RAQIS 2010 – LAPTH Annecy

TBA!

Francesco Ravanini

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA
A.D. 1088

INFN
Istituto Nazionale di Fisica Nucleare

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or…

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To Be Announced !

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Thermodynamic Bethe Ansatz !

Francesco Ravanini

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA
A.D. 1088

INFN
Istituto Nazionale di Fisica Nucleare

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Sine-Gordon TBA and NLIE in Quantum Non-Linear Optics

Francesco Ravanini

ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA A.D. 1088

INFN Istituto Nazionale di Fisica Nucleare

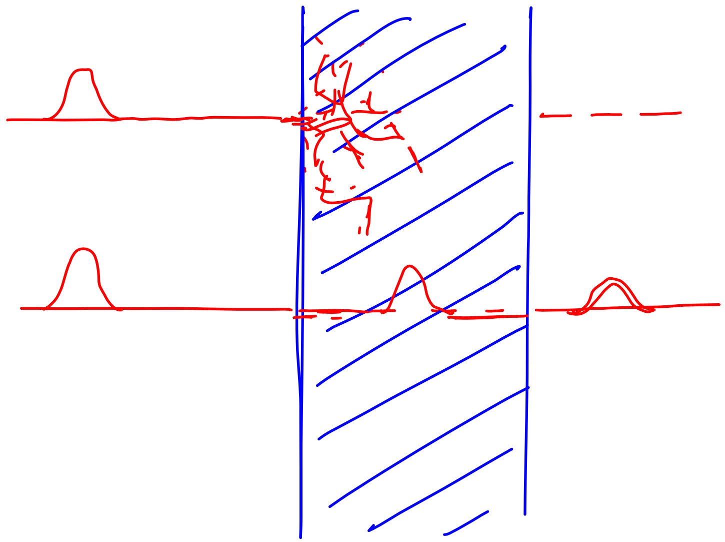
in collaboration with **D. Fioravanti** and **K. Guerrero**

Self Induced Transparency (SIT)

Enlight a dielectic material with monochromatic (laser) light

Input signal:

- coherent pulse
- monocromatic radiation
- short pulse



Dielectric Medium :

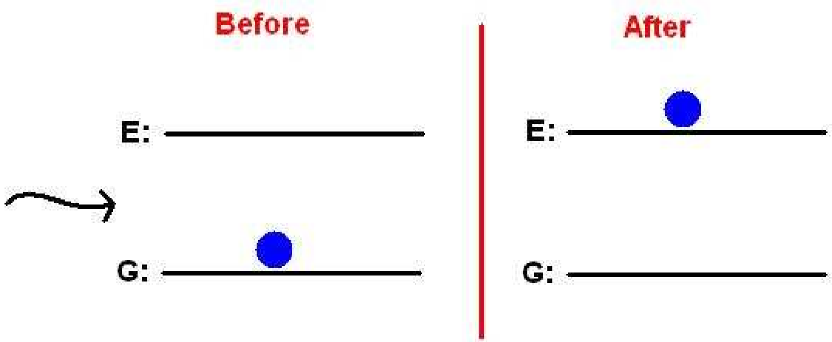
- each atom is a two state system, with energy difference between statates:

Reciprocal conditions:

