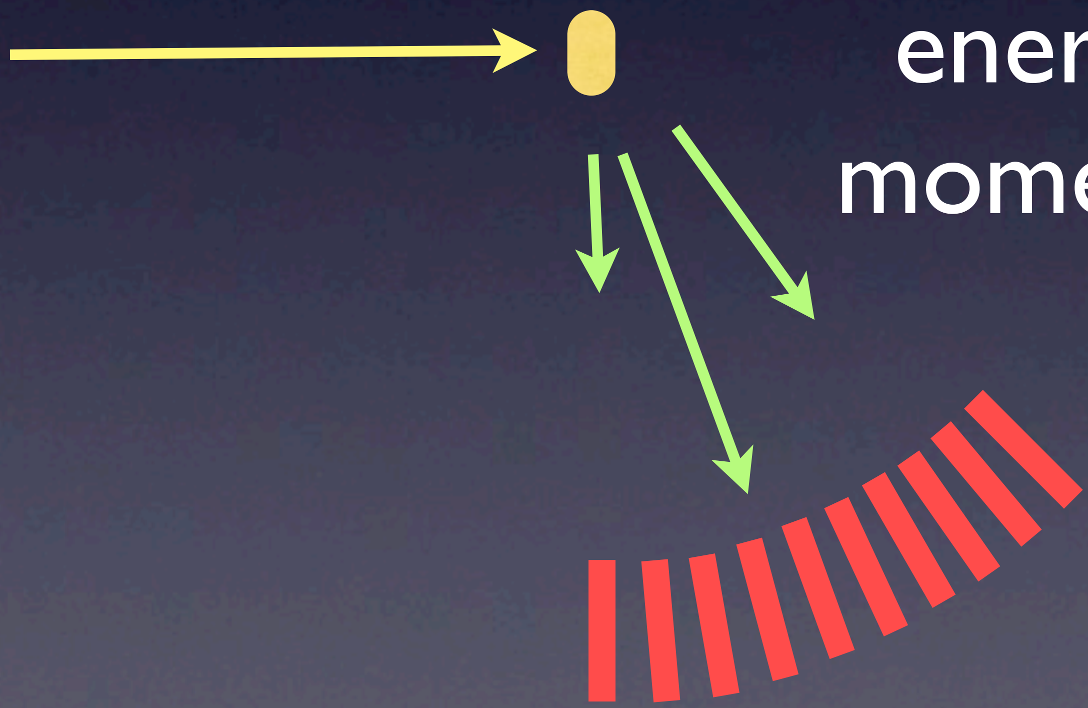

time & direction: energy & momentum

Neutron scattering (HMI, Berlin)

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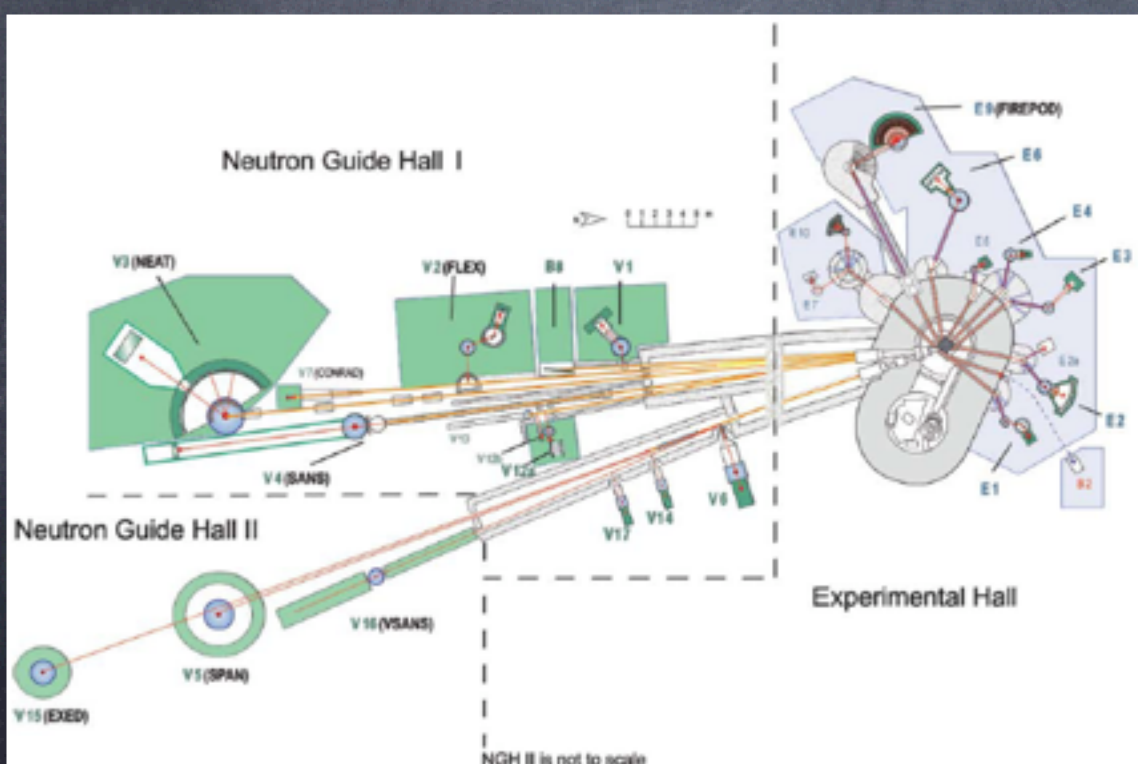

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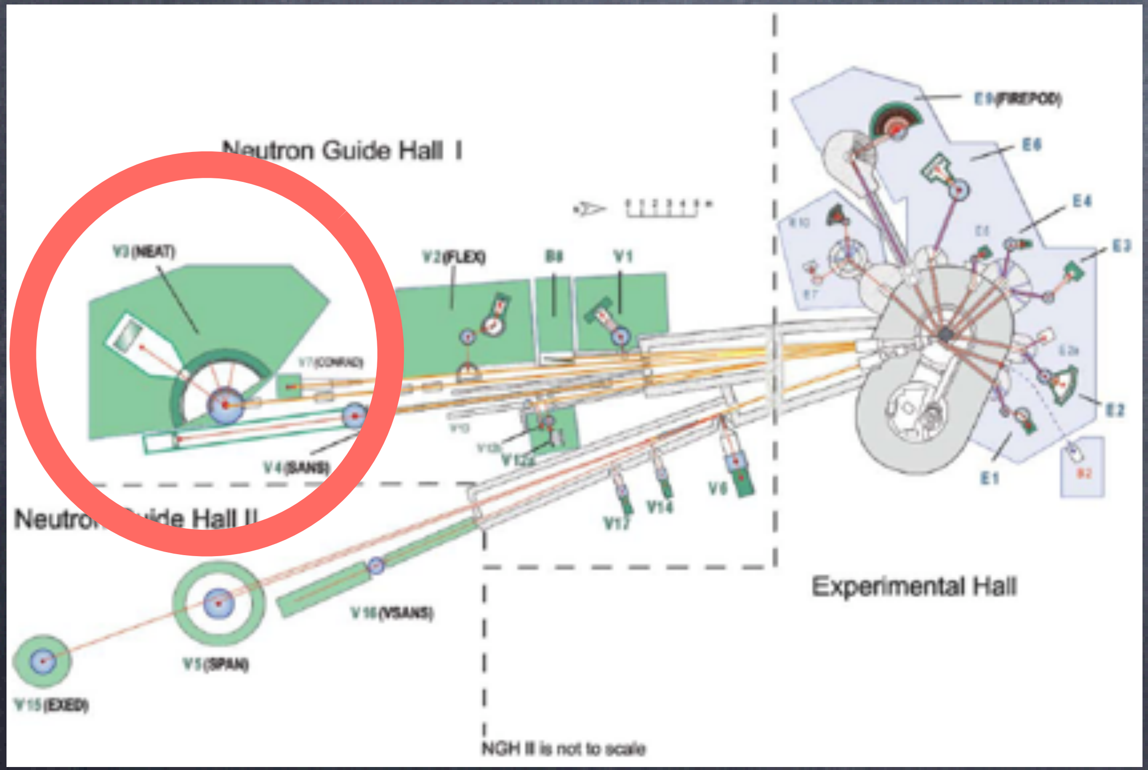
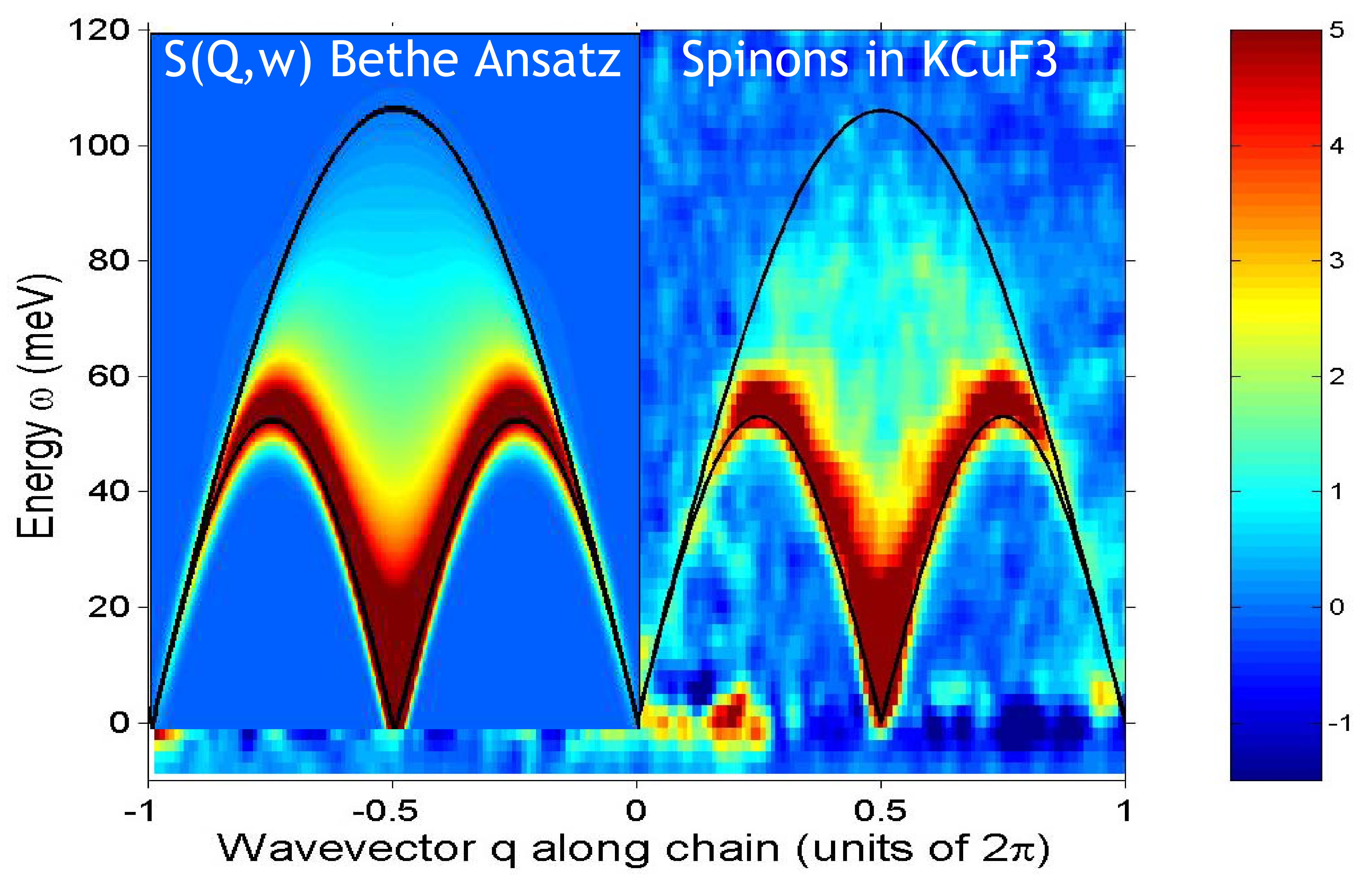
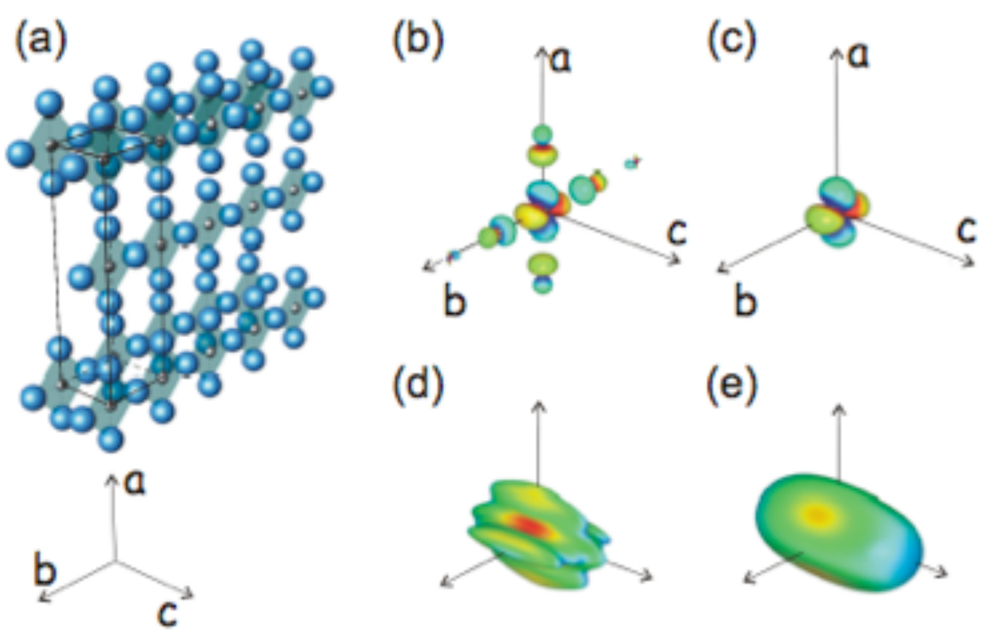
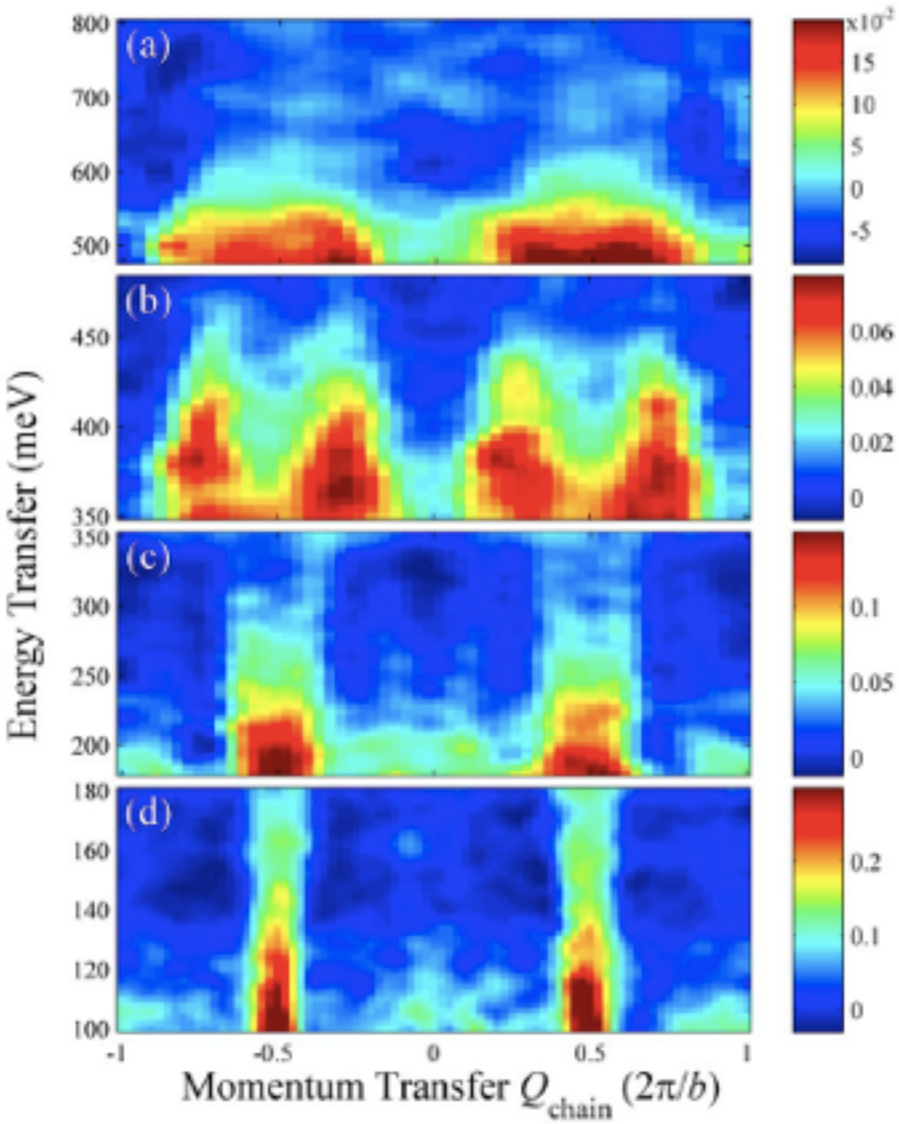


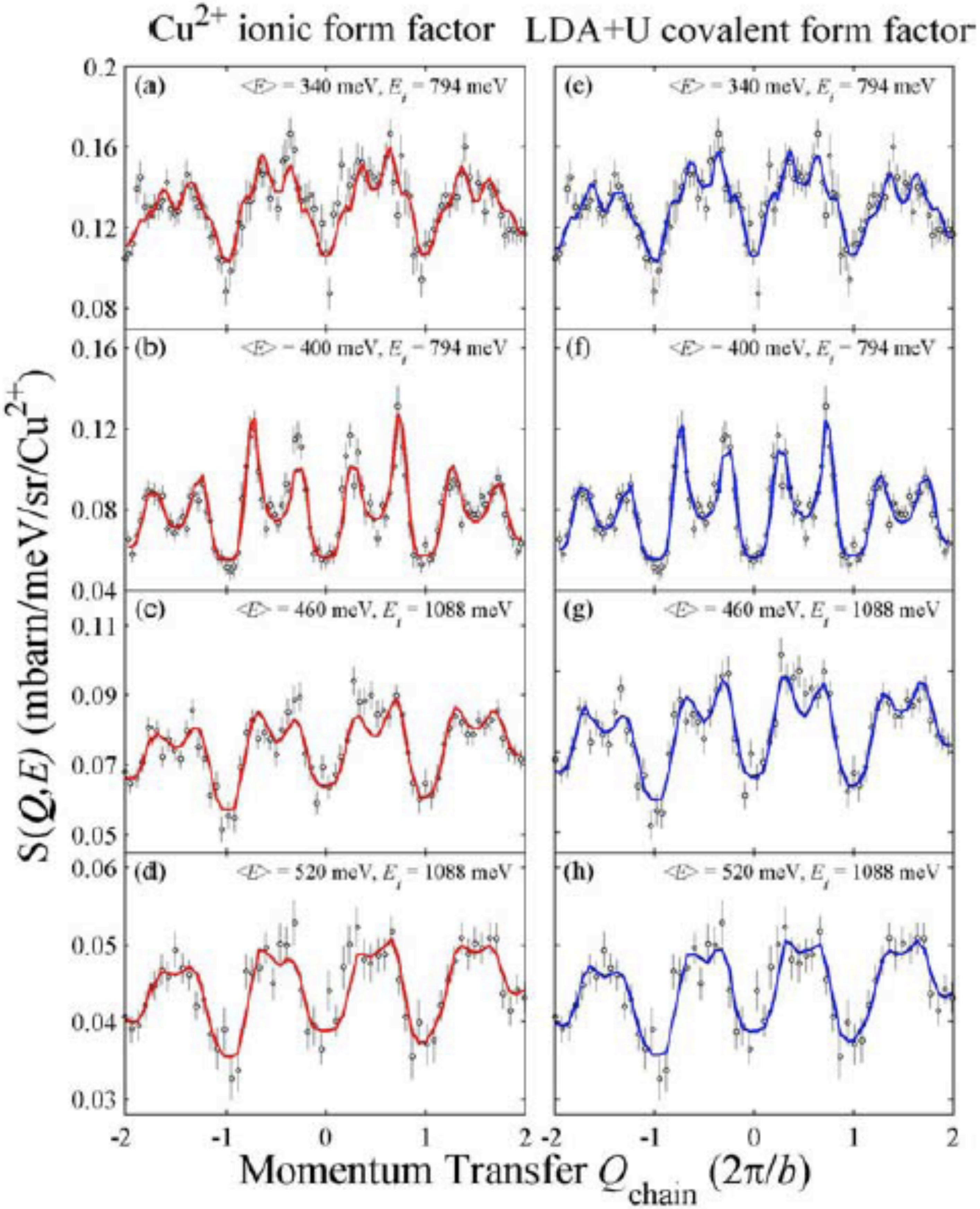
NEAT time-of-flight spectrometer


S(Q,w) Bethe Ansatz
Spinons in KCuF3
Energy ω (meV)
Wavevector q along chain (units of 2π)
120
100
80
60
40
20
0
-1
-0.5
0
0.5
1
5
4
3
2
1
0
-1

