(e^+, e^-) annihilation

(e^+, e^-) annihilation into two photons

$$e^+ + e^- \to \gamma + \gamma$$

(need two γ for momentum conservation, if the e^- is assumed to be free).

Theoretically, (e^+, e^-) annihilation is related to Compton scattering by crossing symmetry:

- incoming $e^+ \leftrightarrow$ outgoing e^-
- outgoing $\gamma \leftrightarrow$ incoming γ

total cross section per atom

The cross-section formula of Heitler is used [Heitl54]:

$$\sigma(Z,E) = \frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

$$\begin{aligned} E &= \text{total energy of the incident positron} \\ \gamma &= E/mc^2 \\ r_e &= \text{classical electron radius} \end{aligned}$$

The cross section decreases with increasing E.

The nonrelativistic limit is:

$$\sigma_{nr}(Z,E) \sim \frac{Z\pi r_e^2}{\beta}$$

Mean free path

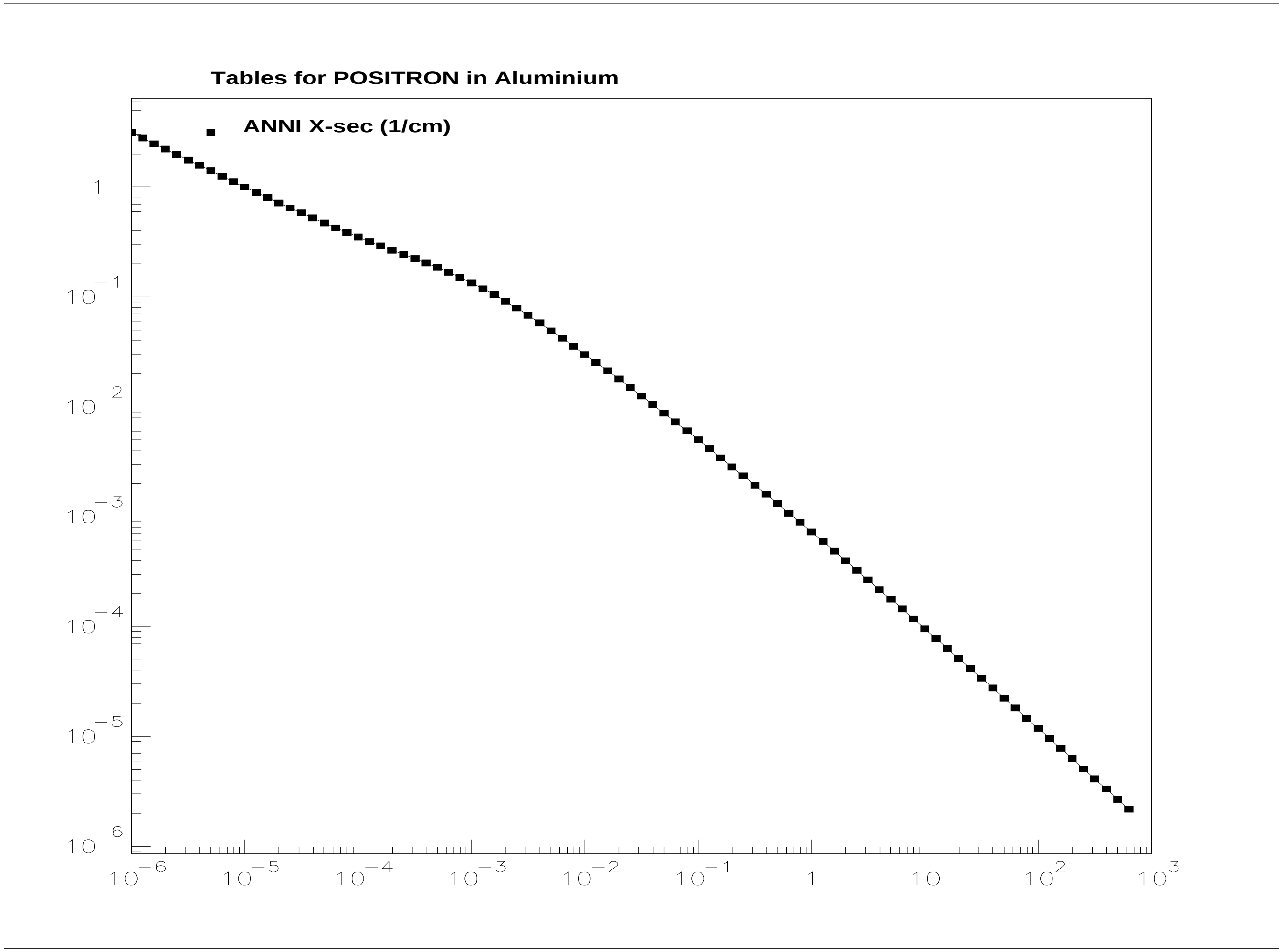
$$\lambda(E) = \left(\sum_i n_{ati} \cdot \sigma(Z_i, E) \right)^{-1}$$

n_{ati} : nb of atoms per volume of the i^{th} element in the material.