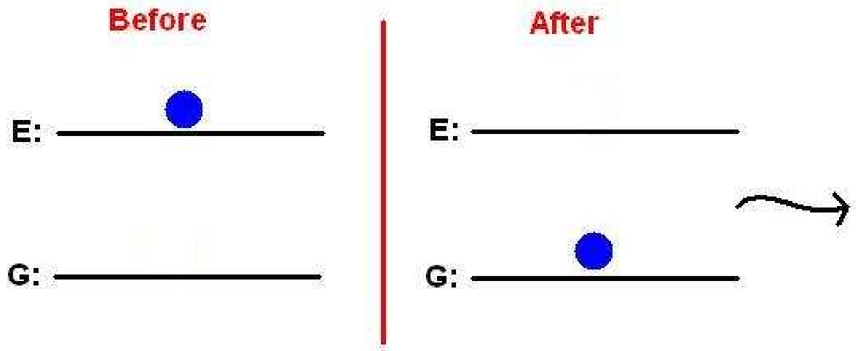
- resonance

- L, dimensions of the dielectric resonant medium
  λ, wavelength

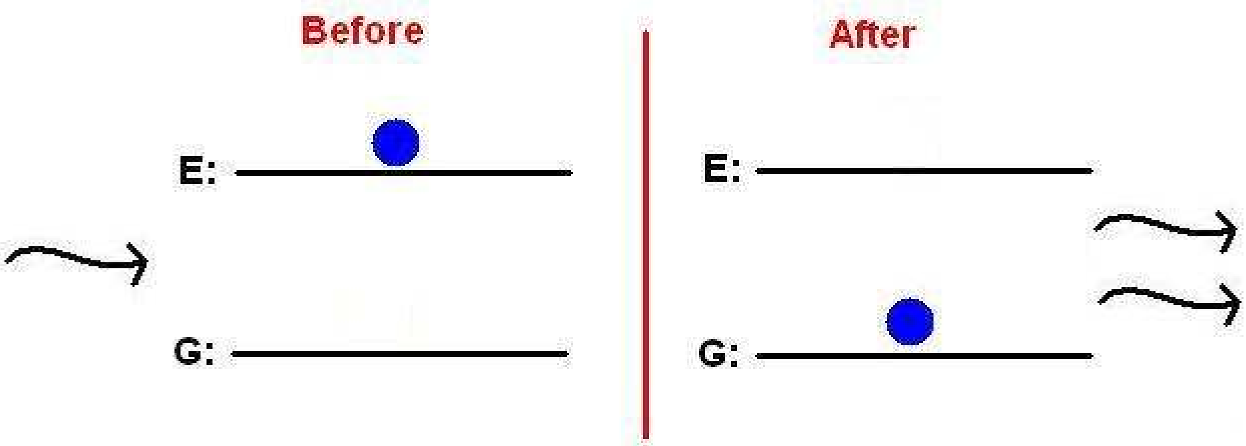
S. McCall and E.L. Hahn 1969.



Absorption



Spontaneous Emission



Stimulated Emission

The radiation energy emitted by the atoms, previously pumped into the excited states, balances the depletion of energy of the first half of the pulse .

The Self Induced Transparency effect occurs when the input energy is bigger than a critical energy.

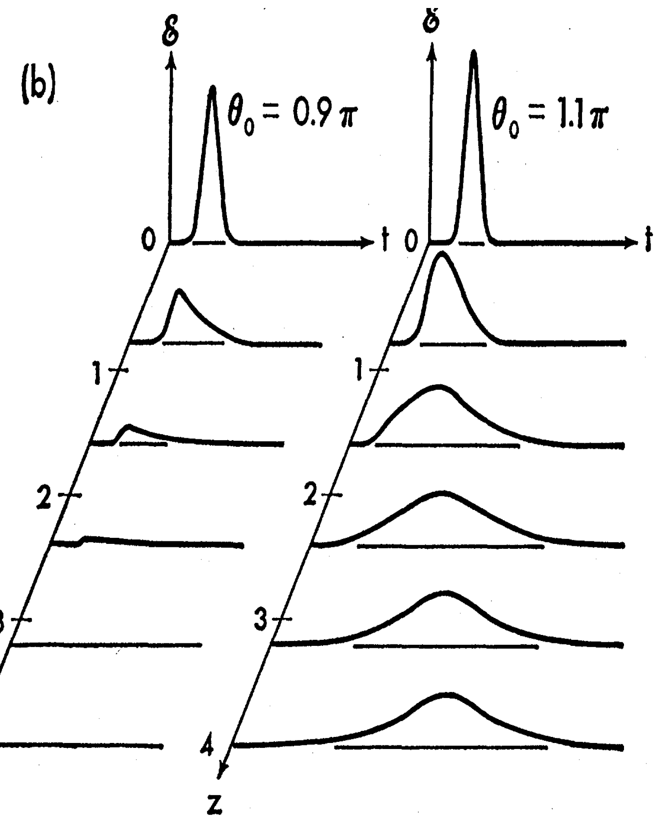
⇩

the medium becomes transparent

The output signal presents:

- anomalous constant energy
- delay
- phase shift
- constant velocity
- constant width

Envelope of the resulting signal: solitonic shape



S. McCall and E.L. Hahn 1969.

Dynamics of the system: classical radiation propagating through a dielectric resonant medium.

Atom: two states system with energy difference:

$$H_0|\psi_2\rangle - H_0|\psi_1\rangle = \hbar\omega_0$$

$$|\psi_1\rangle = \text{gruond state} \qquad |\psi_2\rangle = \text{excited state}$$