Sr_2CuO_3: XXX







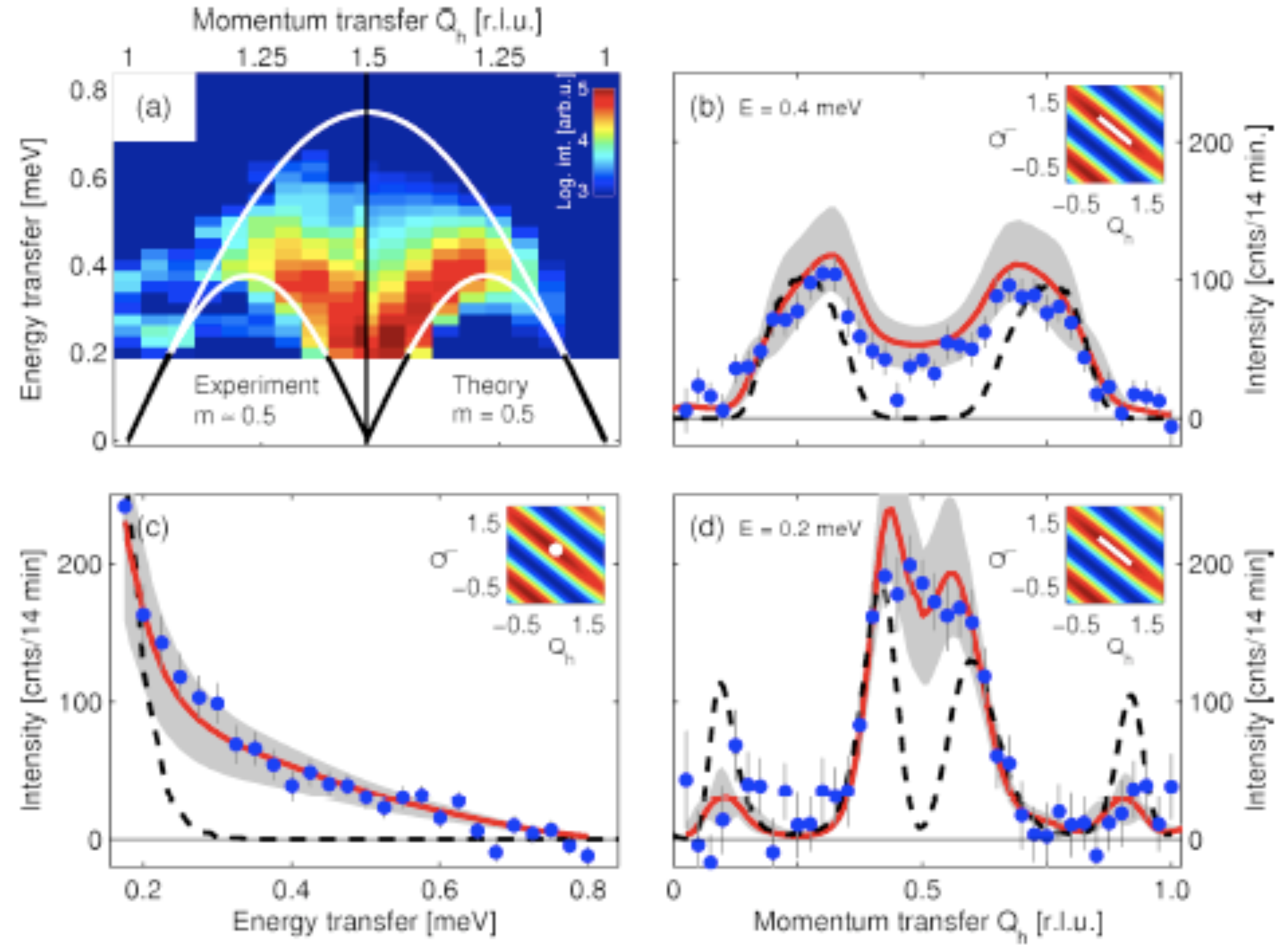
Walters, Perring, JSC, Savici, Gu, Lee, Ku, Zaliznyak, NatPhys 2009

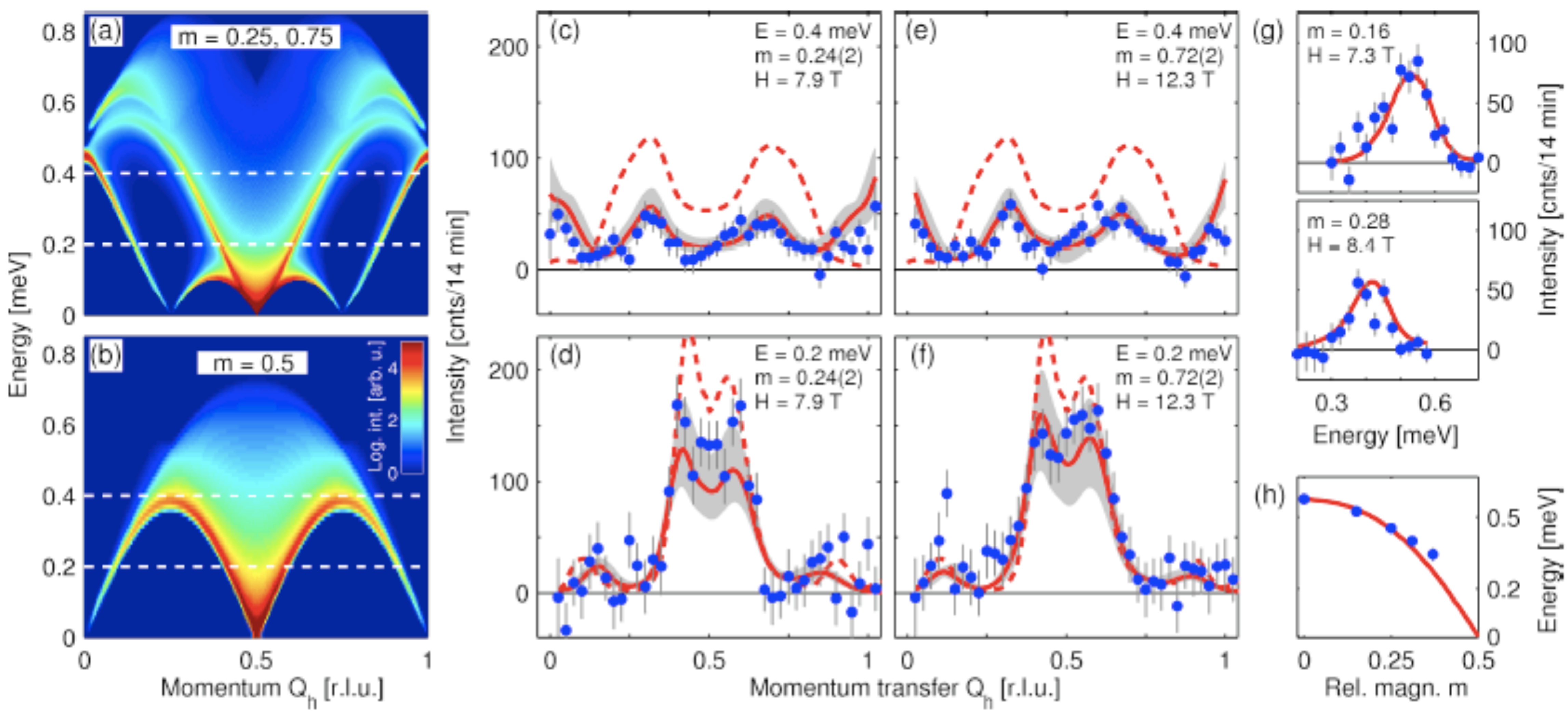
$(C_5D_{12}N)_2CuBr_4$

XXZ AFM at anisotropy $\Delta = 1/2$

B. Thielemann, Ch. Rüegg, H. M. Rønnow, A. M. Läuchli, J.-S. Caux, B. Normand, D. Biner, K. W. Krämer, H.-U. Güdel, J. Stahn, K. Habicht, K. Kiefer, M. Boehm, D. F. McMorrow, J. Mesot,

PRL, 2009

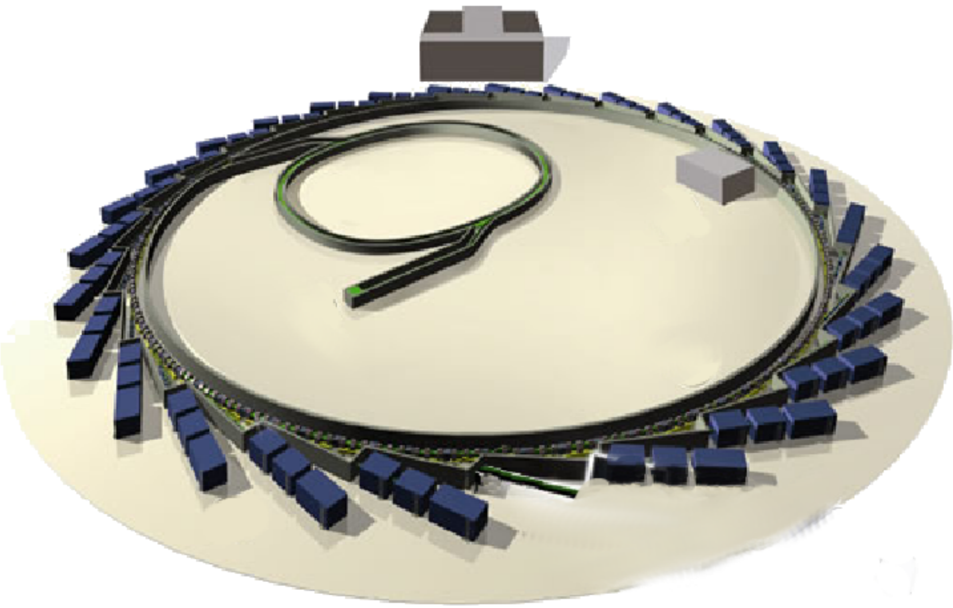


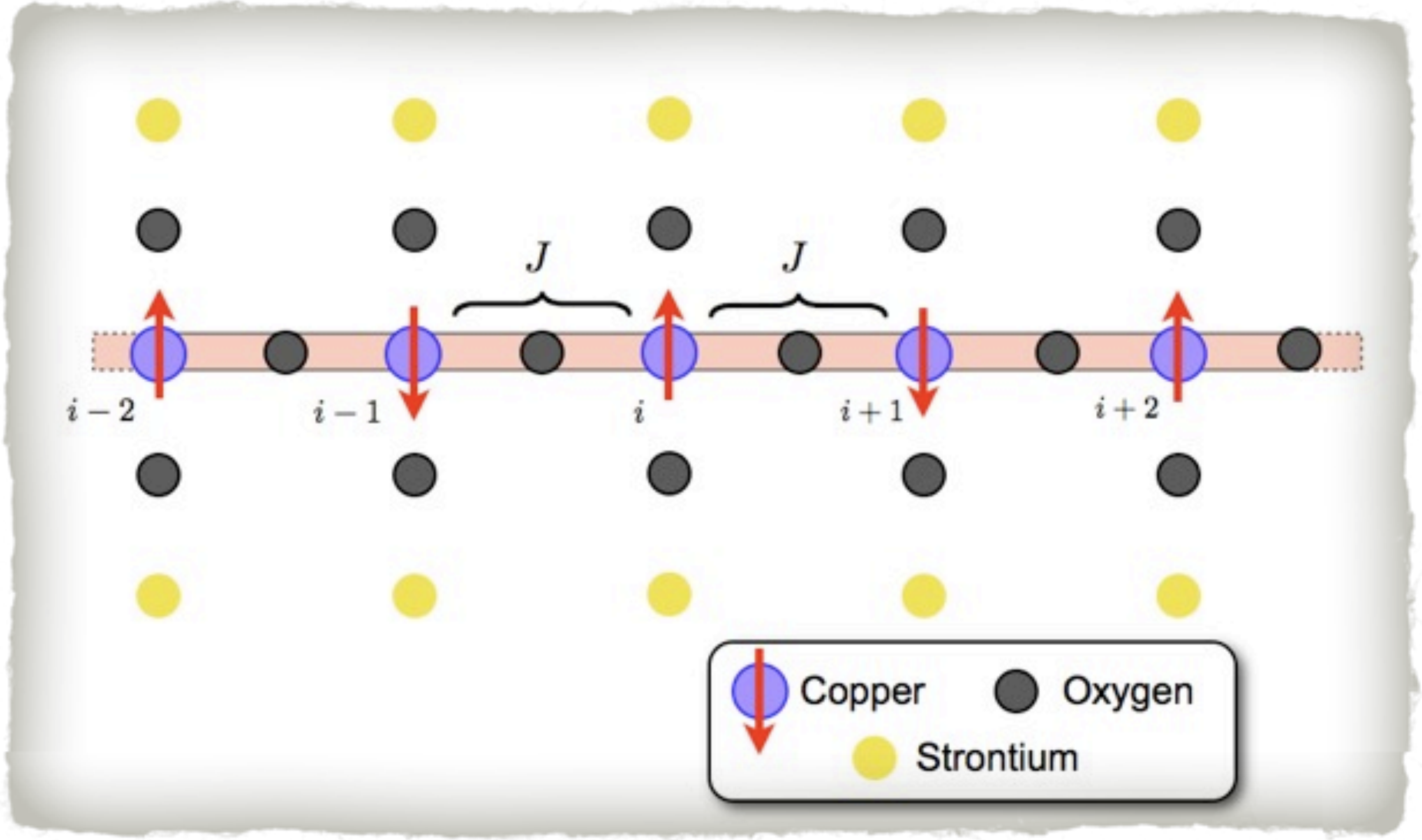


New experimental method: RIXS

(Resonant Inelastic X-ray Scattering)

Synchrotron





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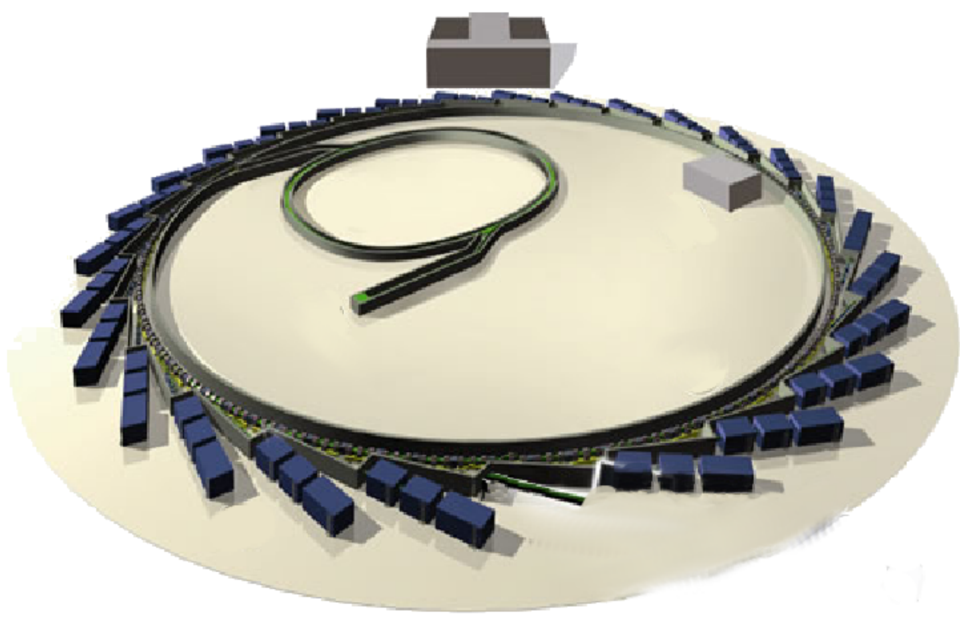

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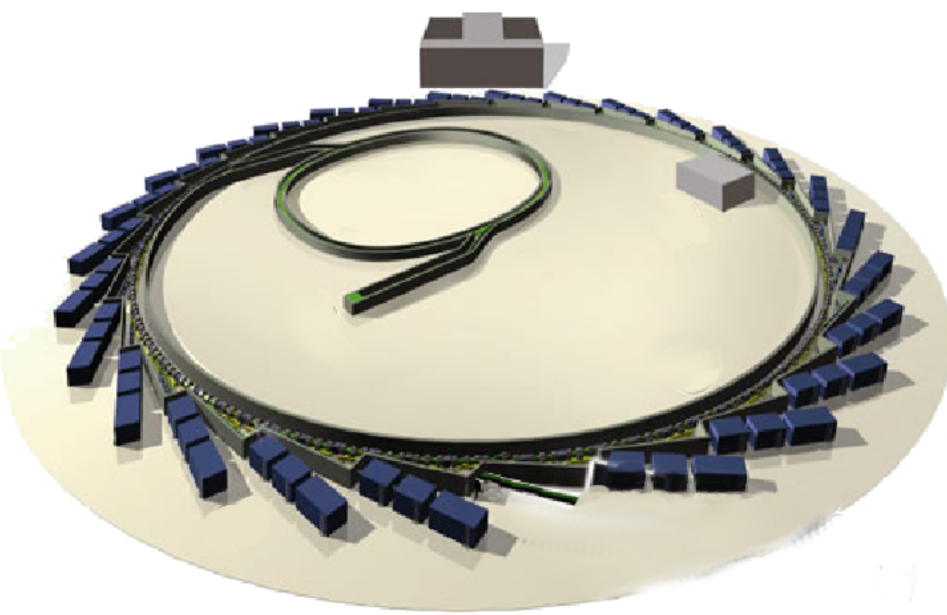
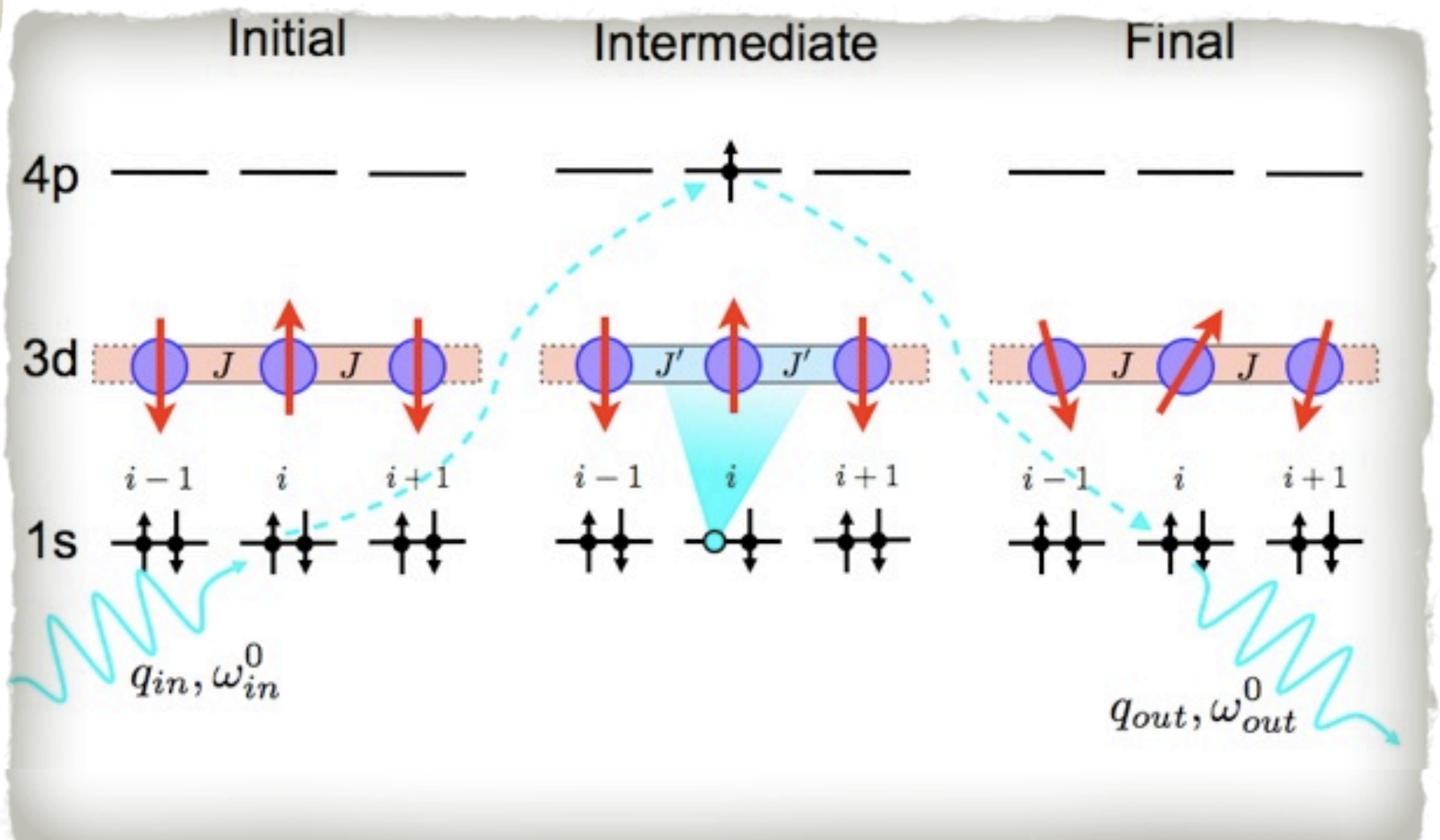


X-ray induces a 1s-4p transition on copper, modifying exchange term



Energy- and momentum-dependent scattering amplitude:

$$S^{RIXS}(k,\omega) = \frac{2\pi}{N} \sum_{\alpha} |\langle \alpha | \sum_j e^{-ikj} S^z_j S^z_{j+1} |GS\rangle|^2 \delta(\omega - E_\alpha + E_0)$$

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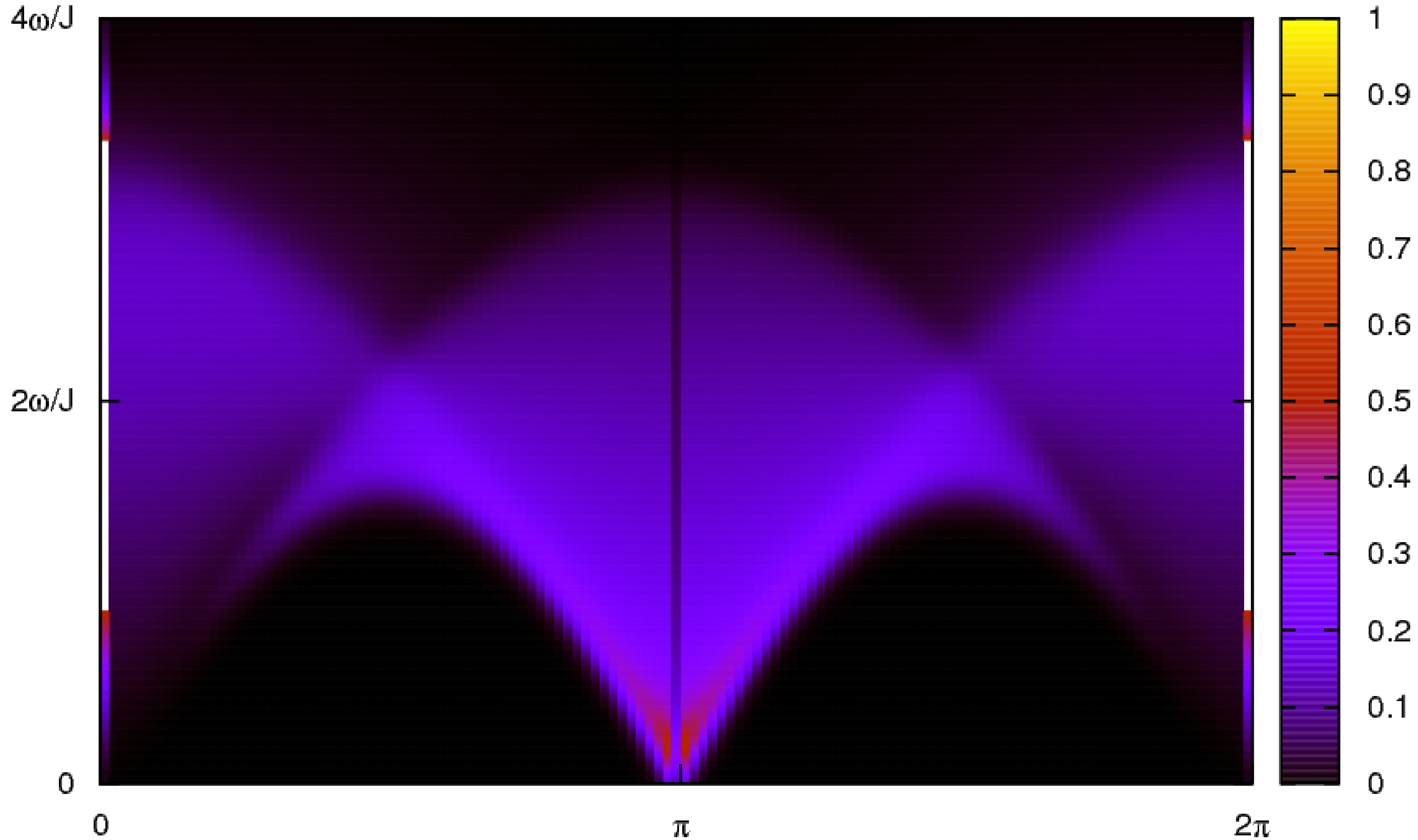