At initialization stage, the function `BuildPhysicsTables()` computes and tabulates :

- crossSectionPerAtom for all elements
- meanFreePath for all materials

number of interactions per cm in Aluminium



positron kinetic energy (GeV)

kinematical limits

$$e^+ \, e^- \rightarrow \gamma_a \, \gamma_b$$

The incident e^+ has a total energy: $E = T + mc^2$

The total available energy is: $E_{tot} = E + mc^2 = E_a + E_b$

Let define the ratio of energy transfered to one photon (says γ_a) :

$$\epsilon = \frac{E_a}{E_{tot}} \equiv \frac{E_a}{T + 2mc^2}$$

Energy-momentum conservation gives :

$$\epsilon_{\min} = \frac{E_a^{\min}}{E_{tot}} = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma - 1}{\gamma + 1}}\right] \quad \epsilon_{\max} = \frac{E_a^{\max}}{E_{tot}} = \frac{1}{2}\left[1 + \sqrt{\frac{\gamma - 1}{\gamma + 1}}\right]$$

Therefore the range of ϵ is:

$$\epsilon \in [\epsilon_{\min}, \; \epsilon_{\max}] \equiv [\epsilon_{\min}, \; 1 - \epsilon_{\min}]$$

sample the gamma energy

The differential cross-section is :

$$\frac{d\sigma(Z,\epsilon)}{d\epsilon} = \frac{Z\pi r_e^2}{\gamma - 1}\ \frac{1}{\epsilon}\ \left[1 + \frac{2\gamma}{(\gamma+1)^2} - \epsilon - \frac{1}{(\gamma+1)^2}\frac{1}{\epsilon}\right]$$

The formula can be factorized :

$$\frac{d\sigma(Z,\epsilon)}{d\epsilon} = \frac{Z\pi r_e^2}{\gamma - 1}\ N\ f(\epsilon)\ g(\epsilon)$$

where:

$$N = \ln(\epsilon_{\max}/\epsilon_{\min})$$

$$f(\epsilon) = \frac{1}{N\ \epsilon}$$

$$g(\epsilon) = \left[1 + \frac{2\gamma}{(\gamma+1)^2} - \epsilon - \frac{1}{(\gamma+1)^2}\frac{1}{\epsilon}\right] \equiv 1 - \epsilon + \frac{2\gamma\epsilon - 1}{\epsilon(\gamma+1)^2}$$

Given 2 random numbers $r_a, r_b \in [0, 1[$:

1. sample ϵ from $f(\epsilon)$: $\epsilon = \epsilon_{\min} \left[\frac{\epsilon_{\max}}{\epsilon_{\min}} \right]^{r_a}$
2. reject ϵ if $g(\epsilon) < r_b$

Then the photon energies are :

$$E_a = \epsilon \ E_{tot} \qquad E_b = (1 - \epsilon) \ E_{tot}$$

The function `PostStepDoIt()` sample ϵ and compute the final kinematic

compute the final kinematic

Lets be θ the angle between the incident e^+ and γ_a.

From the energy-momentum conservation :

$$\cos\theta = \frac{1}{Pc}\left[T + mc^2\frac{2\epsilon - 1}{\epsilon}\right] = \frac{\epsilon(\gamma + 1) - 1}{\epsilon\sqrt{\gamma^2 - 1}}$$

The azimuthal angle, ϕ, is generated isotropically.

The momentum vector of the photons, $\vec{P_{\gamma_a}}$ and $\vec{P_{\gamma_b}}$ are computed from the energy-momentum conservation and transformed to the global coordinate system.