Atomic Hamiltonian in the presence of electromagnetic radiation

$$H_{atom} = H_0 - \vec{p}\cdot\vec{E}$$

$\vec{p}$ Electric momentum dipole operator

Electric field

$$\vec{E} = \hat{n}E(x,t) \quad \text{with} \quad \hat{n}\cdot\hat{x} = 0$$

Assuming spherical symmetry, the only non null elements of the operator:

$$\langle\psi_2|\vec{p}\cdot\vec{E}|\psi_1\rangle = \langle\psi_1|\vec{p}\cdot\vec{E}|\psi_2\rangle^*$$

can be parameterized as follows:

$$\langle\psi_2|\vec{p}\cdot\vec{E}|\psi_1\rangle = pE\exp(-i\alpha)$$

$$\text{where } p = |\vec{p}|,\ E = |\vec{E}|$$

Atomic Hamiltonian in terms of Pauli matrices:

$$H_{atom} = -\frac{1}{2}\hbar\omega_0\sigma_3 - E(x,t)(p_1\sigma_1 + p_2\sigma_2)$$

$$p_1 = p\cos(\alpha)$$
$$p_2 = p\sin(\alpha)$$

Plane wave electric field:

$$E(x,t) = \mathcal{E}(x,t)\cos(\omega t - kx)$$

$\mathcal{E}$ = wave packet envelope (slowly varying)

$$\partial_t \mathcal{E} \ll \omega\mathcal{E} \quad , \quad \partial_x \mathcal{E} \ll k\mathcal{E}$$

Dipole momentum per unit volume $\vec{P} = \hat{x}n\left(p_1\langle\sigma_1\rangle + p_2\langle\sigma_2\rangle\right)$

n = number of atoms per unit volume

Dynamics of the system from Maxwell equations

$$\left(\partial_x^2 - \frac{1}{\bar{c}^2}\partial_t^2\right)E(x,t) = \frac{4\pi}{c^2}\partial_t P(x,t)$$

$$\bar{c} \equiv c\sqrt{\varepsilon}$$

Dynamic equation for polarization from Schrödinger equation for the atom

$$i\hbar\partial_t\langle\sigma_i\rangle = \langle\psi|[\sigma_i, H_{atom}]|\psi\rangle$$

Parallel and perpendicular expectation values of Pauli matrices:

$$\langle\sigma_\parallel\rangle \equiv \langle\sigma_1\rangle\cos(\omega t-kx)+\langle\sigma_2\rangle\sin(\omega t-kx)$$
$$\langle\sigma_\perp\rangle \equiv -\langle\sigma_1\rangle\sin(\omega t-kx)+\langle\sigma_2\rangle\cos(\omega t-kx)$$

Equation of motion indicating how total induced polarization depends on time:

$$\begin{cases} \partial_t\langle\sigma_\parallel\rangle = (\omega-\omega_0)\langle\sigma_\perp\rangle \\ \partial_t\langle\sigma_\perp\rangle = -\dfrac{\mathcal{E}(x,t)}{\hbar}p\langle\sigma_3\rangle + (\omega-\omega_0)\langle\sigma_\parallel\rangle \\ \partial_t\langle\sigma_\perp\rangle = \dfrac{\mathcal{E}(x,t)}{\hbar}p\langle\sigma_\perp\rangle \end{cases}$$

↓

$$\frac{2\omega}{\bar{c}}\left[\left(\partial_x+\frac{1}{\bar{c}}\partial_t\right)\mathcal{E}(x,t)\right]\sin(\omega t-kx)=\frac{2\pi}{c^2}np\omega\bar{c}\left[\sin(\omega t-kx)\langle\sigma_\perp\rangle\right]$$

Initial state of atoms: ground state $\Longrightarrow$ $\langle \sigma_3 \rangle = 1$

Resonant conditions $\omega = \omega_0$ $\Longrightarrow$ $\langle \sigma_{\parallel} \rangle = 0$

$\Longrightarrow$ $\sum_i \langle \sigma_i \rangle = 1,$

Parametrize $\begin{cases} \langle \sigma_3 \rangle = \cos(\beta_{cl}\phi(x,t)) \\ \langle \sigma_\perp \rangle = \sin(\beta_{cl}\phi(x,t)) \end{cases}$ $\Longrightarrow$

$$\partial_t \phi(x,t) = -\frac{p}{\beta_{cl}\hbar}\mathcal{E}(x,t)$$

- Assume that the envelope and the total polarization slowly vary w.r.t. ω
- Express the polarization per unit volume in terms of $\langle \sigma_\perp \rangle$
- Recall the inital condition $\langle \sigma^3 \rangle = 1$
- Define $x' = 2x - \overline{c}t$

to obtain….