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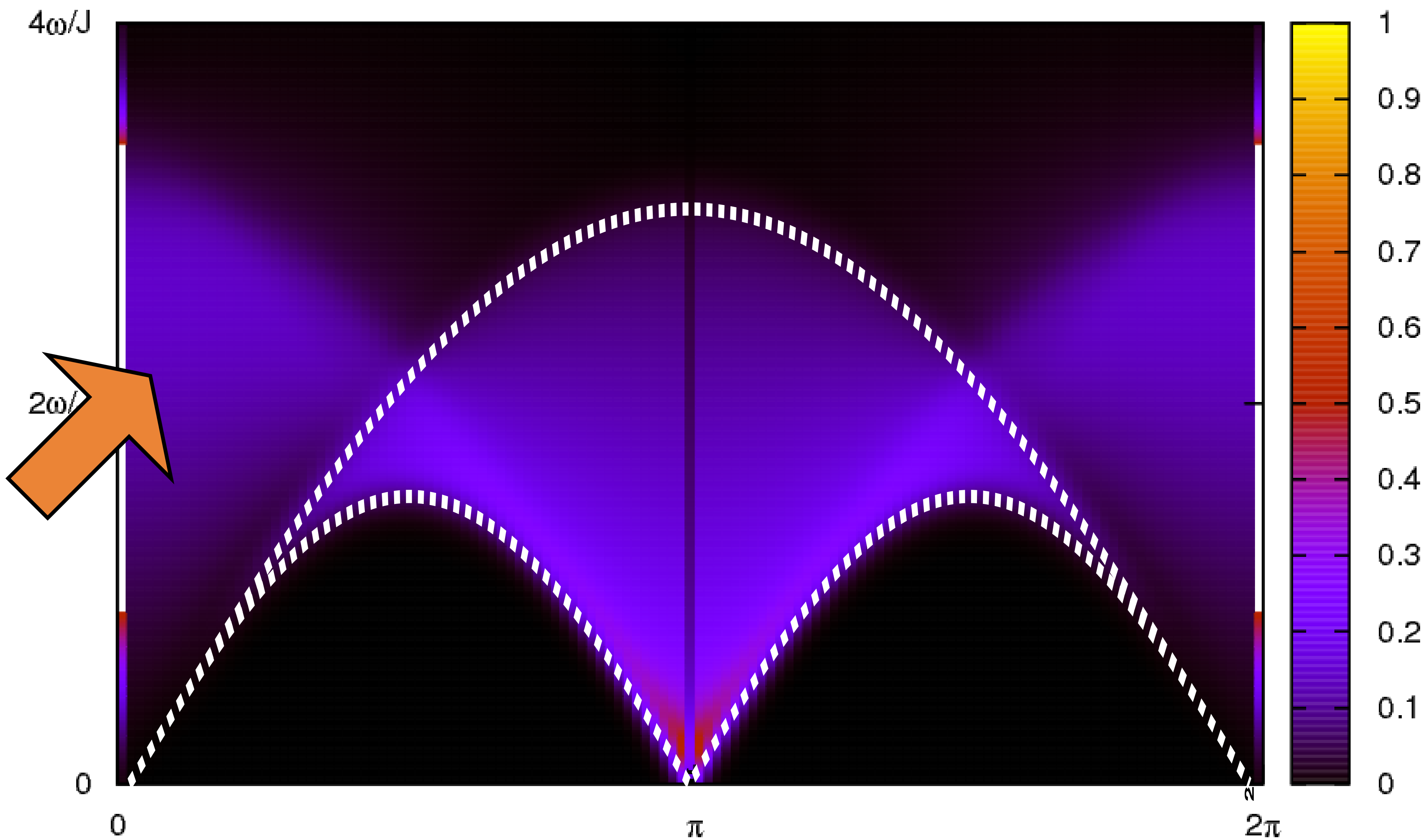
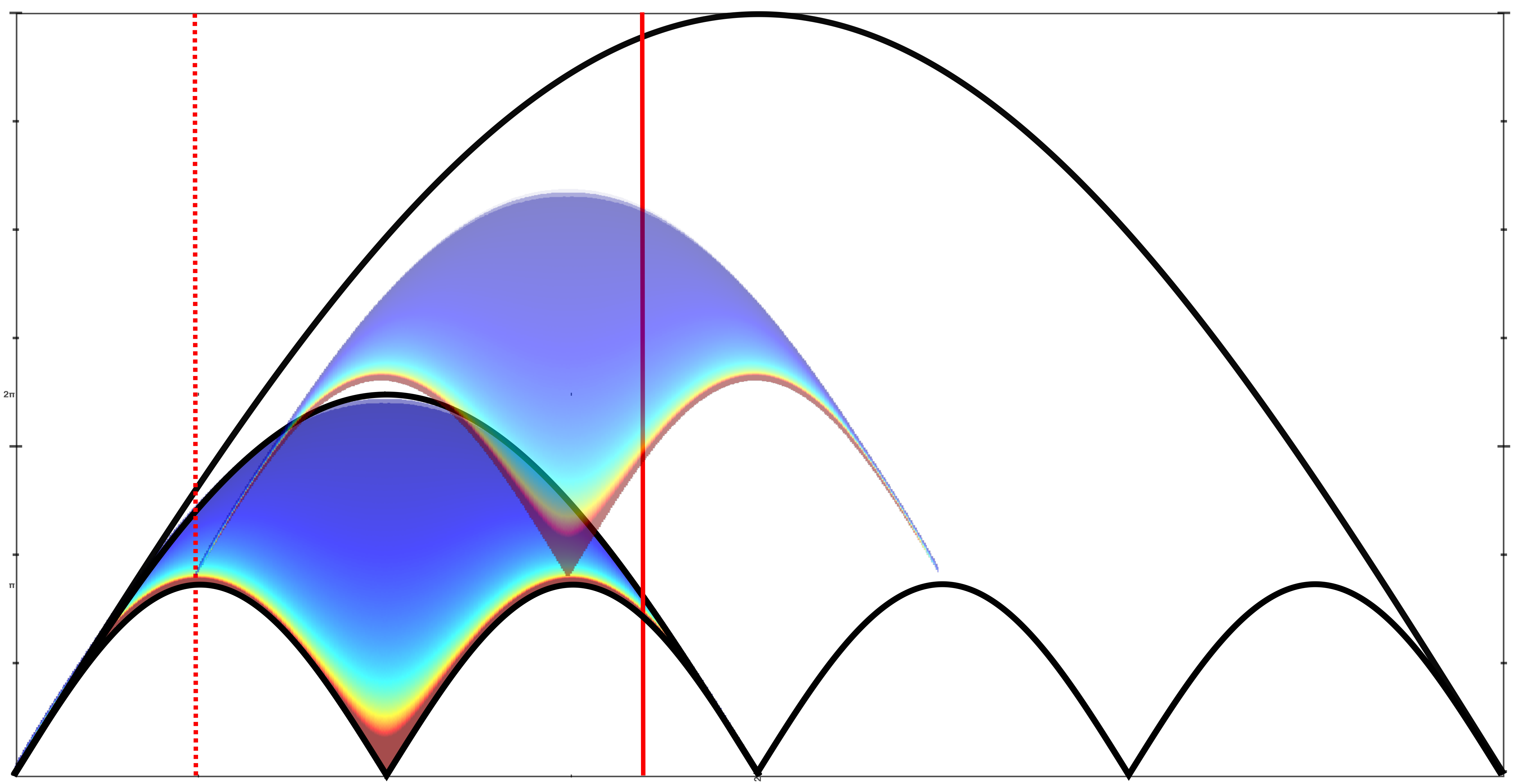


RIXS reveals 4-spinon states !

RIXS response: intuitive picture

‘Two-step’ process: $$\langle \alpha | S^z_j S^z_{j+1} | GS \rangle$$

$$|GS\rangle \rightarrow S^z_p |GS\rangle \rightarrow S^z_{q-p}\, S^z_p |GS\rangle$$



The Richardson model

$$H_{BCS} = \sum_{\substack{\alpha=1 \\ \sigma=+,-}}^{N} \frac{\varepsilon_\alpha}{2} c^\dagger_{\alpha\sigma} c_{\alpha\sigma} - g \sum_{\alpha,\beta=1}^{N} c^\dagger_{\alpha+} c^\dagger_{\alpha-} c_{\beta-} c_{\beta+}$$

(R. W. Richardson, 1963; R. W. Richardon & N. Sherman, 1964)

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Exactly solvable in canonical ensemble.

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Pseudospin representation: $S^z_\alpha = b^\dagger_\alpha b_\alpha - 1/2, \quad S^-_\alpha = b_\alpha, \quad S^+_\alpha = b^\dagger_\alpha$

$$b_\alpha = c_{\alpha-} c_{\alpha+}, \qquad b^\dagger_\alpha = c^\dagger_{\alpha+} c^\dagger_{\alpha-}$$

$$H = \sum_{\alpha=1}^{N} \varepsilon_\alpha S^z_\alpha - g \sum_{\alpha,\beta=1}^{N} S^+_\alpha S^-_\beta$$

Solving the Richardson equations

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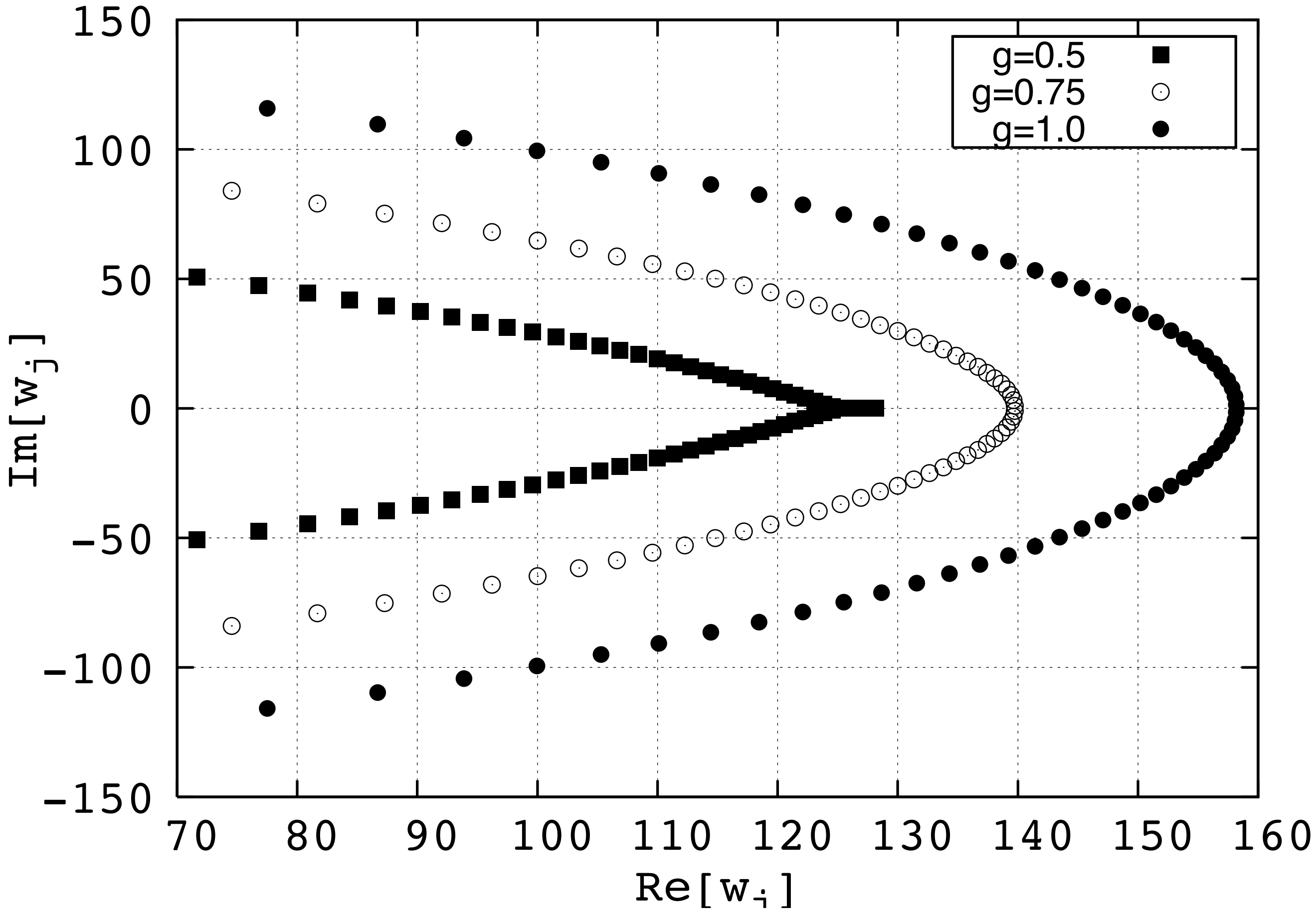
(relatively) straightforward for the ground state



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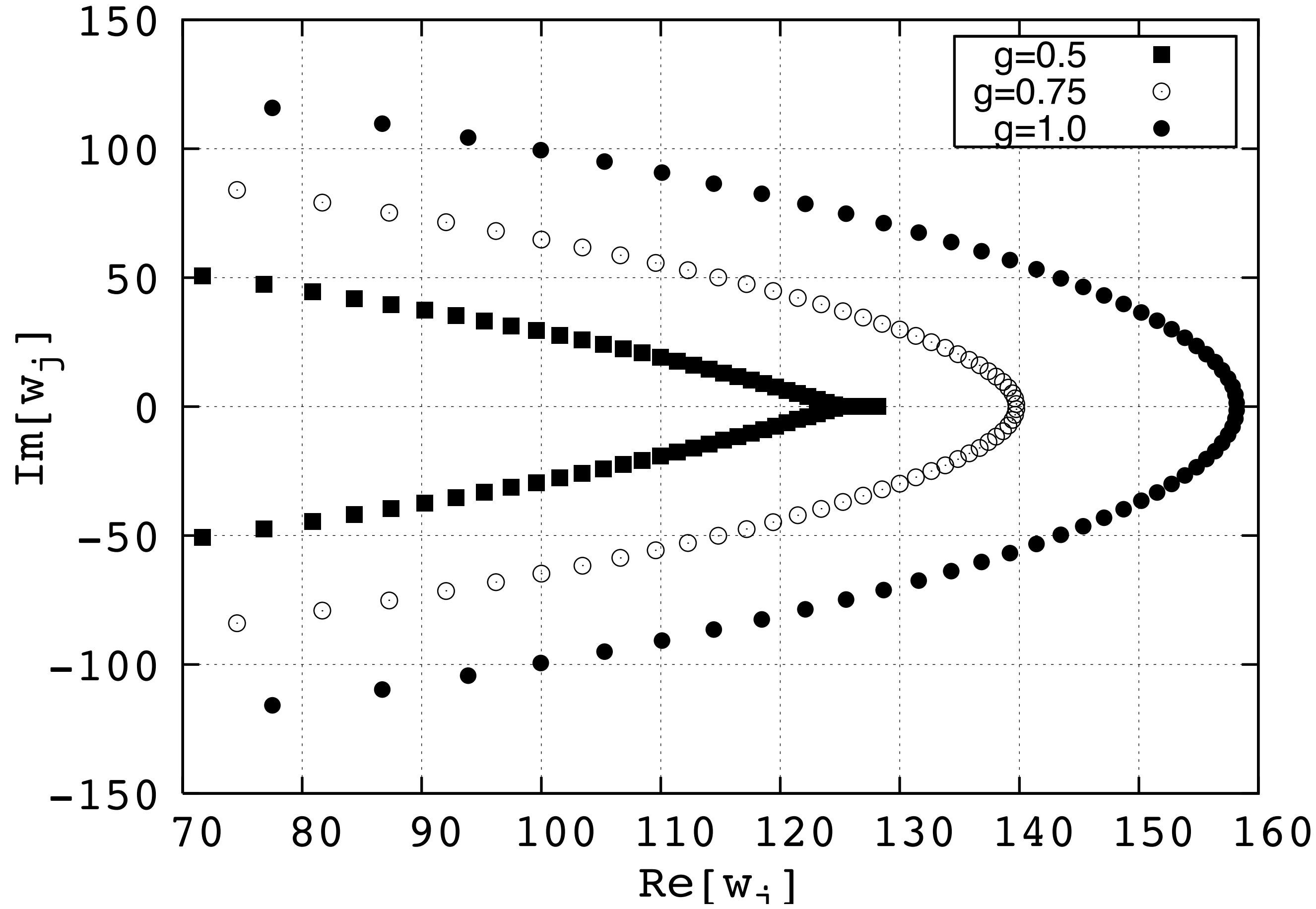

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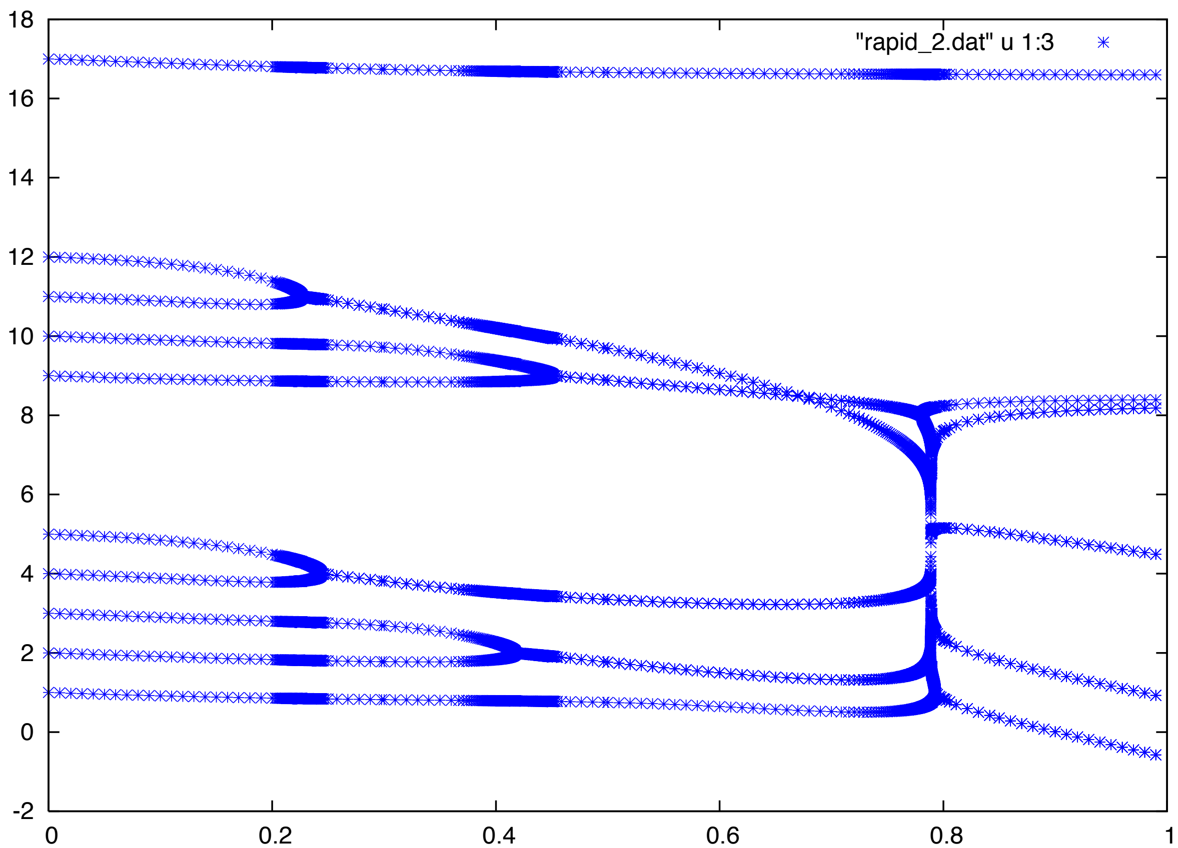

For excited states: can become a real challenge !!