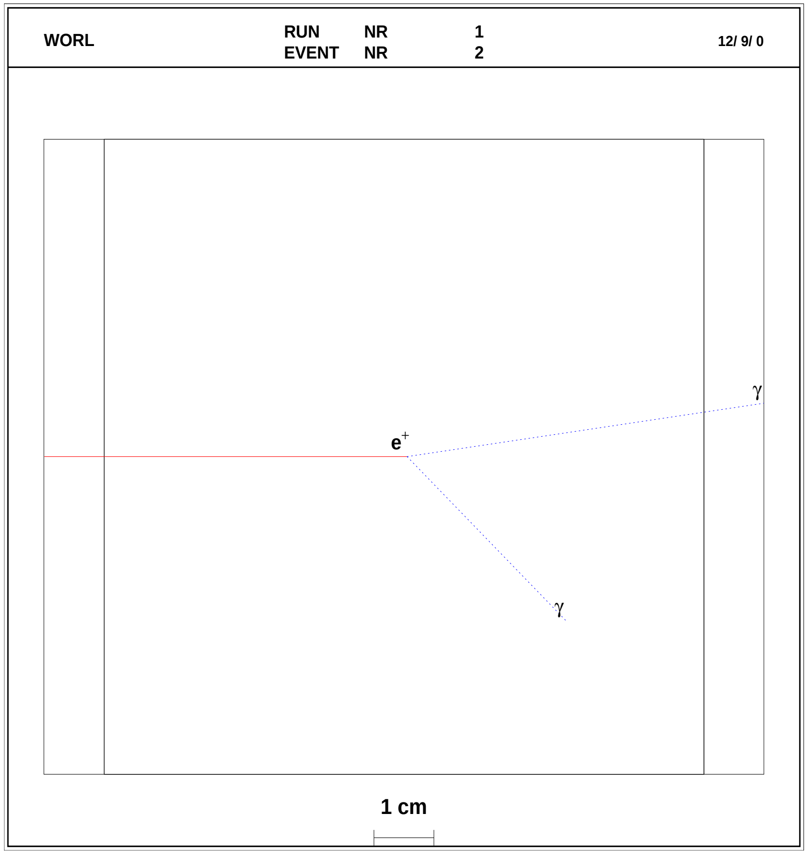
The annihilation in fly is not the dominant process. Most of the time the positron comes at rest and does a *positronium* with the electron.

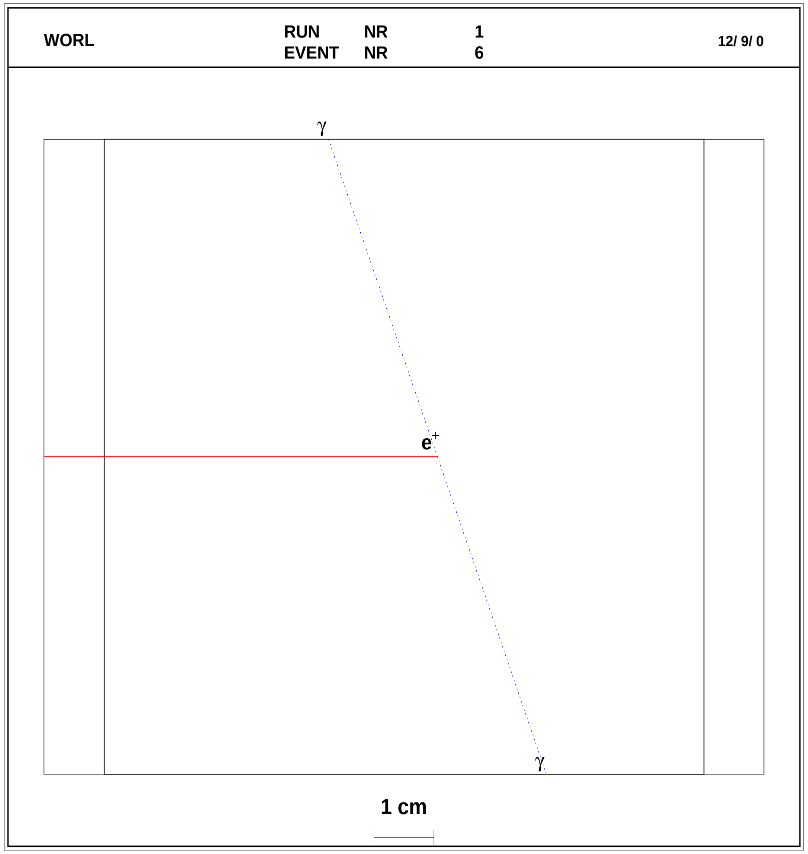
The positronium decays in two-photon (in 0.125 *nanosec*) or three-photon state (in 142 *nanosec*.)

The function `AtRestDoIt` treates this case. It generates two photons with energy $E_\gamma = mc^2$. The angular distribution is isotropic.

The (e^+, e^-) can also annihilate in a single photon: the other photon is absorbed by the recoil mucleus. However this mechanism is suppressed by a factor α^4.

e^+ 30 MeV in 10 cm Aluminium. Annihilation in fly (left), at rest (right).





References

[Heitl54] W. Heitler. *The Quantum Theory of Radiation, Clarendon Press, Oxford* (1954)