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The sine-Gordon equation in 1+1 dimensions

$$\left(\partial_t^2 - \bar{c}^2 \partial_x^2\right)\phi(x,t) = -\frac{\mu^2}{\beta_{cl}} \sin\left(\beta_{cl}\phi(x,t)\right)$$

$$\partial_t \phi(x,t) = -\frac{p}{\beta_{cl}\hbar}\mathcal{E}(x,t)$$

$$\mu^2 = \frac{2\pi n p^2 \omega}{\hbar \varepsilon_0}$$

p	magnitude of the dipole momentum components
n	number of atoms per unit volume
ε_0	vacuum dielectric constant
β_{cl}	parameter (see below)
$\mathcal{E}(x,t)$	electric field envelope

↓

Classical sine-Gordon model

Inverse Scattering Method → Exact solutions: solitons, antisolitons, breathers

Ablowitz, Kaup, Newell, and Segur 1973

Dynamics of the system: quantized radiation

Consider the kinetic terms of sine-Gordon and Maxwell actions:

$$S_{sG} = \frac{1}{\bar{c}} \int dx' dt \left[\frac{1}{2} (\partial_t \phi)^2 \right] \qquad S_{Maxwell} = \frac{1}{\bar{c}} \int d^3 x dt \left[\frac{1}{2} \left(\partial_t \vec{A} \right)^2 \right]$$

$$\vec{A} = \text{vector potential} \quad \Rightarrow \quad \vec{E} = -\frac{1}{c} \partial_t \vec{A}$$

So φ is the envelope of A → Kinetic term of the Maxwell action

$$S_{Maxwell} = \mathcal{A} \left(\frac{\beta_{cl} \hbar}{p} \right)^2 \int dx dt \left[\frac{1}{2} (\partial_t \varphi)^2 \right] \qquad \mathcal{A} = \text{cross sectional area of the pulse}$$

Identification between sine-Gordon and Maxwell actions gives

$$\beta_{cl}^{\ 2} = \frac{4 p^2 \sqrt{\varepsilon_0}}{\mathcal{A} \hbar^2 c}$$

A. Le Clair 1995

Dynamics with quantized electromagnetic field:

↓

Sine-Gordon quantum field Theory!!

$$S_{SG} = \frac{1}{\bar{c}} \int dxdt \left[\frac{1}{2} (\partial_t \varphi)^2 - \frac{\bar{c}}{2} (\partial_x \varphi)^2 + \frac{\mu^2}{\beta^2} \cos(\beta\varphi) \right]$$

Normalized sine-Gordon field $\varphi \to \sqrt{\hbar}\varphi$

Coupling constant: $\beta^2 = \hbar \beta_{cl}{}^2$

In the McCall and Hahn experiment $\beta^2 \ll 1$ → Deep attractive regime

Recently also higher β but always attractve

Quantum spectrum

Rapidity

$$E(\theta) = m\cosh\theta \qquad P(\theta) = m\sinh\theta \qquad v(\theta) = \tanh\theta$$

$$x' = 2x + \bar{c}t \quad \Rightarrow \quad E'(\theta) = me^{-\theta} \quad , \quad v'(\theta) = 1 + \tanh\theta$$

1-st breather = envelope photon (collective state of photons)

n-th breather = bound state of n 1-st breather

soliton – antisoliton = nonperturbative state of envelope photons

$$m_s = c(\beta)\left(\frac{\mu\hbar}{\bar{c}^2}\right)^{p-1}\Lambda^{-p} \qquad , \qquad p = \frac{\beta^2}{8\pi - \beta^2}$$

$$m_n = 2m_s \sin\frac{\pi n p}{2}$$

Intensity of the outcoming pulse

$$\left.\begin{aligned} I &= \frac{\bar{c}}{4\pi\omega}\int_0^{\omega} E^2(x,t)dt \\ I_{env} &= \frac{\bar{c}}{4\pi\omega}\int_0^{\omega} \mathcal{E}^2(x,t)dt \end{aligned}\right\} \Rightarrow I = \frac{I_{env}}{2}$$

Computable in the Sine-Gordon particle picture

$$I_{env} = \sum_a \int_0^{\infty} \rho_a(\theta)\left[E_a(\theta)v_a(\theta) + E_a(-\theta)v_a(-\theta)\right]d\theta$$

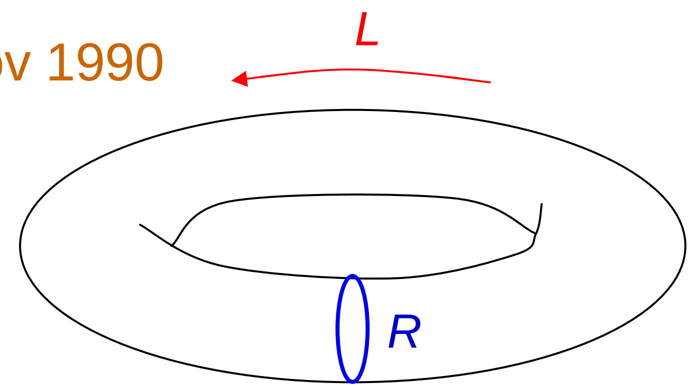
$\rho_a(\theta) =$ density of particles of type a and rapidity θ

$$I_{env} = \sum_a \frac{m_a \bar{c}^3}{2}\int_0^{\infty} \frac{\rho_a(\theta)}{\cosh\theta}d\theta$$