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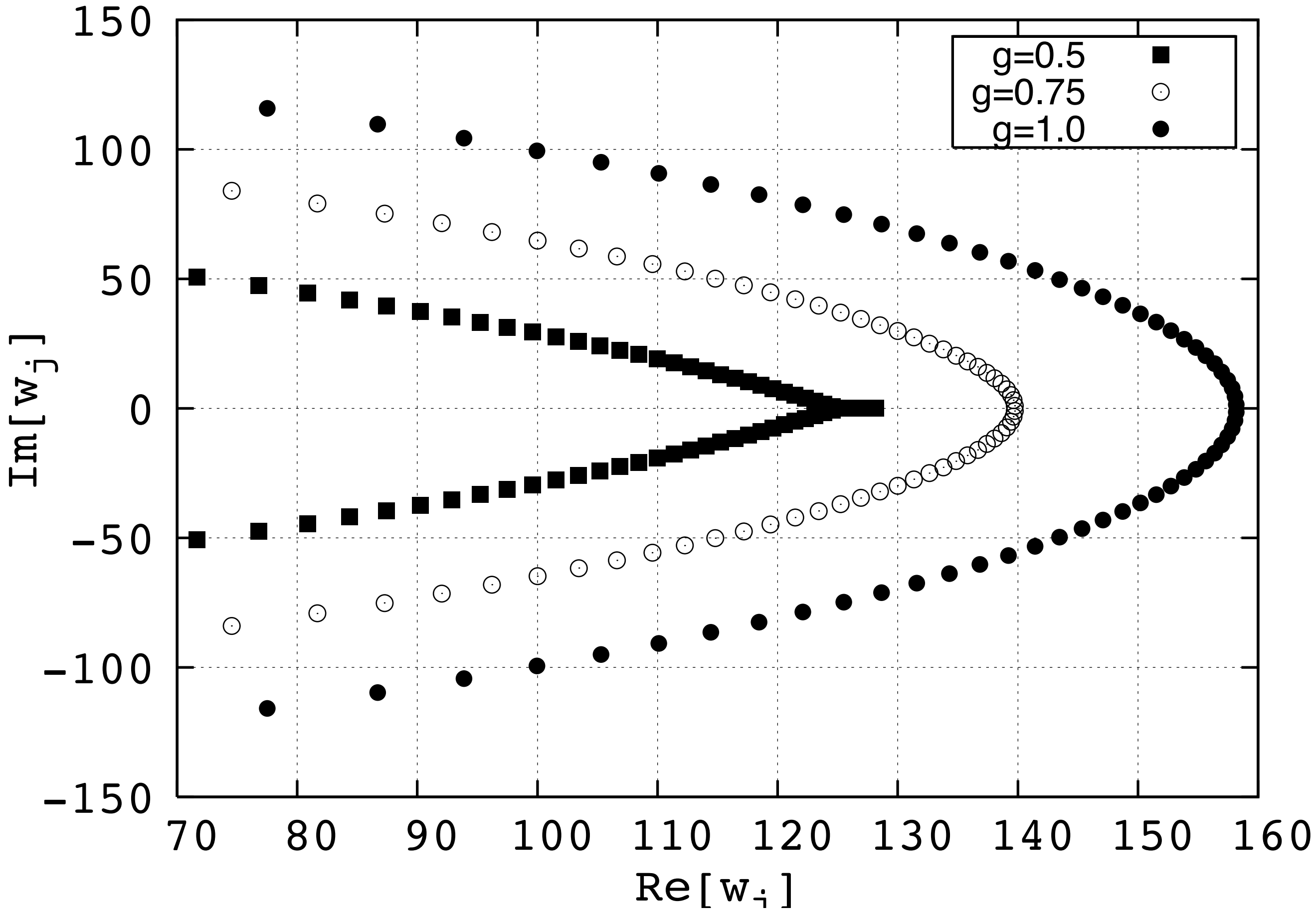
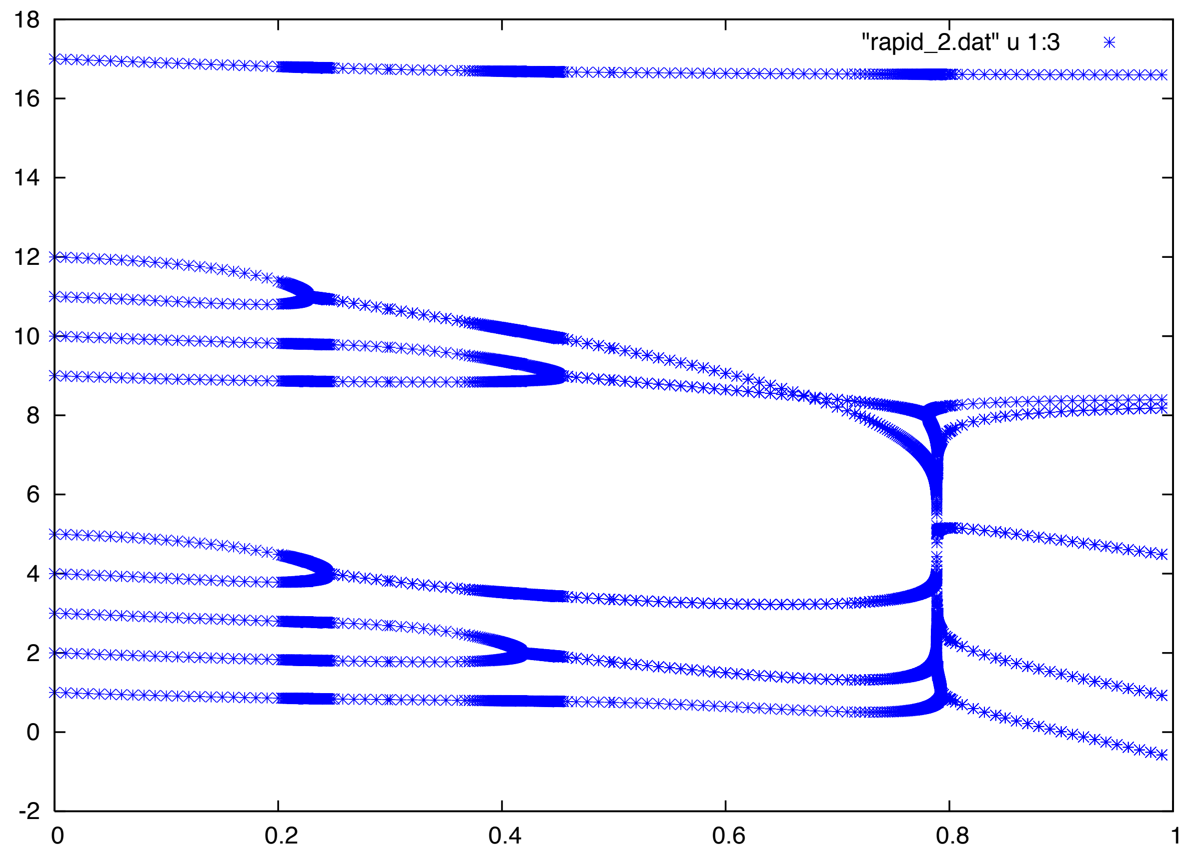




For excited states: can become a real challenge !!

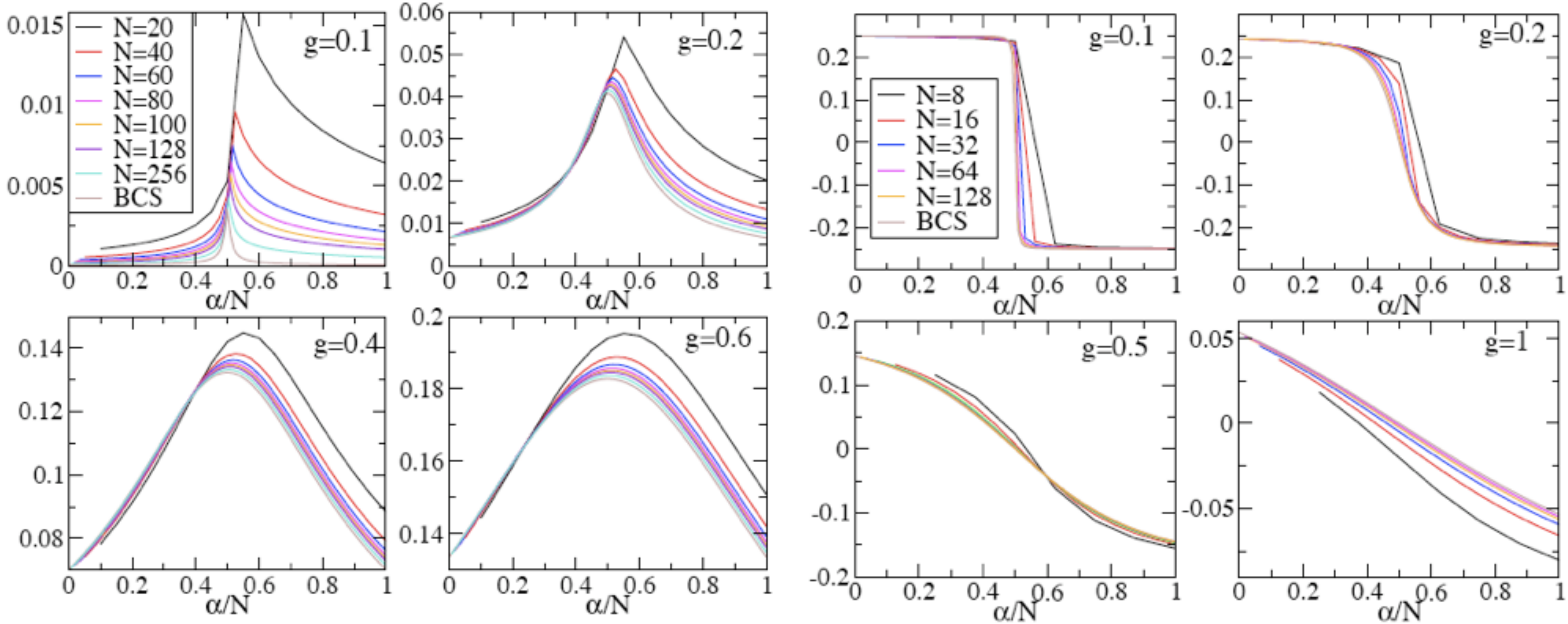
(Richardson, 1964; Schechter, Imry, Levinson & von Delft, 2001; von Delft & Ralph, 2001; Yuzbashyan, Baytin & Altshuler, 2003; Roman, Sierra & Dukelsky, 2003; Snyman & Geyer, 2006; Sambataro, 2007)

The Richardson model: (static) correlation functions

(A. Faribault, P. Calabrese & J-S C, PRB 2008)

(Following up on ABA work by J. von Delft & R. Poghossian, 2002
and H.-Q. Zhou, J. Links, R. H. McKenzie & M. D. Gould, 2002-3)

$\langle S_1^- S_\alpha^+ \rangle$ $\langle S_1^z S_\alpha^z \rangle$



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→ ***incredibly efficient basis for many physically relevant correlations***

Part 2:

Quench dynamics

Quenches: some trivialities

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Sudden change of interaction parameter

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(Barouch & McCoy, ..., Calabrese & Cardy, ... Cazalilla, Lamacraft, Klich, Lannert & Refael, Barmettler & al, ...)

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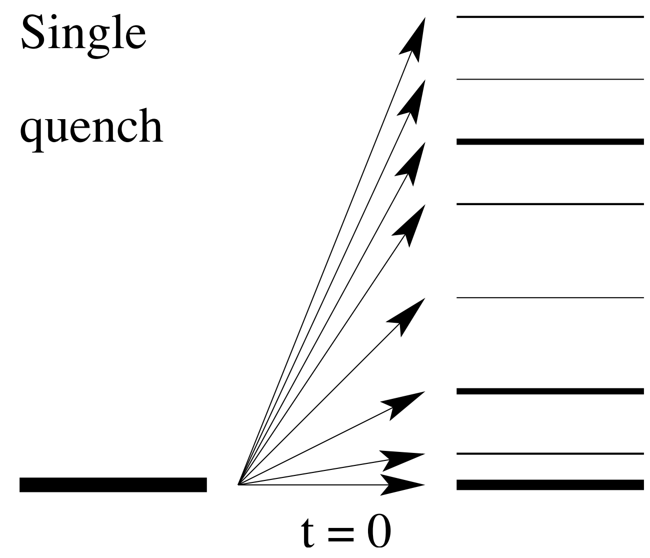
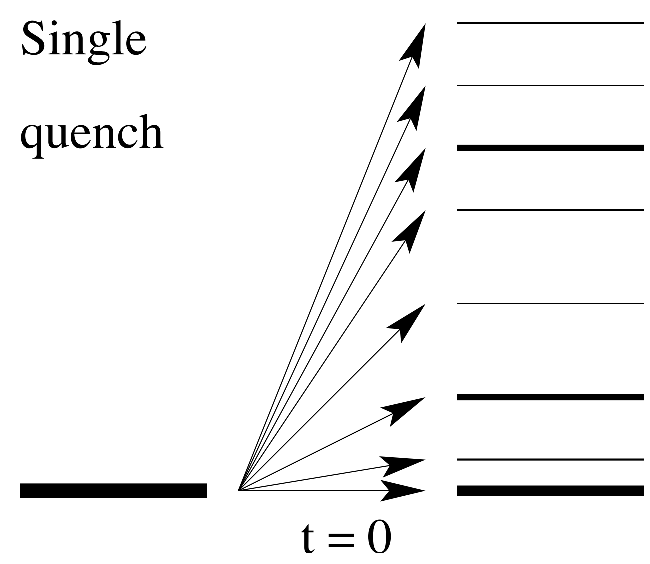
Cazalilla, Lamacraft, Klich, Lannert & Refael,
Barmettler & al, ...)



Quenches: some trivialities

Sudden change of interaction parameter

(Barouch & McCoy, ..., Calabrese & Cardy, ... Cazalilla, Lamacraft, Klich, Lannert & Refael, Barmettler & al, ...)



At quench time: $$|\Psi_g^0\rangle = \sum_\alpha |\Psi_{g'}^\alpha\rangle\langle\Psi_{g'}^\alpha|\Psi_g^0\rangle \equiv \sum_\alpha M_{g'g}^{\alpha 0}|\Psi_{g'}^\alpha\rangle$$

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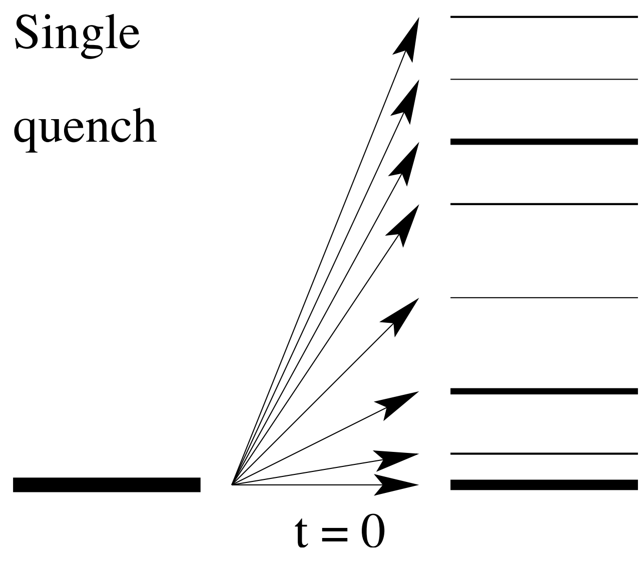

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Subsequent time evolution:

$$|\Psi(t)\rangle = \sum_\alpha M_{g'g}^{\alpha 0} e^{-i\omega_{g'}^\alpha t}|\Psi_{g'}^\alpha\rangle$$

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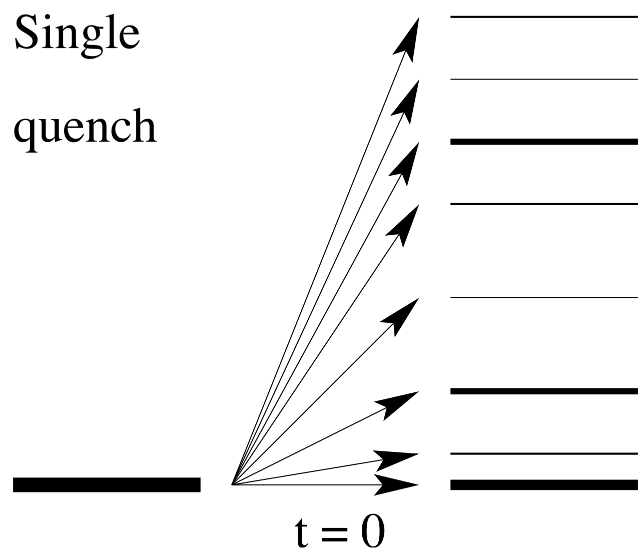

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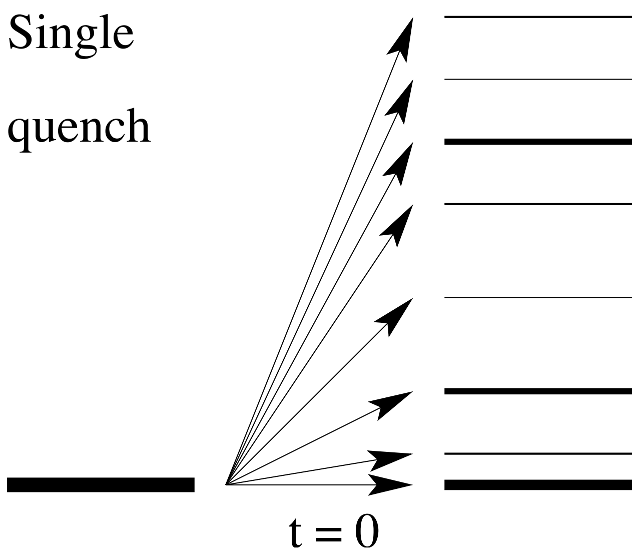

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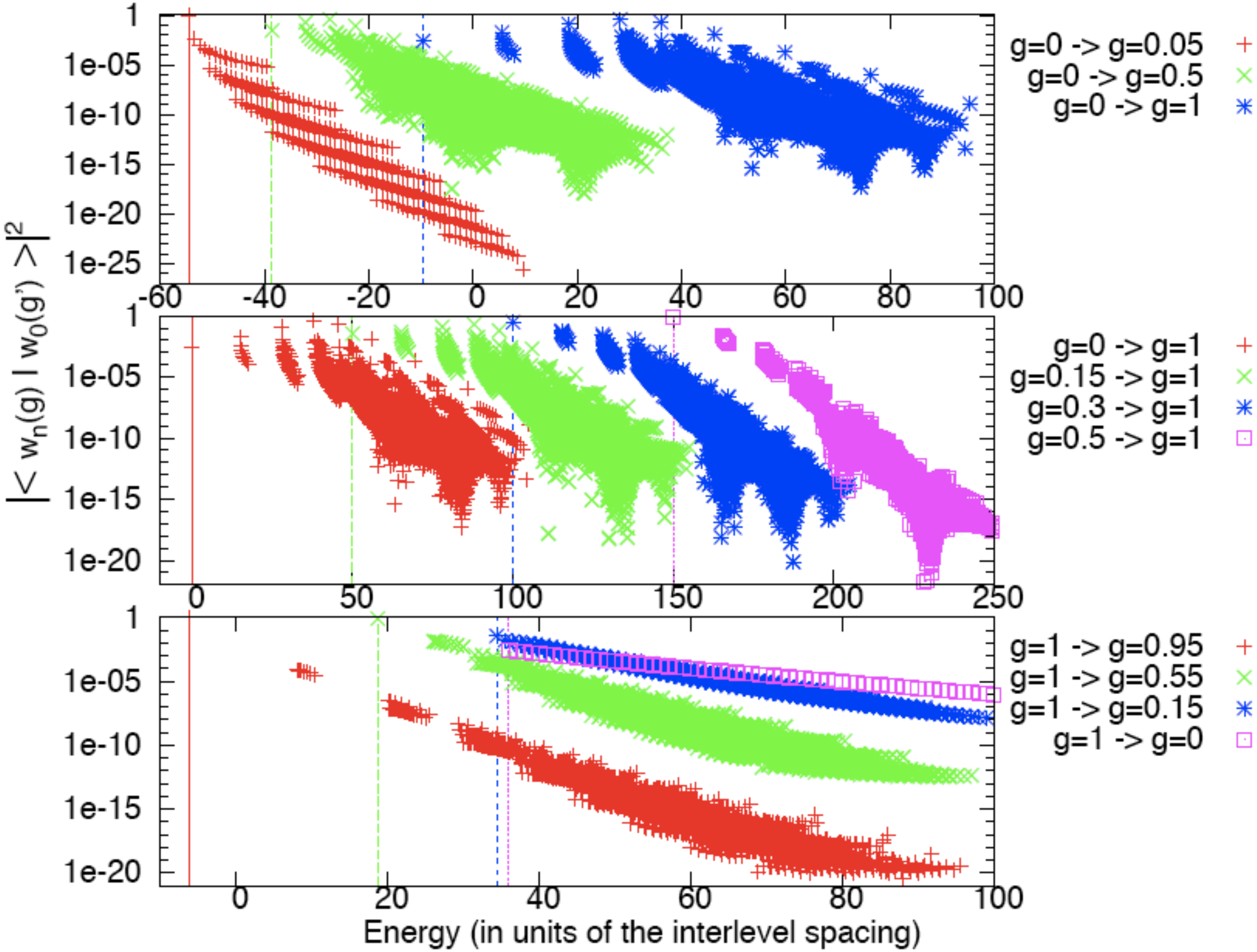
We know how to calculate the quench matrix for the Richardson model !!