Density ρ_a computable in the framework of **TBA**

T.B.A. (Thermodynamic Bethe Ansatz)

Yang-Yang 1969 - Al. Zamolodchikov 1990



Integrability (1+1 D)

Thermodynamic Bethe Ansatz

	space	time	space	time
	R	$L \to \infty$	$L \to \infty$	$R = T^{-1}$
	finite size effect		thermodynamics	
Hamiltonian:	$H_R(R) = \int_0^R T_{ll}\, dr$		$H_L = \int_0^\infty T_{rr}\, dl$	
Hilbert space:	$\mathcal{R}$		$\mathcal{L}$	
Partition function	$\mathrm{Tr}_{\mathcal{R}}[e^{-LH_R(R)}]$	$=$	$\mathrm{Tr}_{\mathcal{L}}[e^{-RH_L}]$	
	$\approx e^{-RE_0(R)}$		$= e^{-LRf(R)}$	

Scaling function of the vacuum (Casimir effect)

$$E_0(R) = Rf(R) \quad \Rightarrow \quad c(r) = -\frac{6}{\pi} f(r) \quad , \quad r = MR$$

Form of S-matrix

$$S_{0,j}(\theta-\theta_j) = \sigma(\theta-\theta_j) R_{o,j}(\theta-\theta_j) = (\text{dressing factor}) \times (\text{R-matrix})$$

Compute *f(r)*: Dynamics dictated by **Bethe-Yang eqs**. in the thermodynamic limit

$$e^{ipR} \prod_{j=1}^{N} S(\theta-\theta_j) = 1 \Rightarrow \begin{cases} e^{-ir\sinh\theta} = \prod_j \sigma(\theta-\theta_j) \cdot \mathbf{T}(\theta \mid \{\theta_j\}) \\ \mathbf{T}(\theta \mid \{\theta_j\}) = \mathrm{Tr}_0 \left(\prod_{j=1}^{N} R_{0,j}(\theta-\theta_j) \right) = \text{color transfer matrix} \end{cases}$$

Diagonalize color transfer matrix by Bethe ansatz (Al. Zamolodchikov, 1991)

Example: Sine-Gordon – S-matrix (Zam-Zam, 1979) has a dressing factor

$$\sigma(\theta) = \exp \int_{-\infty}^{+\infty} \frac{dk}{2\pi k} \frac{\sinh\left(\frac{\pi k}{2}(p+1)\right)}{2\sinh\left(\frac{\pi k}{2}p\right)\cosh\left(\frac{\pi k}{2}\right)}$$

$$p = \frac{\beta^2}{8\pi - \beta^2}, \quad \gamma = \frac{\pi}{p+1}$$

and a matrix part coinciding with the XXZ spin ½ R-matrix

$$R(\theta) = \frac{1}{\sinh(i\gamma - \theta)} \begin{pmatrix} \sinh(i\gamma-\theta) & & & \\ & \sinh\theta & i\sin i\gamma & \\ & i\sin i\gamma & \sinh\theta & \\ & & & \sinh(i\gamma-\theta) \end{pmatrix}$$

The color transfer matrix is diagonalized by the fully inhomogeneous XXZ spin ½ Bethe ansatz

Bethe equations $$\prod_{r=1}^{N} s_{1/2}(\theta_j - \theta_r) = \prod_{k=1}^{M} s_1(\theta_j - \theta_k)$$

where $$s_\nu(x) = \frac{\sinh \frac{\gamma}{\pi}(x + i\pi\nu)}{\sinh \frac{\gamma}{\pi}(x - i\pi\nu)}$$

Eigenvalues $$\Lambda_{\{\theta_j\}}(\theta \,|\, \{\theta_r\}) = \prod_{j=1}^{M} s_{1/2}(\theta + \theta_j)$$

$$+ \text{ terms vanishing for } N \to \infty$$

Full set of Bethe-Yang equations for Sine-Gordon given by eigenvalues coupled to the exp(iPL) term and Bethe equations for the Bethe roots (magnon excitations)

$$\theta_n, \theta_r = \text{particle rapidities} \qquad u_j, u_k = \text{magnons}$$

$$e^{-ir\sinh\theta_n} = \prod_{r=1}^{N} \sigma(\theta_n - \theta_r) \prod_{j=1}^{M} s_{1/2}(\theta_n - u_j)$$

$$1 = \prod_{r=1}^{N} s_{1/2}(u_j - \theta_r) \prod_{k=1}^{M} s_1(u_j - u_k)^{-1}$$

String hypotesis for the Bethe roots: in the thermodynamic limit the Bethe roots tend to organize as follows:

$$u_{j,q}^{(n)\pm} = u_j^{(n)\pm} + \frac{i\pi}{2}(n+1-2q) \qquad q = 1,\ldots,n$$

$$u_j^{(n)+} = u_j^{(n)} = n-\text{string centres (not necessarily roots)} \in \mathbb{R}$$

$$u_j^{(n)-} = u_j^{(n)} + \frac{i\pi}{2}(p+1) \qquad \text{(strings of the second kind)}$$

In the thermodynamic limit the number of roots tends to infinity and they become dense.

Intorduce density for each type of n-string and for the corresponding holes

$$\rho_n(\theta) = \text{density of centres of strings of type } n$$
$$\overline{\rho}_n(\theta) = \text{density of centres of holes of type } n$$

Logs of Bethe-Yang equations give coupled integral eqs. for the densities

$$\rho_n(\theta) + \overline{\rho}_n(\theta) = \nu_n(\theta) + \sum_m K_{n,m} * \rho_m(\theta)$$
$$\text{where} \qquad \nu_n(\theta) = \delta_{n,0} r \cosh\theta = \text{driving term}$$

From density: compute energy, entropy and free energy of the system.
Minimum of free energy gives the conditions (TBA equations)

$$\nu_n(\theta) = \varepsilon_n(\theta) - \frac{1}{2\pi}\sum_m (K_{nm} * \log(1+e^{-\varepsilon_m}))(\theta)$$

$$y_n(\theta) = e^{\varepsilon(\theta)} = \frac{\bar{\rho}_n(\theta)}{\rho_n(\theta)} \qquad K_{nm}(\theta) = \frac{1}{\cosh\theta} H_{nm}$$

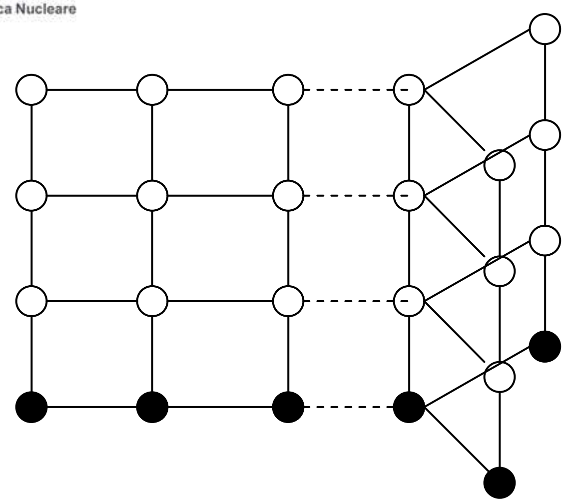
$$H = \text{adiacency matrix of a (magnon) graph } \mathcal{H}$$

If more particles, each type is coupled to an equation with its mass term

$$\varepsilon_a^i = \nu_a^i - \frac{1}{2\pi}\sum_b (K_{ab} * \log(1+e^{\varepsilon_b^i})) + \frac{1}{2\pi}\sum_j H_{ij}(K * \log(1+e^{-\varepsilon_a^j}))$$

$$K(\theta) = \frac{\text{const.}}{\cosh\theta} \qquad K_{ab}(\theta) = -i\frac{d}{d\theta}\log S_{ab}(\theta)$$

$$\nu_a^i(\theta) = \delta^{i,M} m_a R \cosh\theta \qquad \text{(periodic b.c.)}$$



$\mathcal{H}$ (magnons)

FR, 1992

Hollowood, 1994

FR, Tateo, Valleriani, 1993
(Dynkin TBA's)

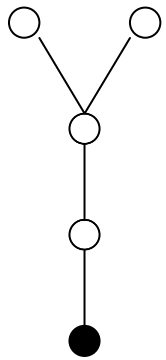
$\mathcal{G}$ (Dynkin) masses $m_a = M\psi_{\mathcal{G}}{}^a$ Perron-Frobenius