Quench matrix elements

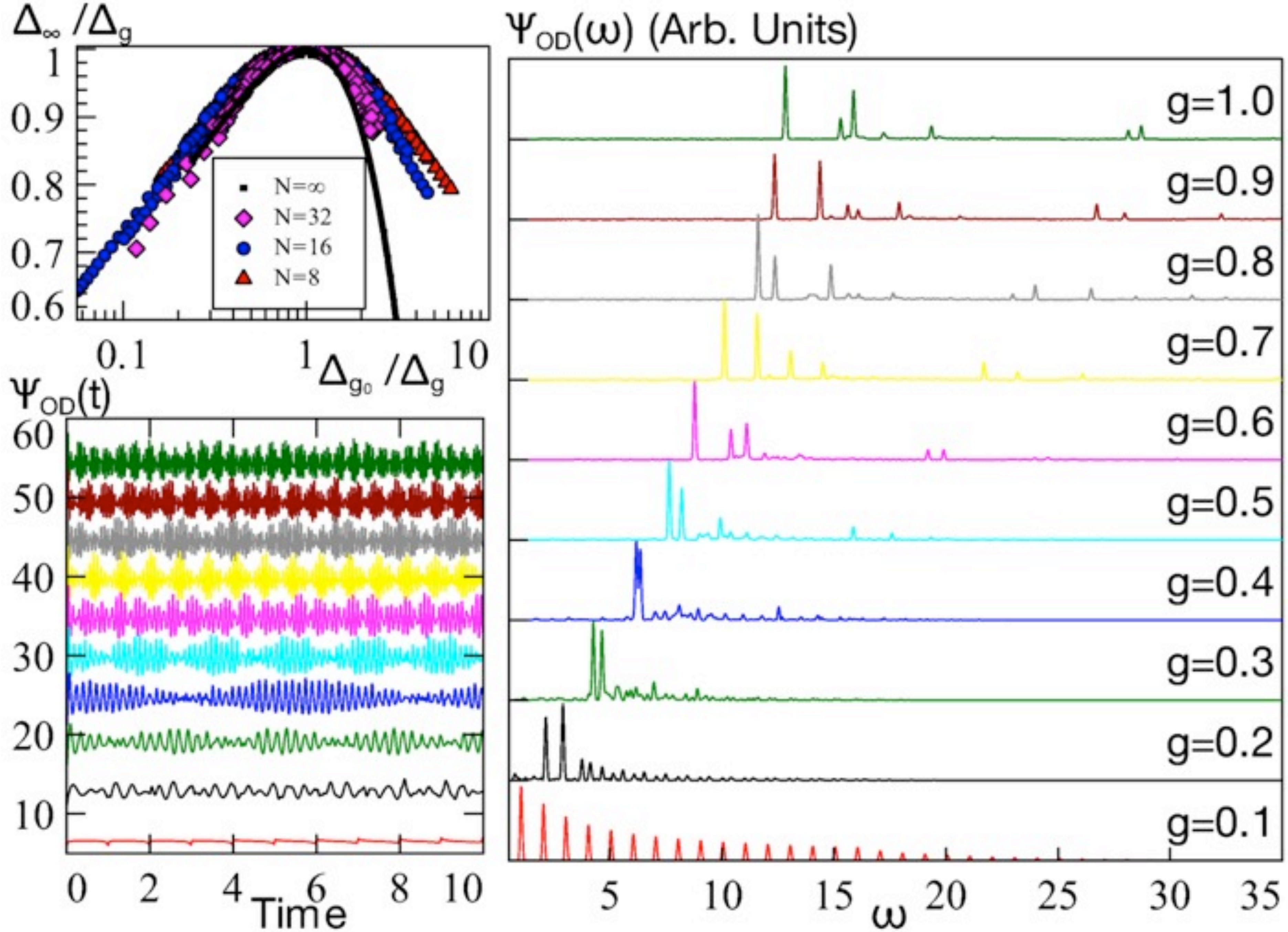


Time dependence of observables

'order parameter' $$\Psi_{OD}(t) \propto \sum_{\alpha,\beta} \langle \psi(t) | S^+_\alpha S^-_\beta | \psi(t) \rangle$$

Plotted against mean-field prediction

(Barankov & Levitov, PRL 2006)



Δ_{g0} gap for initial g

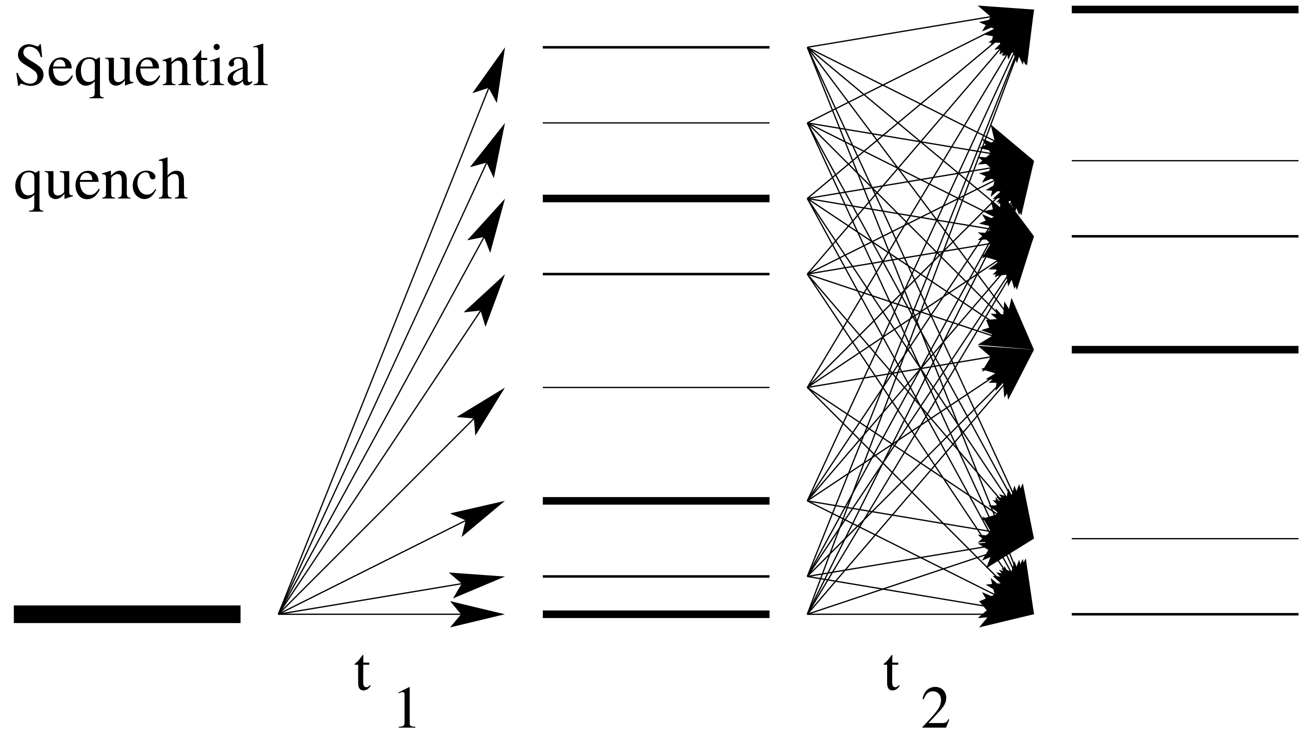
Δ_g gap for final g

Δ_∞ asymptotic gap

Sequential quenches

Sequential quenches

Generic situation, here for 2 quenches:



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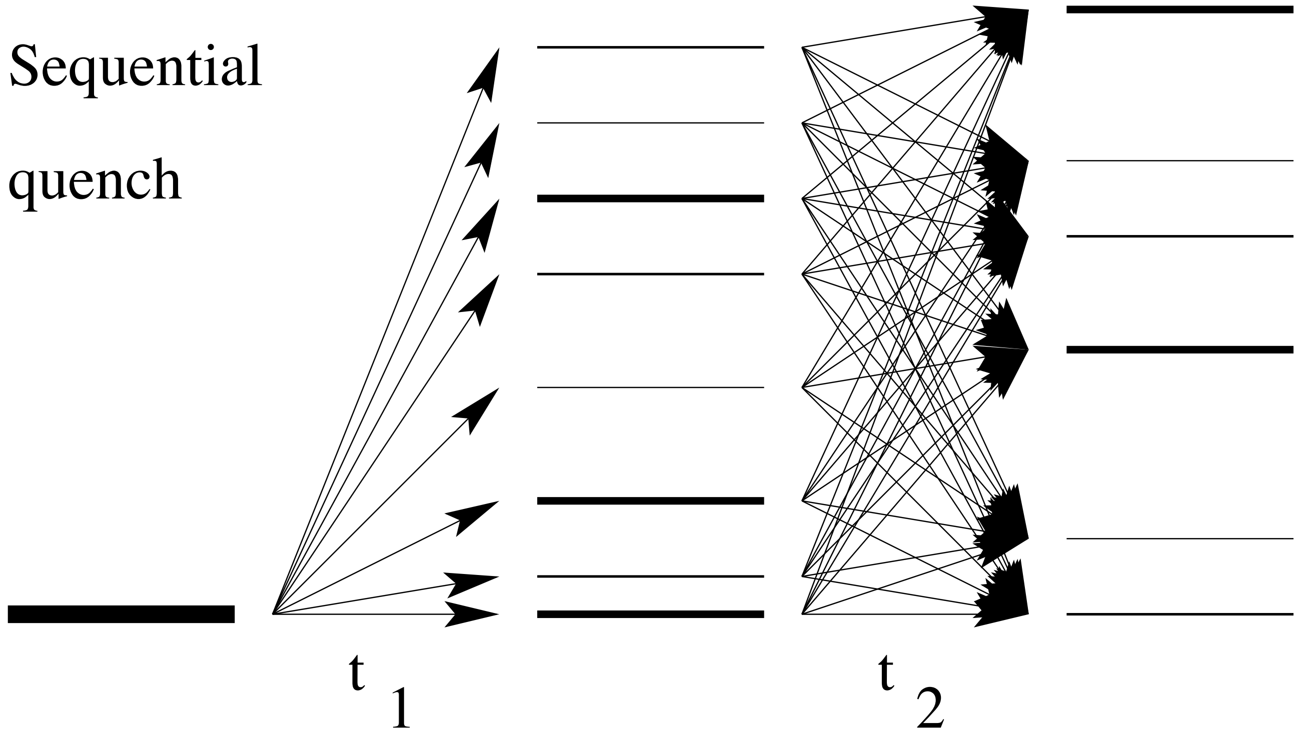

'Quench propagator' for quench-dequench

$$Q_{\beta\alpha}(t_q) = \sum_{\gamma \in \mathcal{H}_{g_1}} M_{g_0 g_1}^{\beta\gamma} M_{g_1 g_0}^{\gamma\alpha} e^{-i\omega_\gamma t_q}$$

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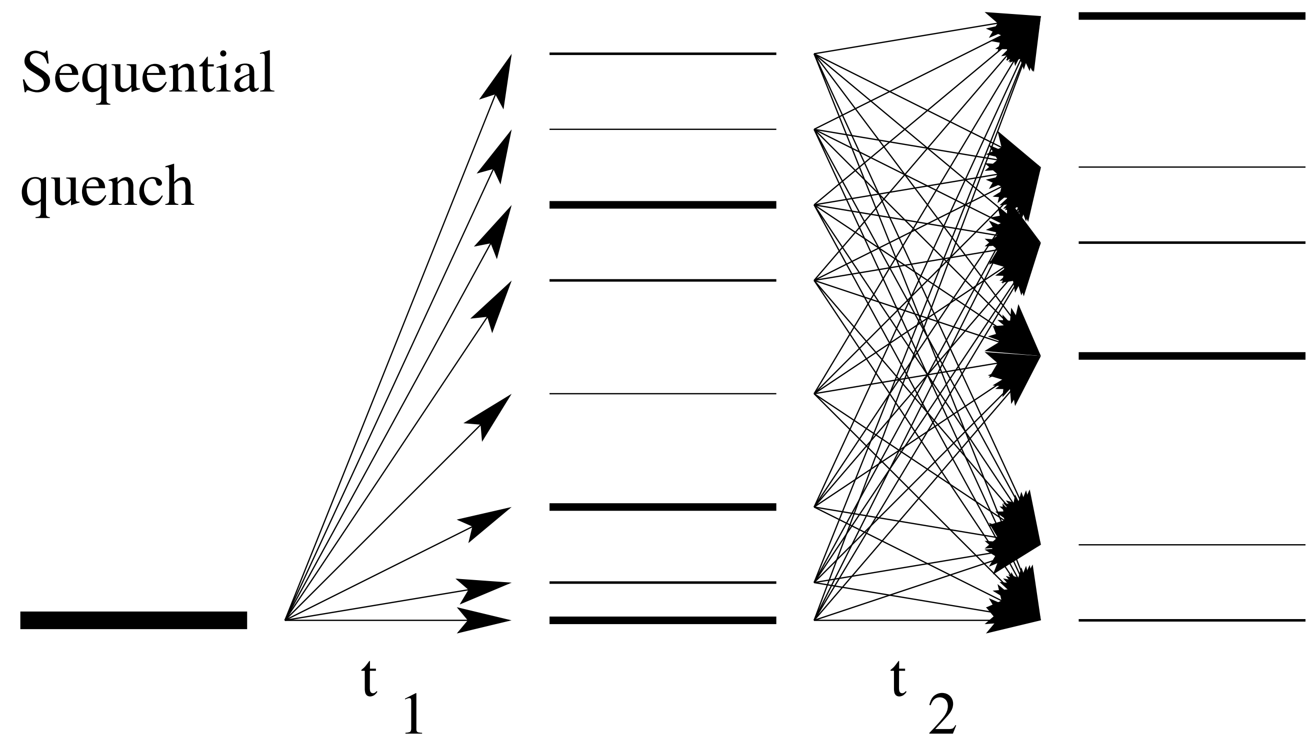

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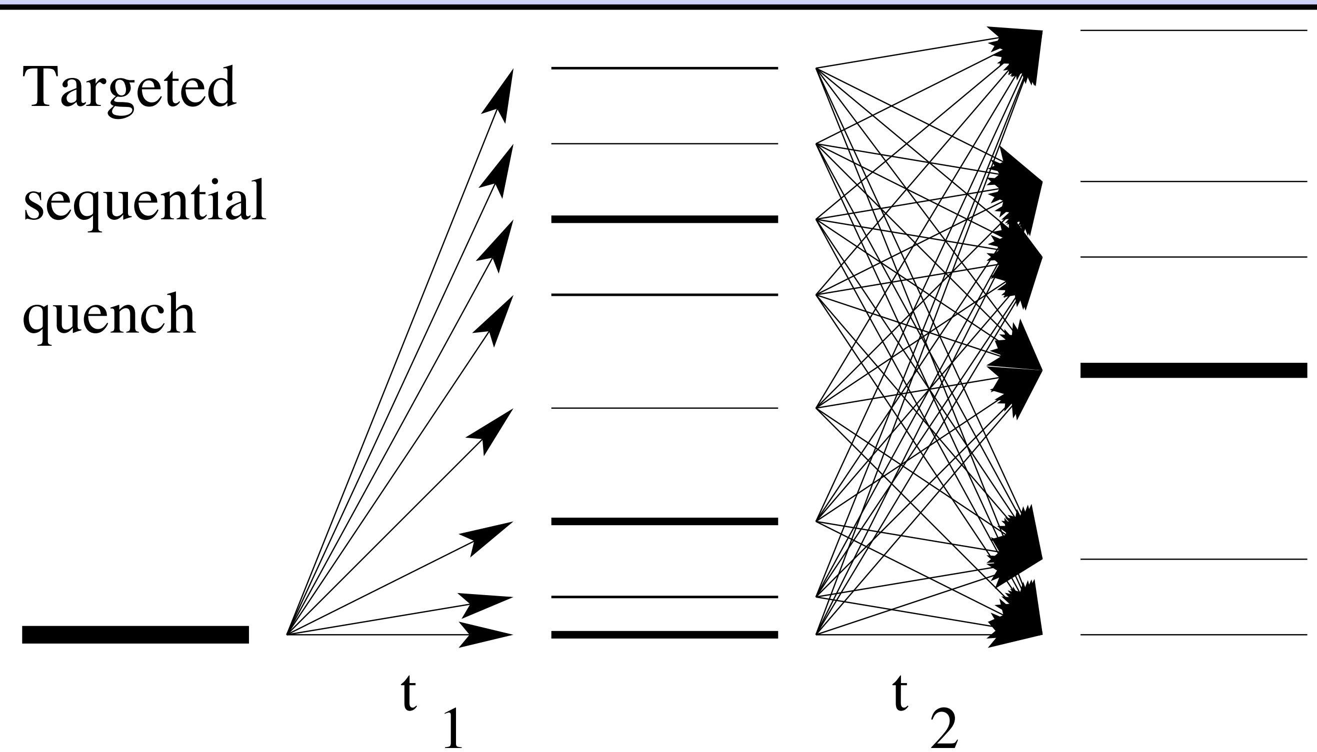
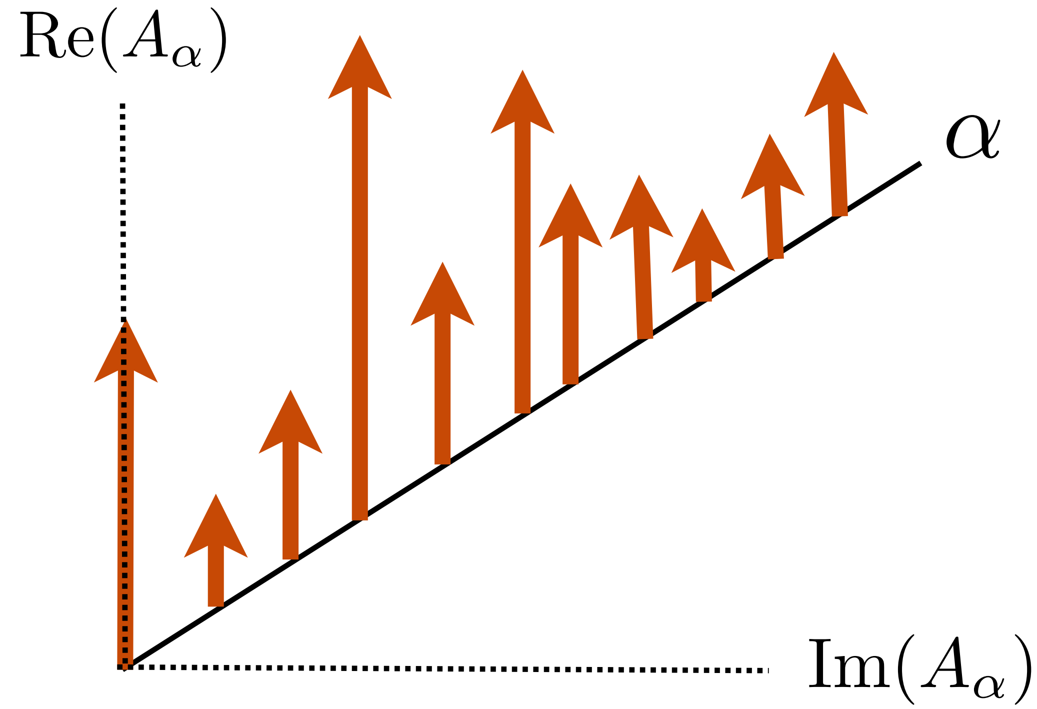
Possible to focus on specific excited states ?



At t = 0, the initial quench populates excited states of H_g



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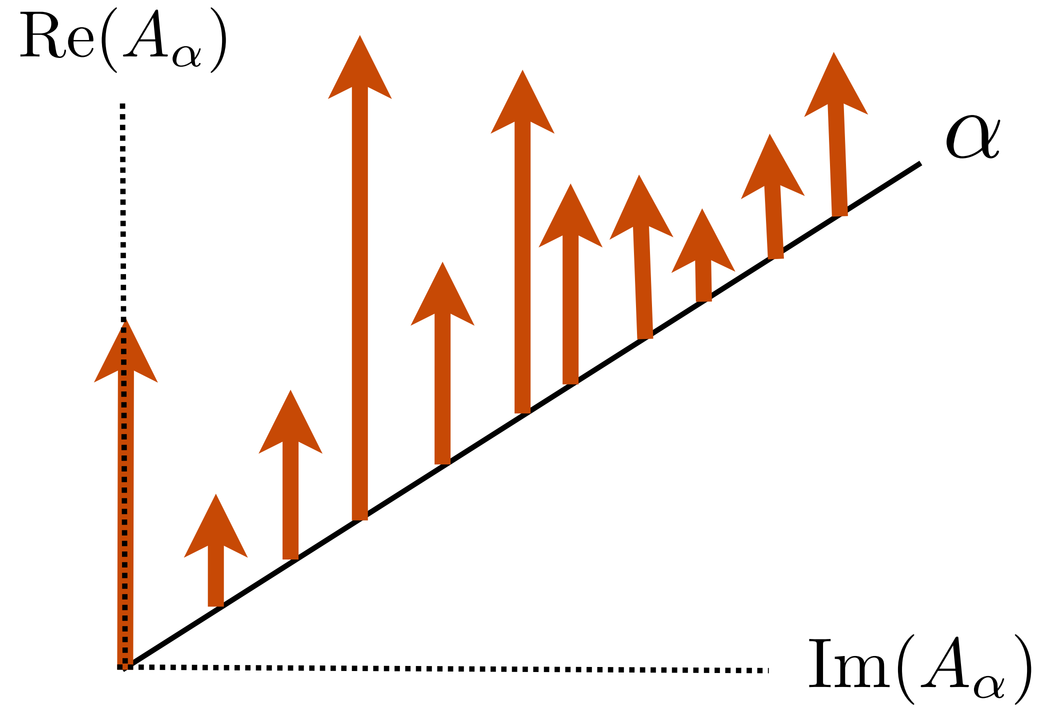
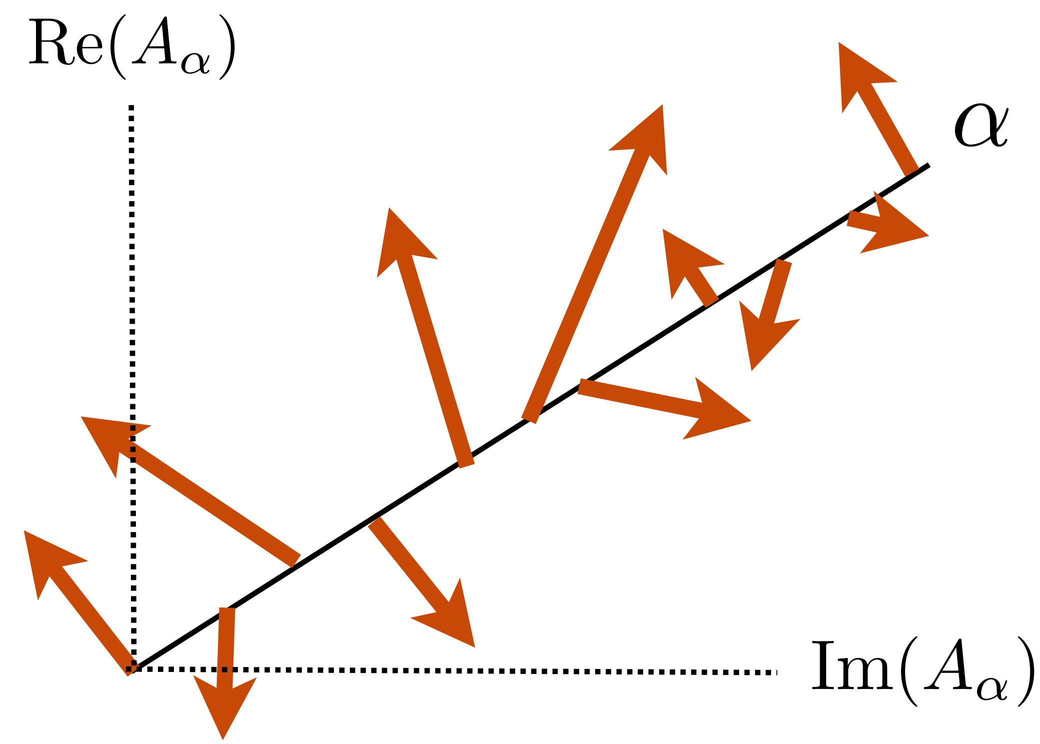