For $\mathcal{G} = A_1$

Al.Zam. 1991-92

$\mathcal{M}_p(\text{Vir}) + \varphi_{13}$

Sine-Gordon at $\dfrac{\beta^2}{8\pi} = \dfrac{p}{p+1},\ p \in \mathbb{N}$

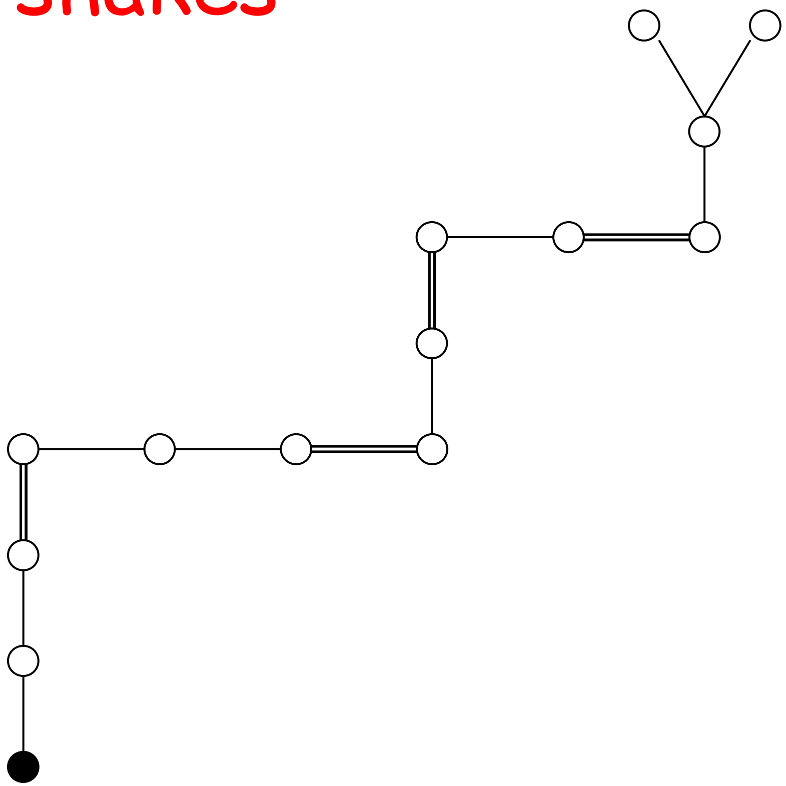


H diagram can be **Dynkin** or **extended Dynkin** (or something **else** ???)
(Quattrini, FR, Tateo, 1993)

Sine-Gordon for general p rational → Continued Fraction

Takahashi, Suzuki (1972) – Mezincescu, Nepomechie (1990) – Tateo (1994)

Tateo snakes



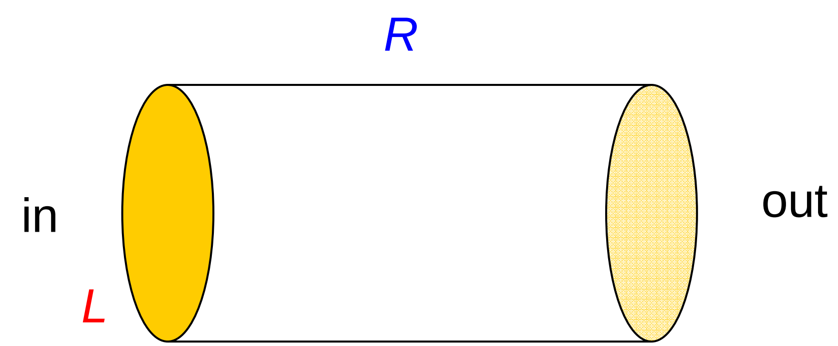
Emission form an excited atom

- initial state = excited state
- final state = ground state

$$\phi_{in} = \frac{\pi}{\beta} \quad , \quad \phi_{fin} = 0$$

after a time $t = iR$ -- L direction periodic, $L \to \infty$

We need TBA with fixed (Dirichlet) boundaries



TBA with non-trivial boundaries

Fixed or free boundaries

Generalization to a spatial region limited by conditions that preserve integrability

K, K'

Reflection matrices off the boundary

Dirichlet boundary conditions:

-diagonal reflection matrices

-topological charge conservation

sine-Gordon fixed boundary conditions: $\psi = \psi_0, \quad t = 0$ with arbitrary ψ_0

- Factorizable boundary scattering theory. (crossing symmetry condition generalized to the boundary reflexion matrices)
- Bethe-Yang equations with Boundary conditions

S. Goshal, Al. Zamolodchicov 1993

TBA equations with Boundary conditions

$$\varepsilon_a = \nu_a - \sum_b \Phi_{ab} * L_b = 0$$

$$\nu_a = 2m_a R\cosh(\theta) - \log\left[K_a\left(K_a'\right)^*\right]$$

LeClair et al. 1995

TBA application to SIT

- Boundary fixed conditions:

  - $t = 0$ atoms in an excited state

  - $t = \bar{t}$ atoms in ground state

- TBA with the particle spectrum:

$$a \in \{1, \dots, N-1, +, -\}$$

The solutions of TBA equations with Boundary conditions for the sine-Gordon field theory, are known in the situation of the restricted values :

$$\frac{\beta^2}{8\pi} \approx \frac{1}{N+1} \quad , \quad N \in \mathbb{N}$$

For those values S matrix is diagonal

Pairs of particles density

$$\rho_a(\theta)$$

Intensity of Emitted radiation

$$I_{env} = \sum_a \frac{m_a \bar{c}^3}{2} \int_0^\infty d\theta \frac{\rho_a(\theta)}{\cosh(\theta)}$$