As the quench lasts, each ‘arrow’ rotates at the appropriate frequency



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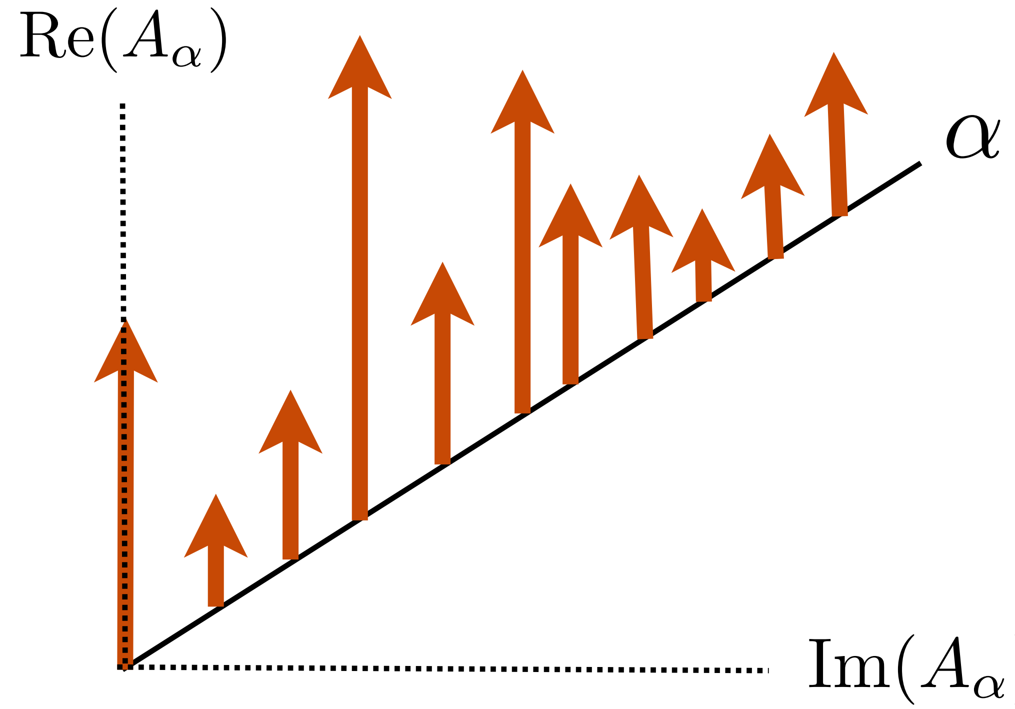

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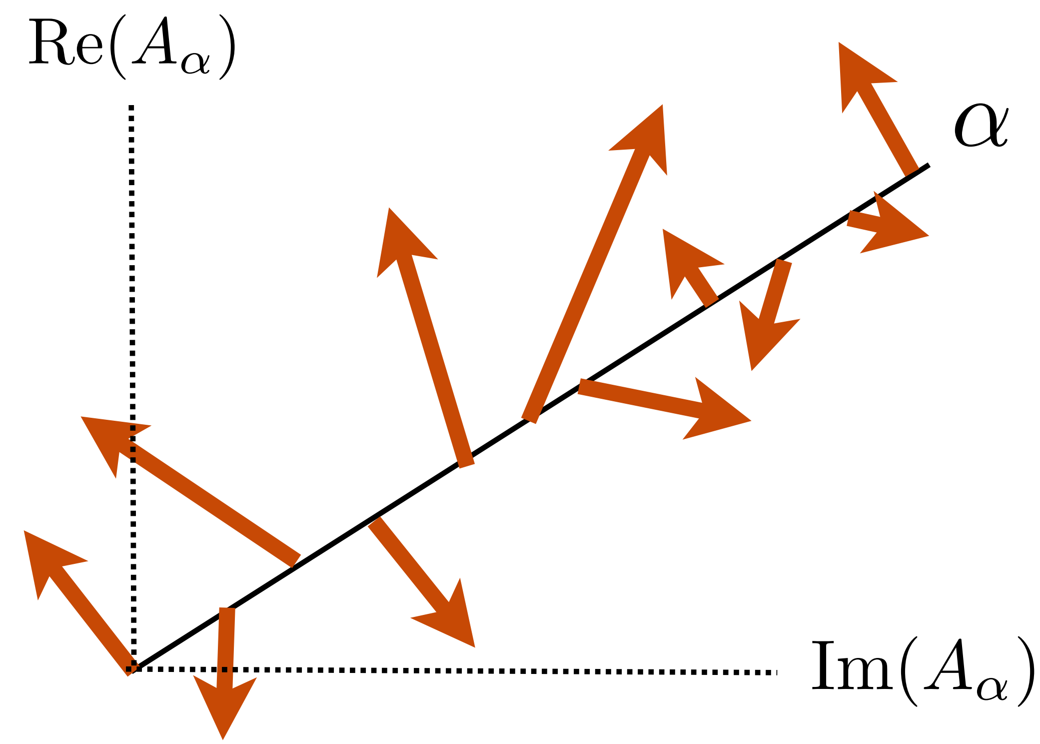

The dequench repopulates states of original Hamiltonian

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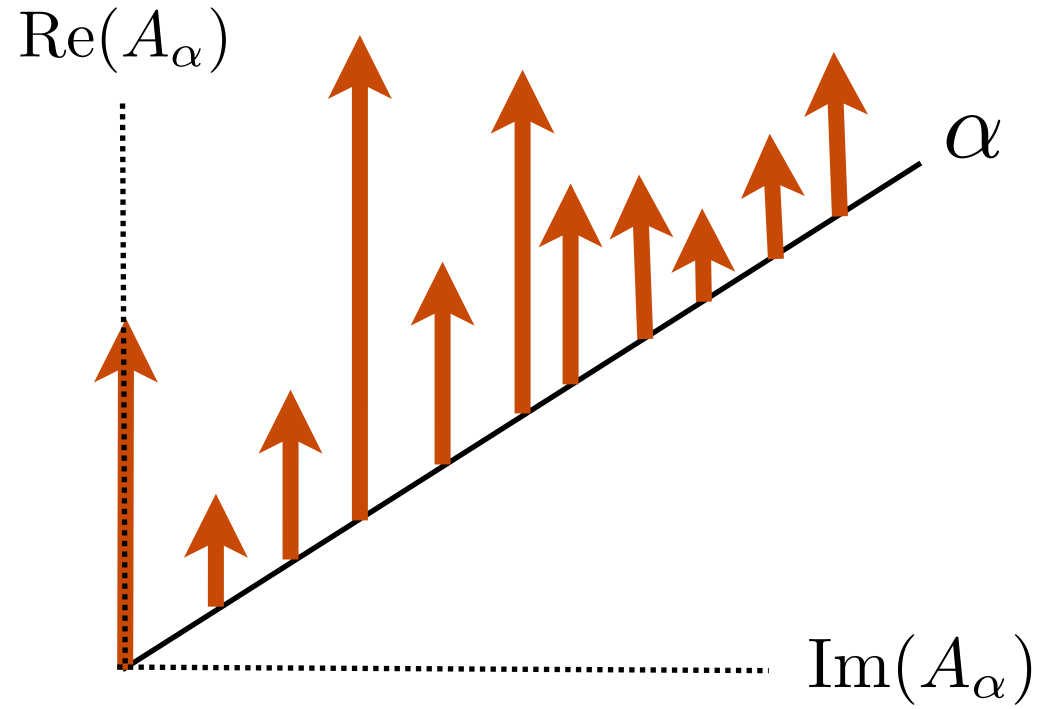

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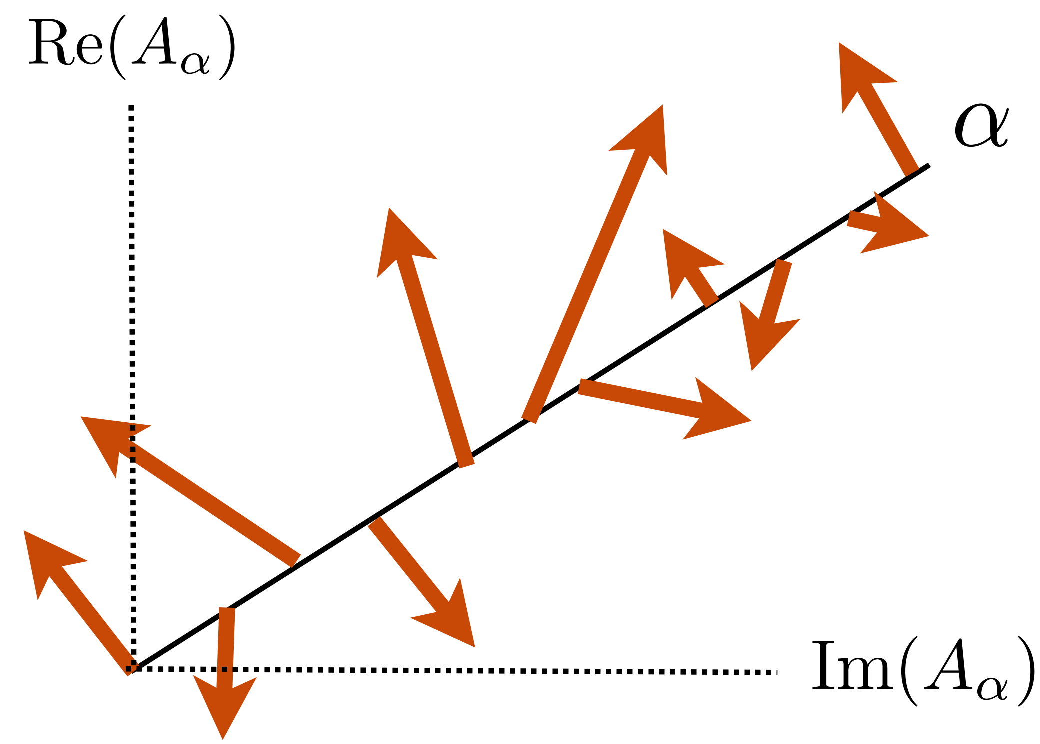

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When arrows 'add up to zero': state destruction

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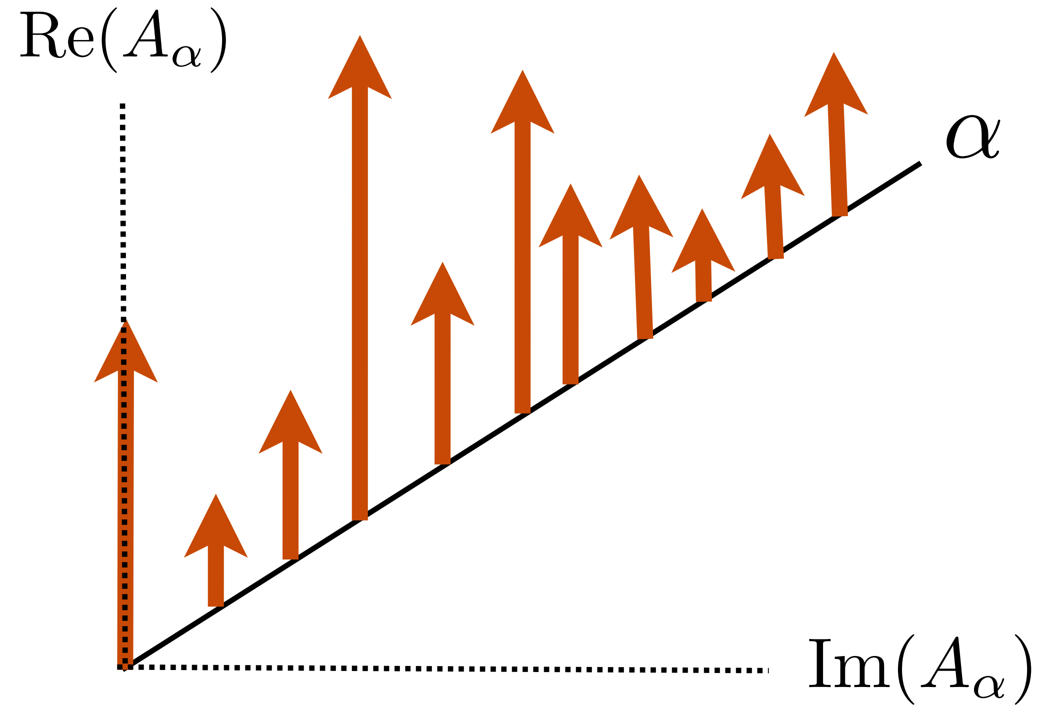

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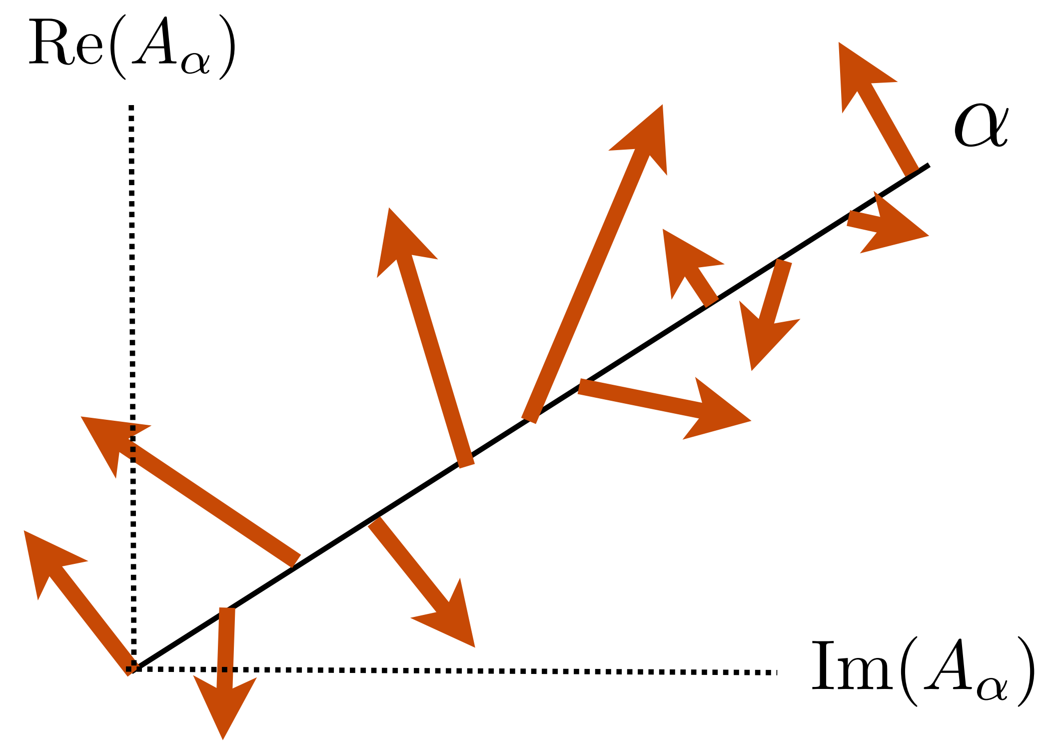

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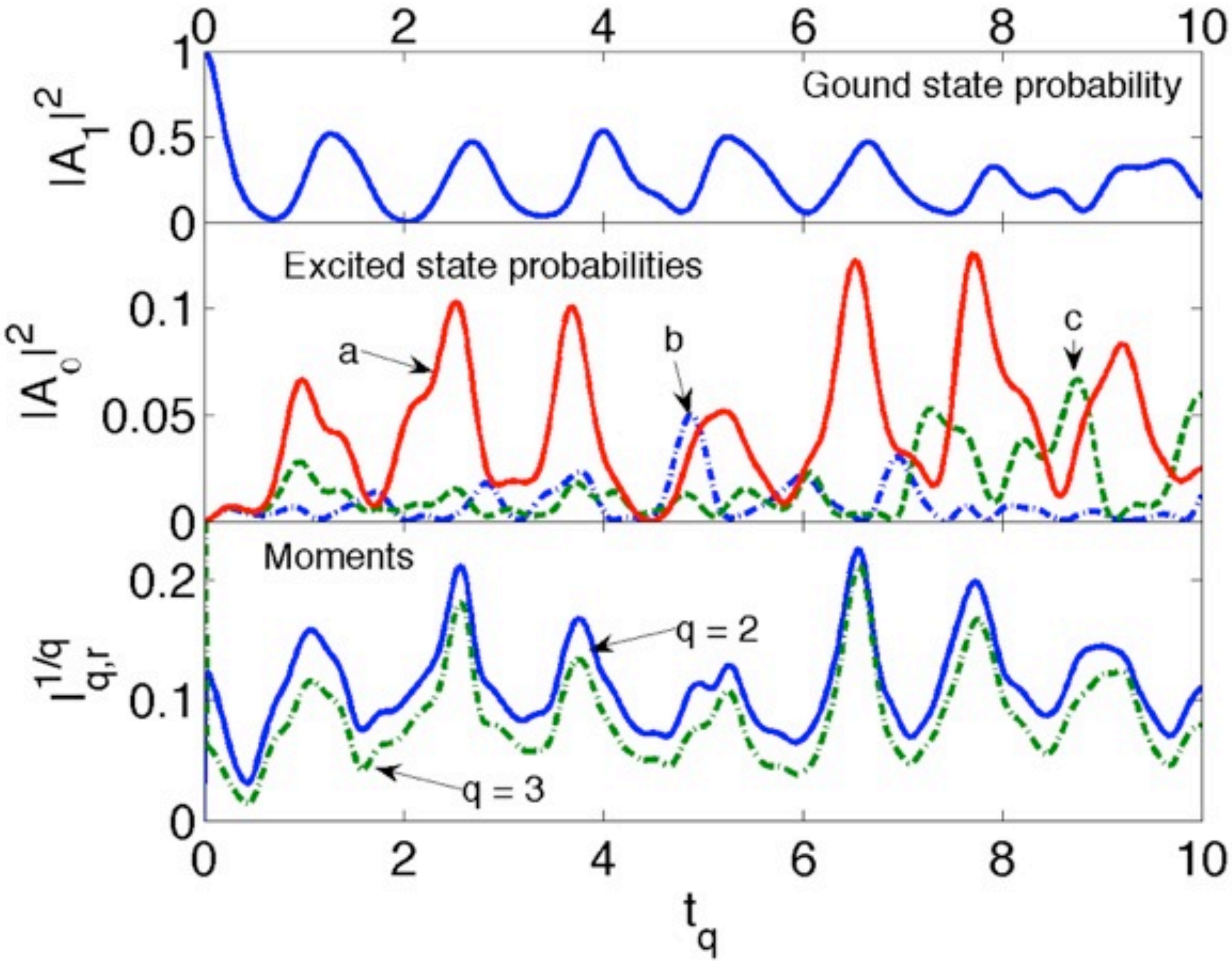
When arrows ‘add up to zero’: state destruction

When arrows realign: state reconstruction

State occupation probabilities after double quench (quench-dequench)

Ground state disappears and reappears ('collapse and revival'); excited states nontrivially weighted



Weight distribution among excited states: look at IPRs

$$I_{q,r} = \sum_{\alpha>0} |A_\alpha|^{2q} / (\sum_{\alpha>0} |A_\alpha|^2)^q$$

Domain wall quenched into XXZ

J. Mossel and JSC, NJP 2010

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Initial state: $$|\phi\rangle = |\underbrace{\downarrow \ldots \downarrow}_{M}\underbrace{\uparrow \ldots \uparrow}_{N-M}\rangle.$$

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Time evolution dictated by

$$H_{XXZ} = J \sum_{j=1}^{N} \left[\frac{1}{2\Delta} \left(S_j^- S_{j+1}^+ + S_j^+ S_{j+1}^- \right) + S_j^z S_{j+1}^z \right]$$

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Solution to Schrödinger eqn: $$|\phi(t)\rangle = \sum_n e^{-iE_n t} Q_n |\Psi_n\rangle$$

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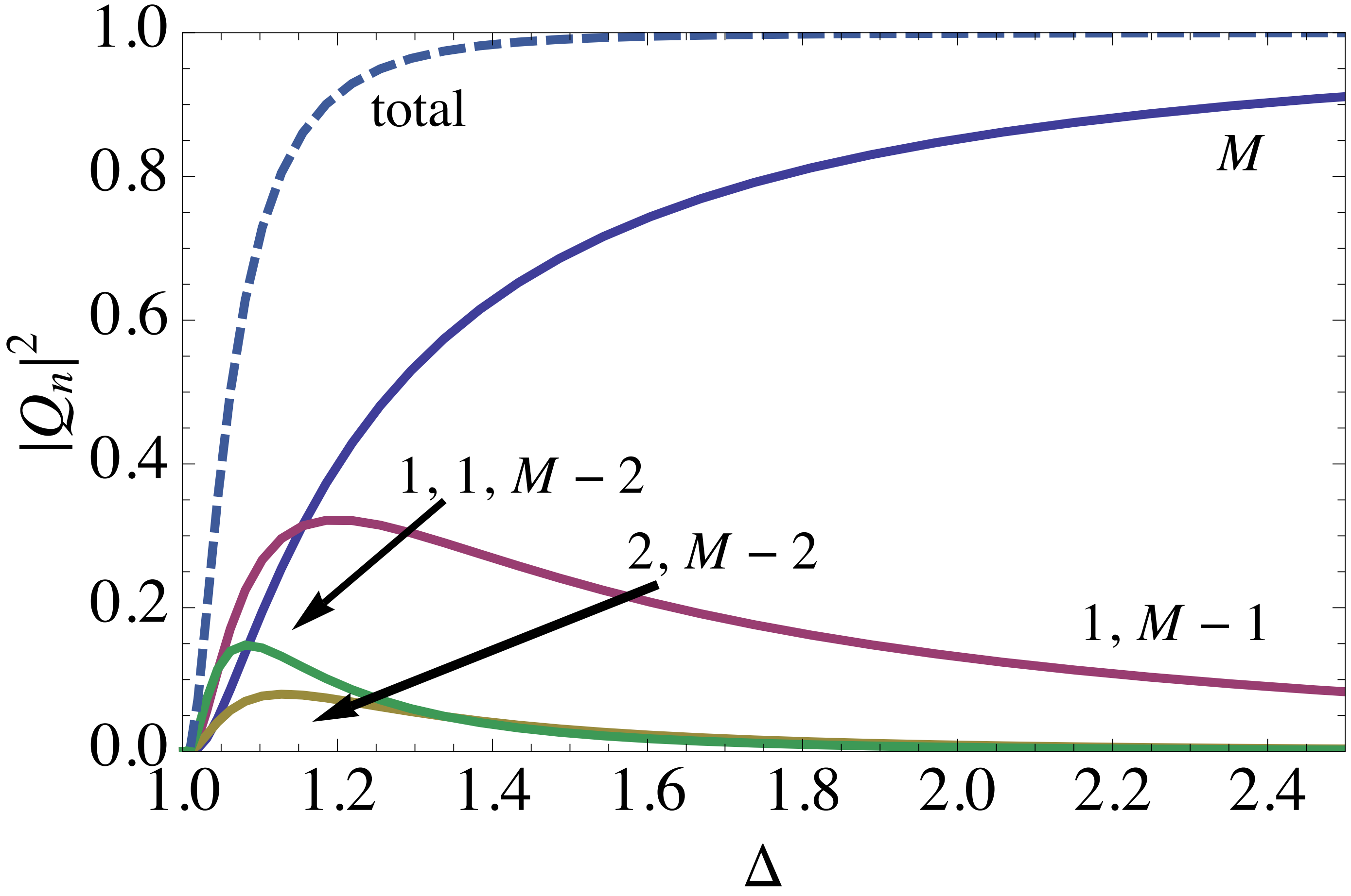
Quench vector elements: $$Q_n \equiv \langle\Psi_n|\phi\rangle \qquad \sum_n |Q_n|^2 = 1$$

Dominant overlaps: with string states



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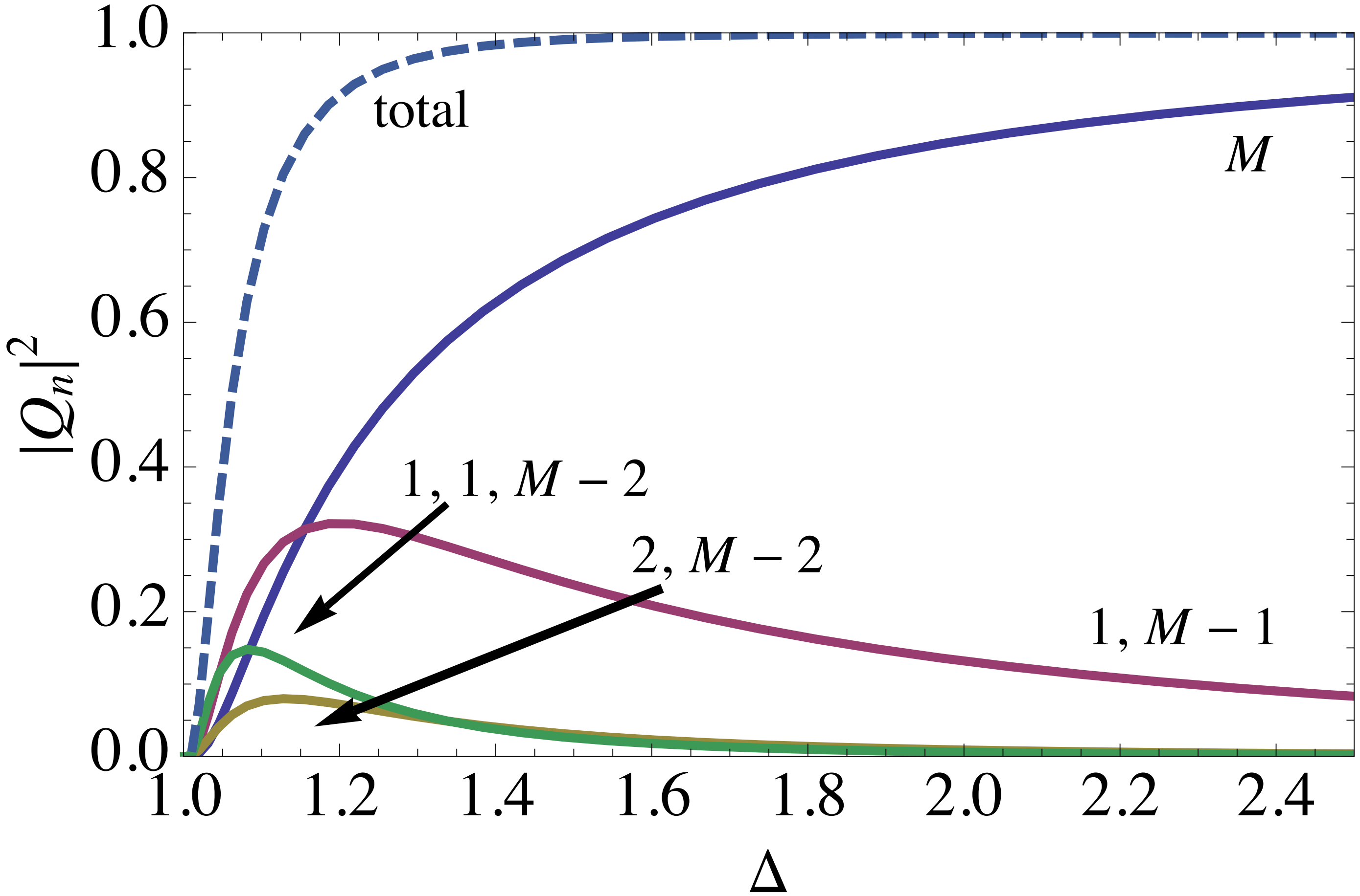
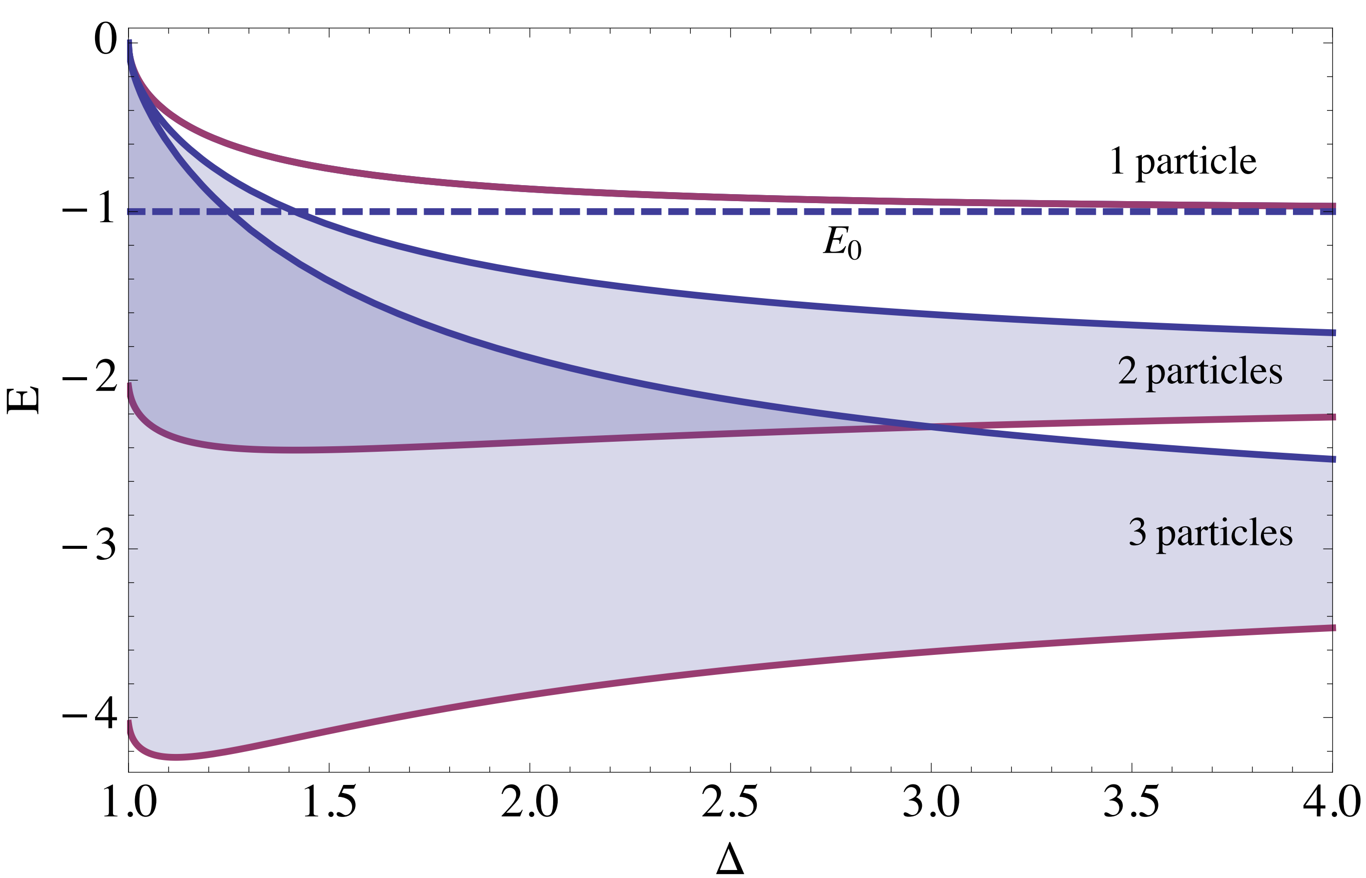
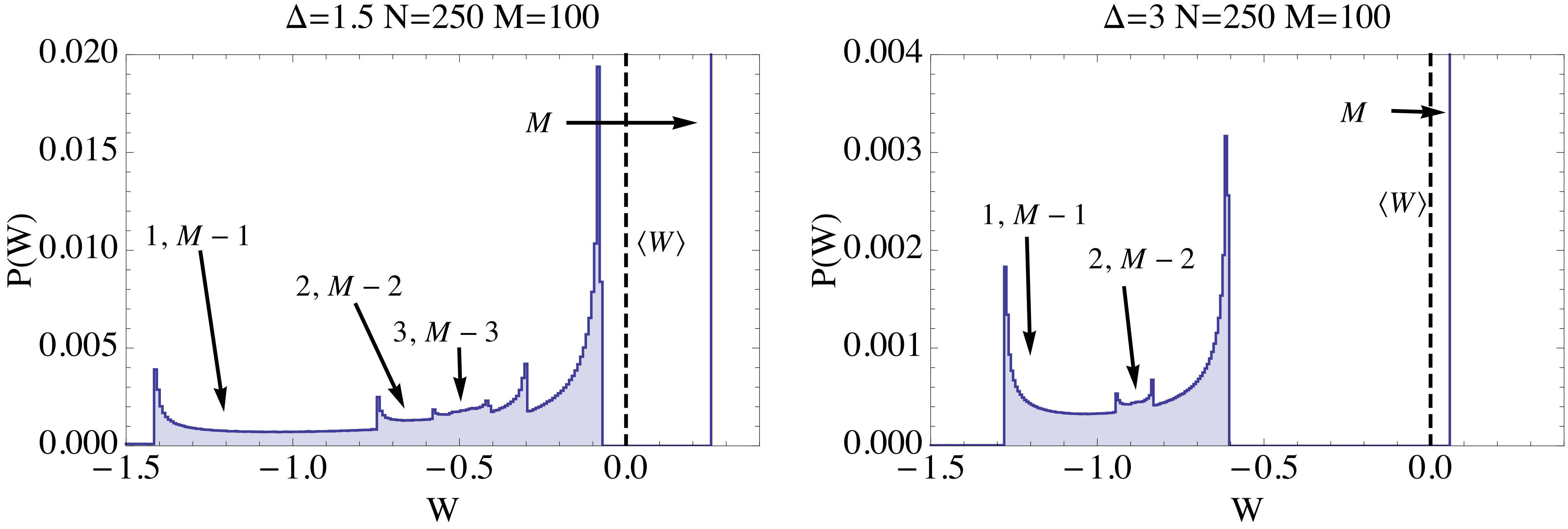




Excitation continua for various state families

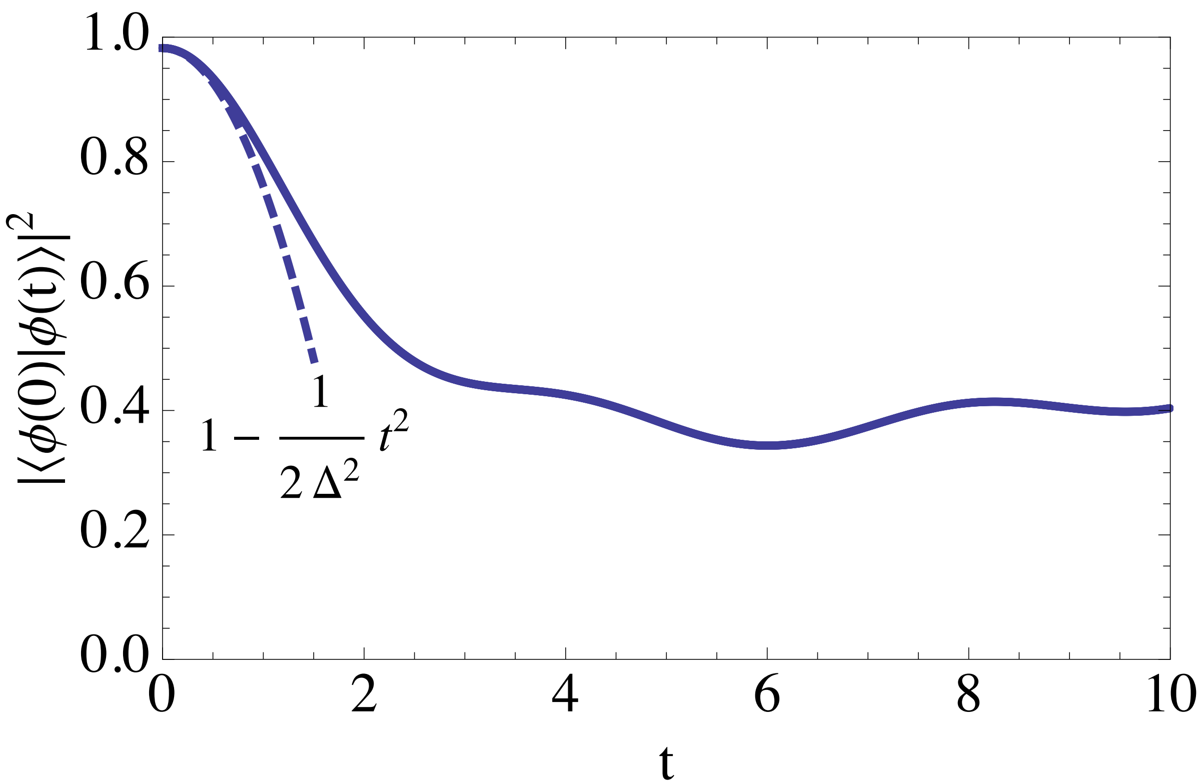
Work probability distribution

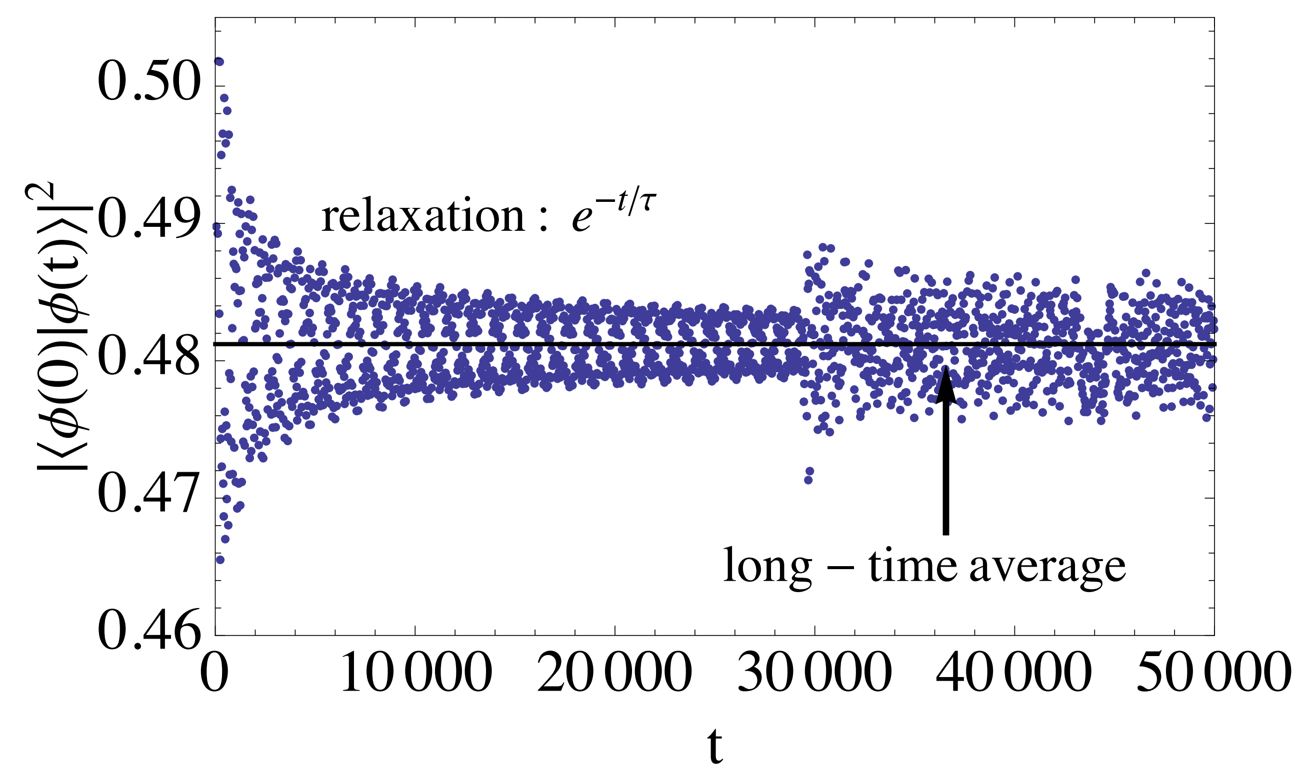
$$P(W) = \sum_n |\langle \phi | \Psi_n \rangle|^2 \delta(W - E_n + E_0)$$



Loschmidt echo

$$\mathcal{L}(t) = \left|\langle\phi|e^{iH_0 t}e^{-iHt}|\phi\rangle\right|^2$$





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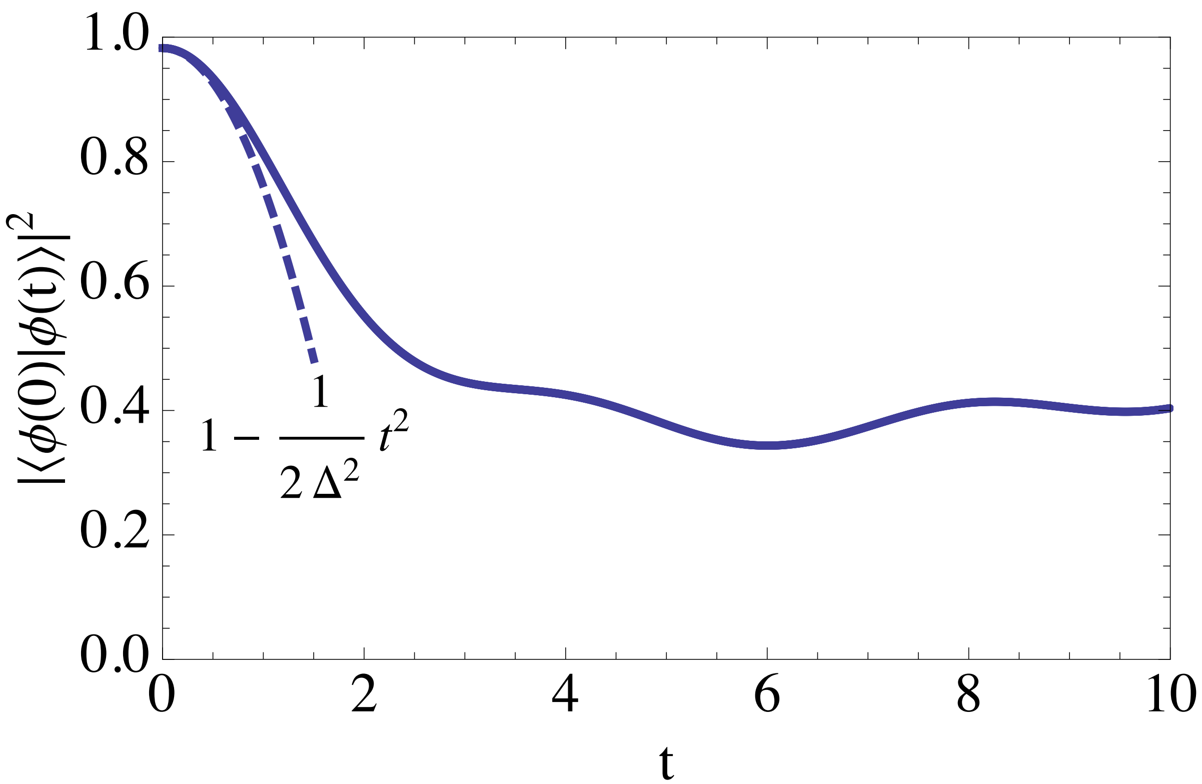