A. Le Clair 1995

In any purely massive TBA

$$\rho_a = \frac{L}{2\pi} \frac{1}{1+e^{\varepsilon_a}} \frac{\partial \varepsilon_a}{\partial R}$$

so at reflectionless points

$$I_{env} = \frac{m\overline{c}^3}{2} \sum_a \psi_a \int d\theta \frac{1}{\cosh\theta} \frac{\partial}{\partial R} \log(1+e^{-\varepsilon_a(\theta)})$$

At non-reflectionless points, the TBA analysis becomes cumbersome, at irrational points TBA is an infinite set of eqs. (useless)

Luckily, there is an alternative: NLIE (or DDV)

(Klumper Batchelor Pearce – Destri DeVega 1991…)

(Bologna Group 1996-…)

$$\begin{cases} h(\theta) = ML\sinh\theta + \sum_k c_k \chi(\theta-\theta_k) + 2\,\mathrm{Im}\int_{\mathbb{R}+i\varepsilon} dx\, G(\theta-x)\log(1+(-)^{\delta} e^{ih(x)}) \\ h(\theta_k) = 2\pi I_k \end{cases}$$

sources

$$\chi(\theta) = 2\pi\int_0^{\theta} G(x)dx \qquad c_k = \begin{cases} 1 & \text{holes} \\ -1 & \text{roots} \end{cases}$$

Kernel

$$G(\theta) = \int \frac{dk}{2\pi} e^{ik\theta} \frac{\sinh(p+1)\frac{\pi k}{2}}{\sinh\frac{\pi pk}{2}\cosh\frac{\pi k}{2}}$$

$$E = M\sum_k c_k \cosh_{(k)}\theta_k + 2\,\mathrm{Im}\int d\theta \sinh\theta \log(1+e^{ih(\theta+i\varepsilon)})$$

Comparison between energy expressions from TBA and Sine-Gordon NLIE at reflectionless points shows that

$$\sum_a \psi_a \log(1+e^{-\varepsilon_a}) = \frac{1}{2}\left[\log\left(1+e^{iZ(\theta+i\pi/2)}\right) - \log\left(1+e^{iZ(\theta-i\pi/2)}\right)\right]$$

At reflectionless points, the intensity can be written also in terms of NLIE

$$I_{env} = \frac{m^2 \bar{c}^3}{4} \int d\theta \frac{1}{\cosh\theta} \frac{\partial}{\partial \ell} \operatorname{Im} \log(1 + e^{iZ(\theta + i\pi/2)})$$

$$\ell = mR = \text{dimensionless scale in NLIE}$$

We conjecture that:

This formula is valid not only at reflectionless points, but it may be extended at all values of *β*

This is not the usual resummation of massless nodes into a massive one (Balog, Hegedus 2003…) à la Hirota (Gromov, Kazakov, Vieira 09)

It is better interpreted as an equivalence between different realizations of the same model: e.g.

$$\mathcal{W}(D_N) \text{ lowest minimal model} + \phi_{id,adj} \quad \Leftrightarrow \quad \text{Sine-Gordon at } \frac{\beta^2}{8\pi} = \frac{1}{N+1}$$

Extensions:

- frequency detuning
- frequency modulation

} of the pulse → **Complex Sine-Gordon** theory

$$S=\frac{1}{2}\int d^2x\left[\partial_\mu\phi\partial^\mu\phi+4\cot^2\phi\partial_\mu\eta\partial^\mu\eta-\frac{\mu^2}{2\beta^2}\cos\beta\phi\right]$$

Difficulties in quantizing the theory

- S-matrix known only for a quantized set of β (N.Dorey, Hollowood)
- TBA known (Miramontes et al.) but NLIE not known
- links with other models (Fateev's Complex Sinh-Gordon, Sausage…)

Equivalent sigma model

SU(2)/U(1) HSG theory

$$S=\int d^2x\left(\frac{\partial_\mu\psi\partial^\mu\psi}{1-\beta^2|\psi|^2}+m^2\psi^2\right)$$

Opens the road to higher HSG theories describing more refined situations:

- **Degeneration** of states of the atom
  - not taken into account in SG formulation
  - usually breaks integrability
    - ex: two-level atom with two-fold degeneration → Double Sine-Gordon
  - but some cases are expressible with higher HSG theories
    - ex: SU(3)/U(2) HSG theory for p→s, s→p, p→p transitions
- **Multi-level systems** break integrability but at some values of parameters
  - ex: 3-level degenerate system → SU(5)/U(4) HSG theory

TBA known, NLIE not known.

This stimulates study of NLIE for HSG theories

THANK YOU