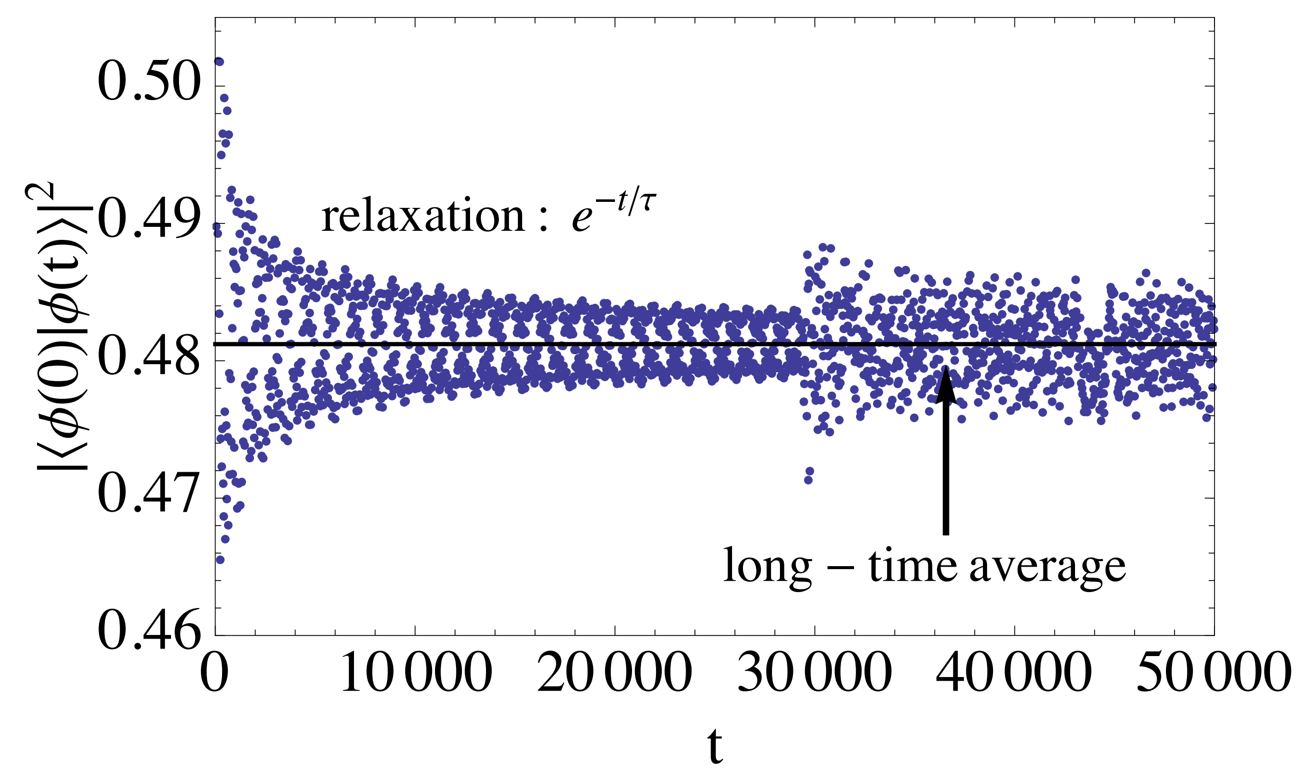

\`Eigenstate thermalization hypothesis'
(Deutsch, Srednicki) does not apply here

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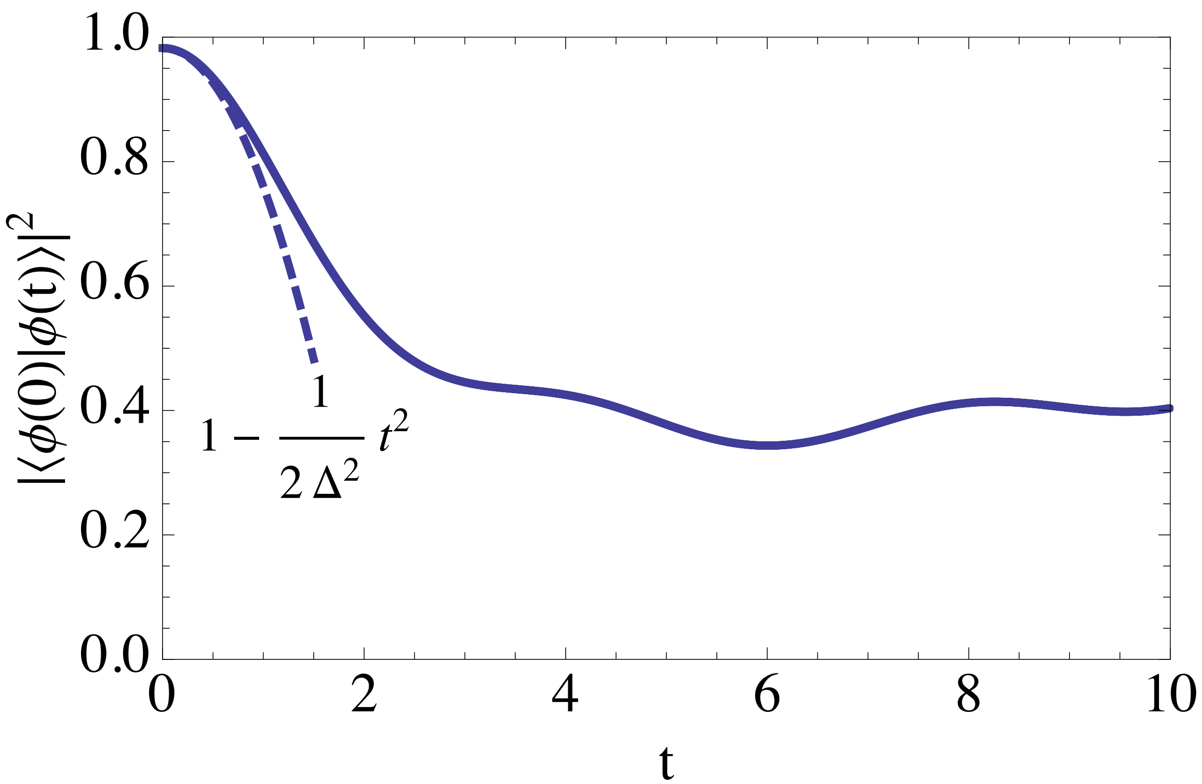

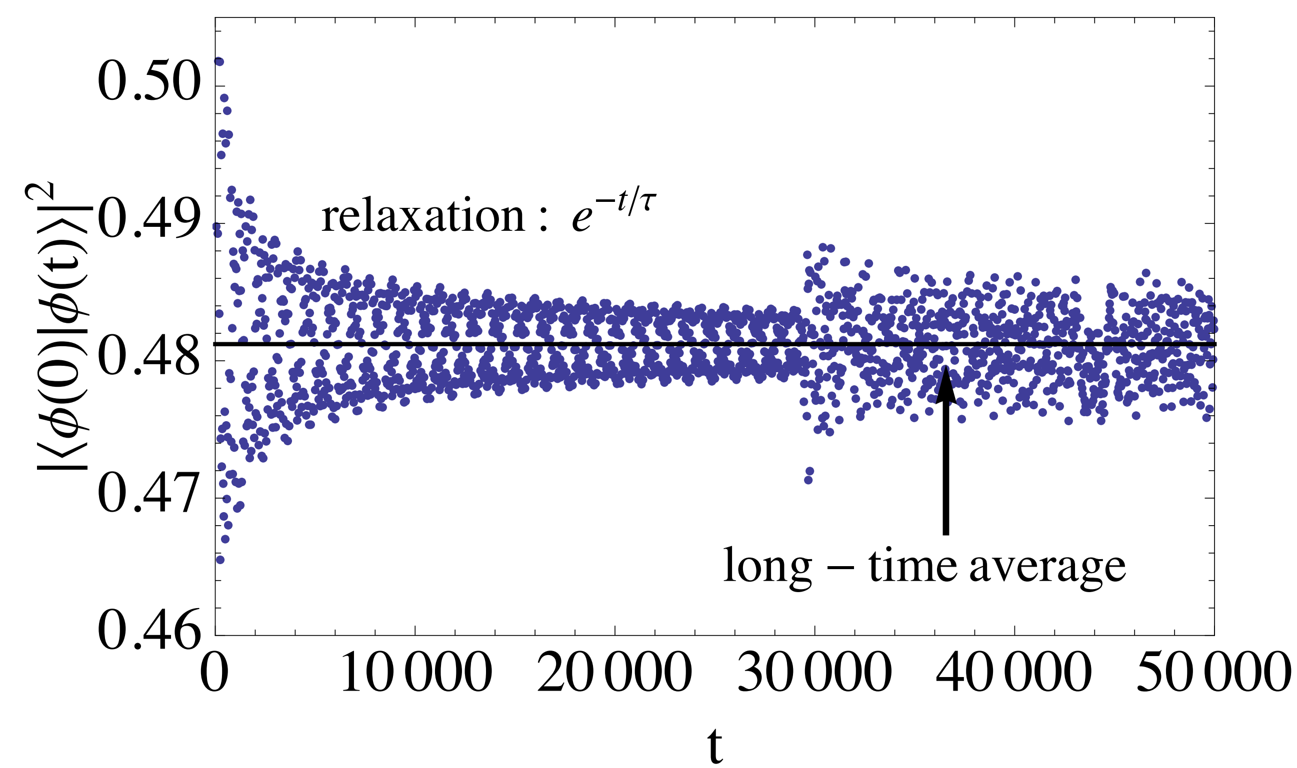

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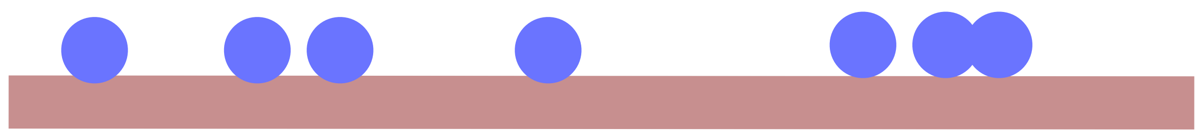
Initial state is \`remembered' at all times

Geometric quenches

J. Mossel, G. Palacios and JSC, 2010

$$t < 0 : x_i \in [0, L_1[$$



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J. Mossel, G. Palacios and JSC, 2010

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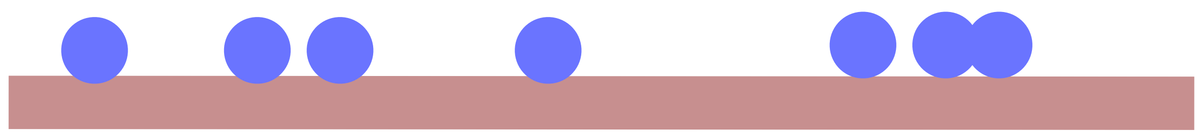

$$t > 0 : x_i \in [0, L_2[$$

Geometric quenches

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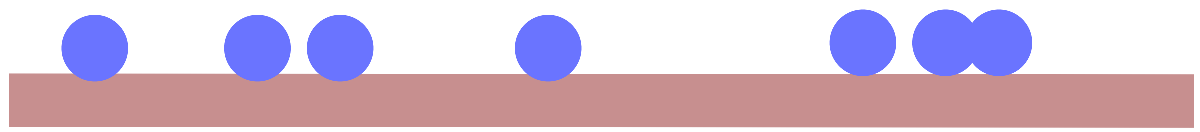

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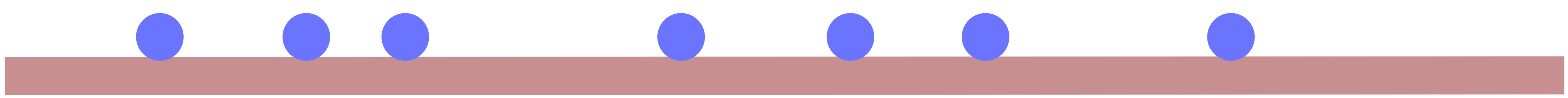

Initial wavefunction: nonlinear mapping

$$\Psi_c^{(1)}(\{x\}|\{\lambda\}_{L_1}) = \begin{cases} \Psi_c^{(2)}(\{x\}|\{\lambda\}_{L_1}), & 0 \leq x_i < L_1, \\ 0 & \text{otherwise} \end{cases}$$

Geometric quenches

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The overlap can in fact be calculated using Slavnov !

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It's just the overlap in the original space domain:

$$\langle \{\lambda_c^{L_1}\} | \{\mu_c^{L_2}\} \rangle = \int_{0 \leq x_1 < x_2 < \ldots \leq L_1} d^N x (\psi_c^{L_1}(\{x_i\}|\{\lambda_i\}))^* \psi_c^{L_2}(\{x_i\}|\{\mu_i\}) = F(\{\lambda\}|\{\mu\})$$

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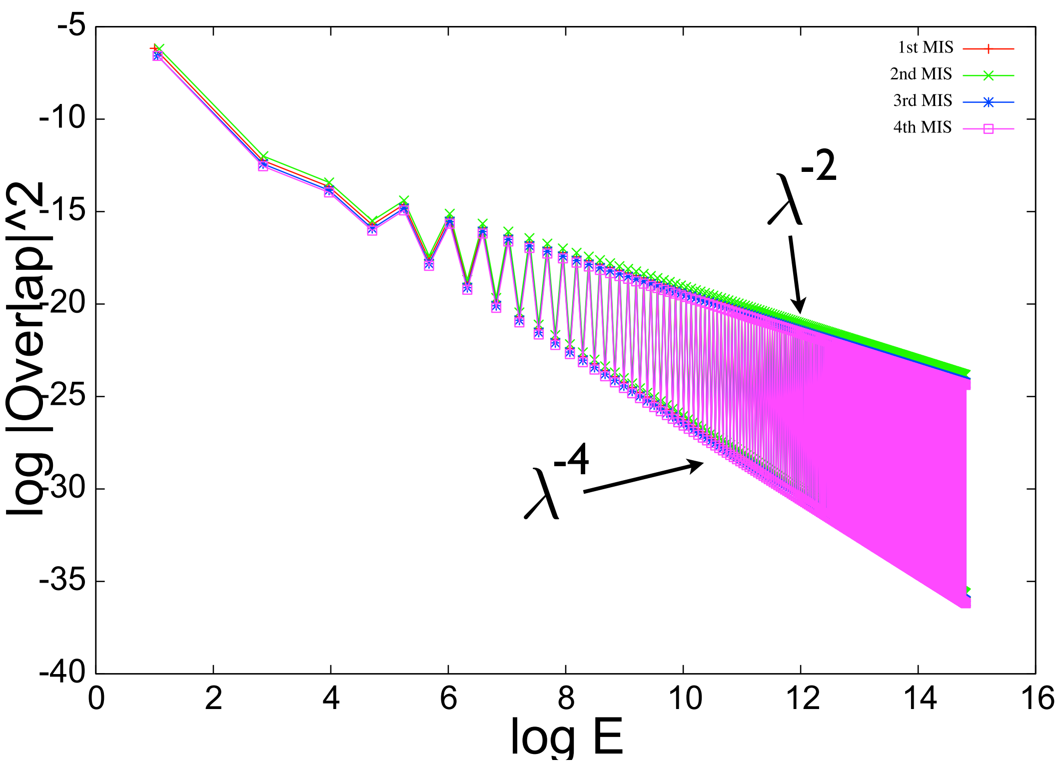

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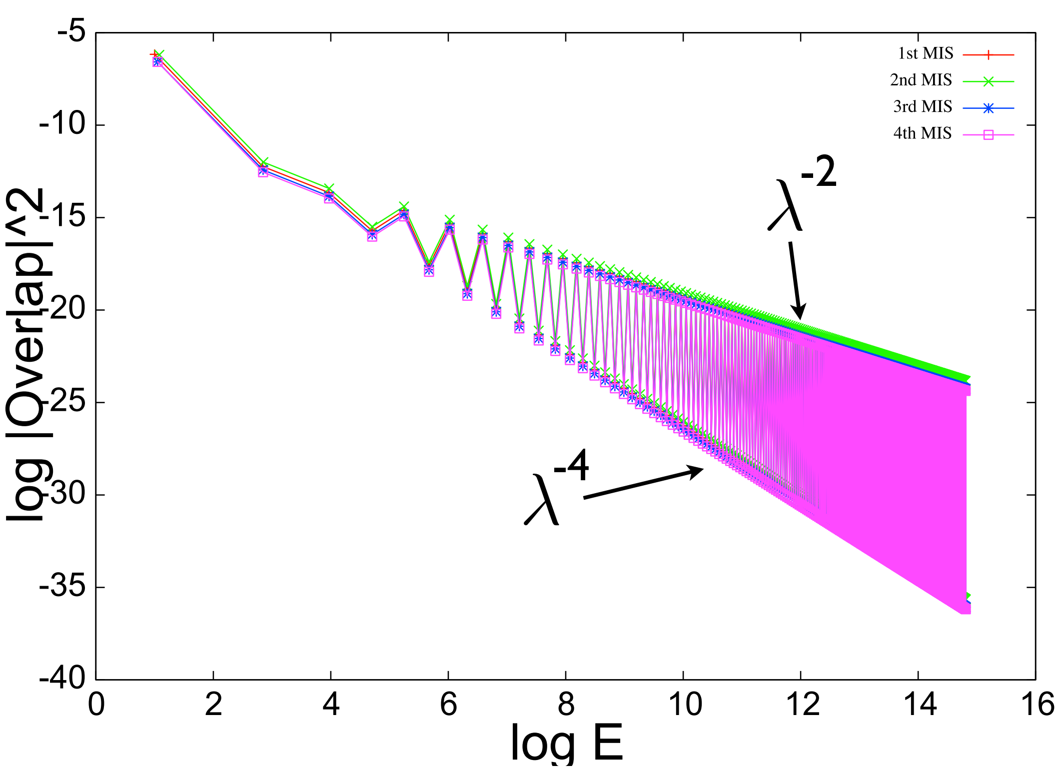
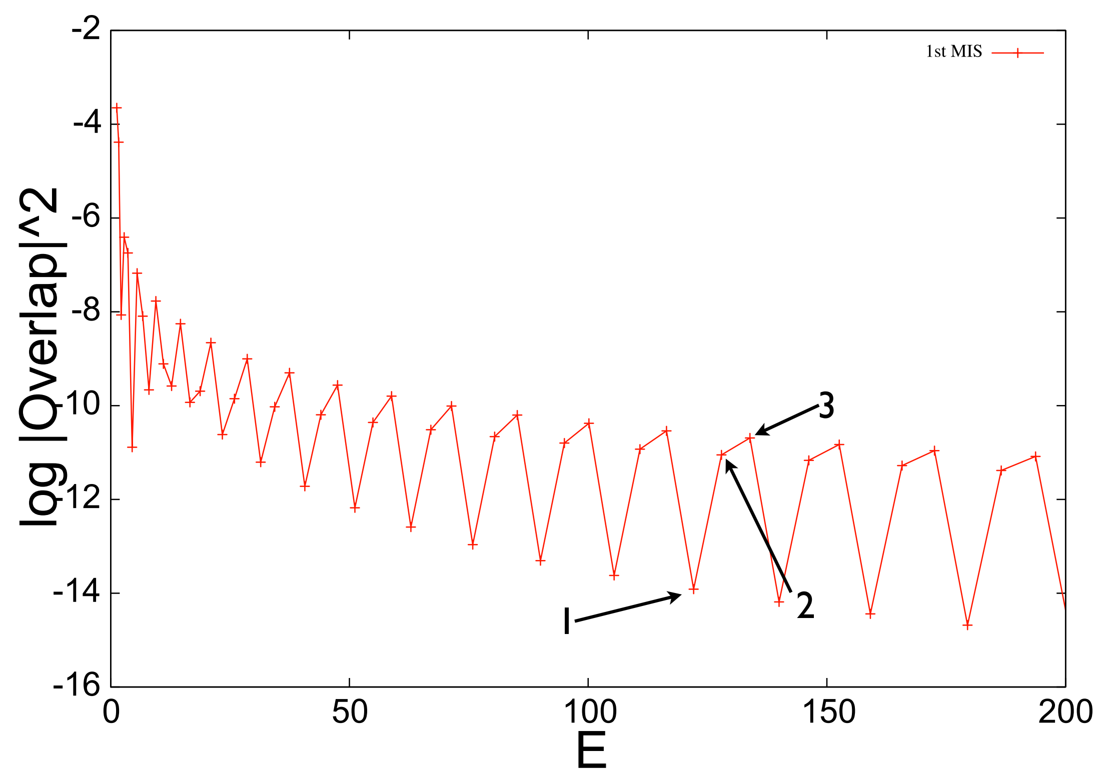
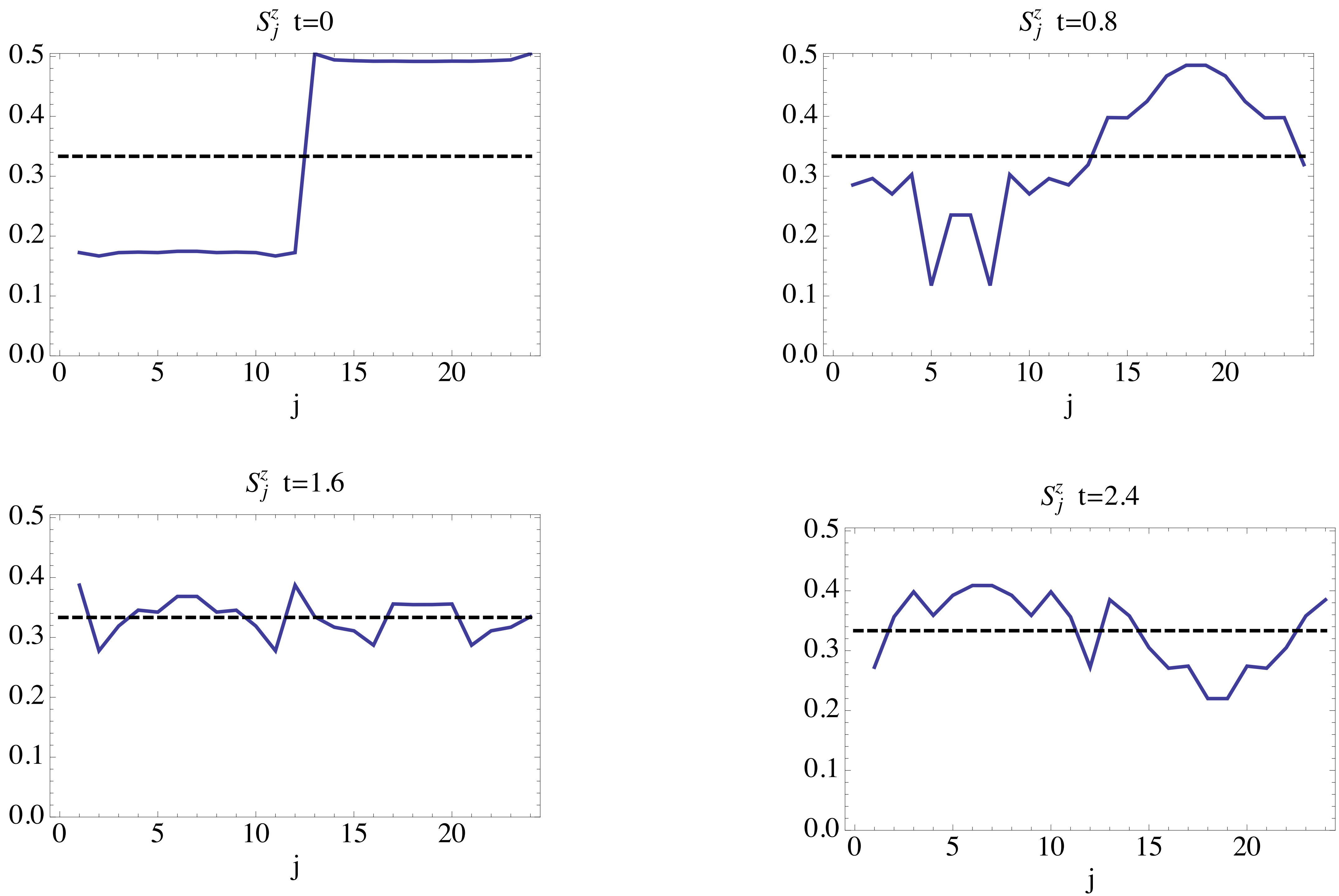




Geometric quench: Heisenberg

‘Release’ M = N/3 from system size N to 2N



Not discussed here...

- Contact with field theory calculations (`Nonlinear Luttinger Liquid' theory)

To do list/work in progress:

- Better classification of solutions to Bethe eqns
- Q group approach: other regimes/polarizations
- Finite temperatures
- Correlations in nested systems
- Quenches from integrability: other cases
- Renormalization from integrable points