Čerenkov radiation

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In a material with refractive index n, a charged particle emits photons if its velocity is greater than the local phase velocity of light.

The charged particle polarizes the atoms along its trajectory. These time dependent dipoles emit electromagnetic radiations.

If $v < c/n$ the dipole distribution is symmetric around the particle position, and the sum of all dipoles vanishes.

If $v > c/n$ the distribution is asymmetric and the total time dependent dipole is non nul, thus radiates.

mechanism of the Čerenkov radiation [Grupen96].

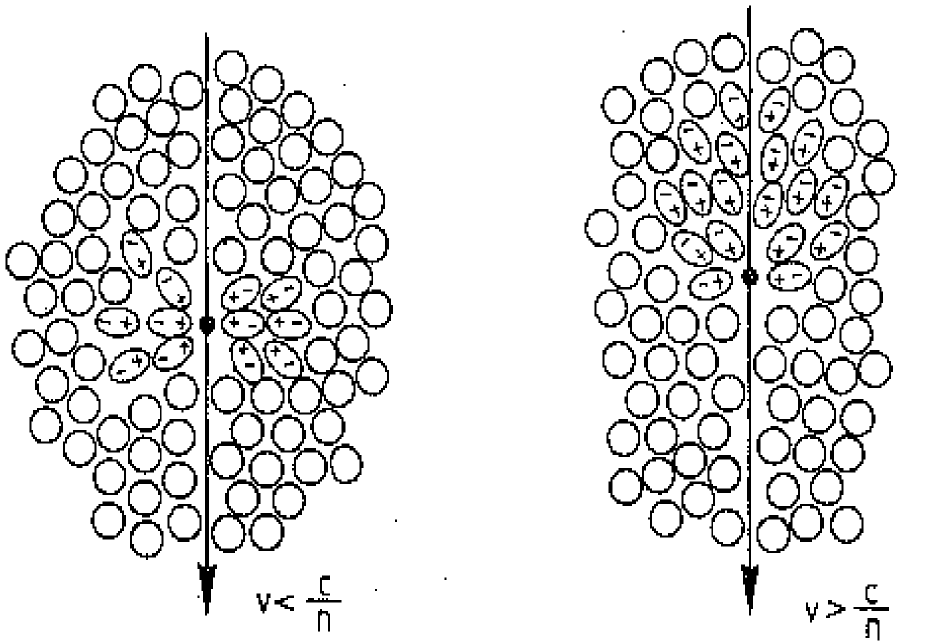
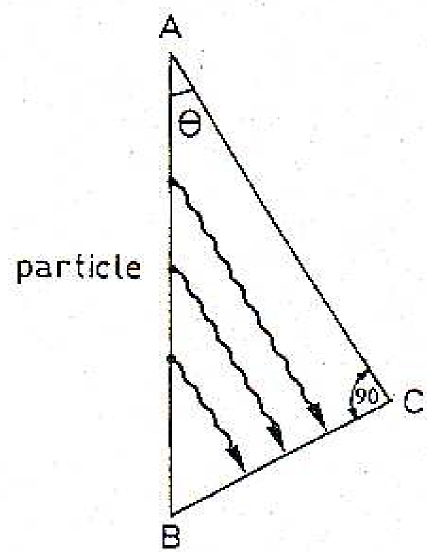


Fig. 6.7. Illustration of the Cherenkov effect [68].



The Huyghens construction gives immediately :

$$\cos\theta = \frac{1}{\beta n}$$

Thus :

$$\frac{1}{n} \leq \beta < 1 \Longrightarrow 0 \leq \theta < \arccos\frac{1}{n}$$

The number of photons produced per unit path length and per energy interval of the photons is

$$\frac{d^2N}{d\epsilon\, dx} = \frac{\alpha z^2}{\hbar c}\ \sin^2\theta = \frac{(\alpha z)^2}{r_e\ mc^2}\ \left[1 - \frac{1}{\beta^2\ n^2(\epsilon)}\right]$$

in which

$$\beta\ n(\epsilon) > 1$$

In the X-ray region $n(\epsilon) \approx 1$. There is no X-ray Čerenkov emission.

The average number of photons produced per unit path length :

$$\frac{dN}{dx} = \frac{(\alpha z)^2}{r_e\ mc^2} \int_{\epsilon_{min}}^{\epsilon_{max}} d\epsilon \left(1 - \frac{1}{\beta^2 n^2(\epsilon)}\right)$$

The number of photons produced per step is calculated from a Poissonian distribution with average value :

$$< n >= \text{StepLength}\ \frac{dN}{dx}$$

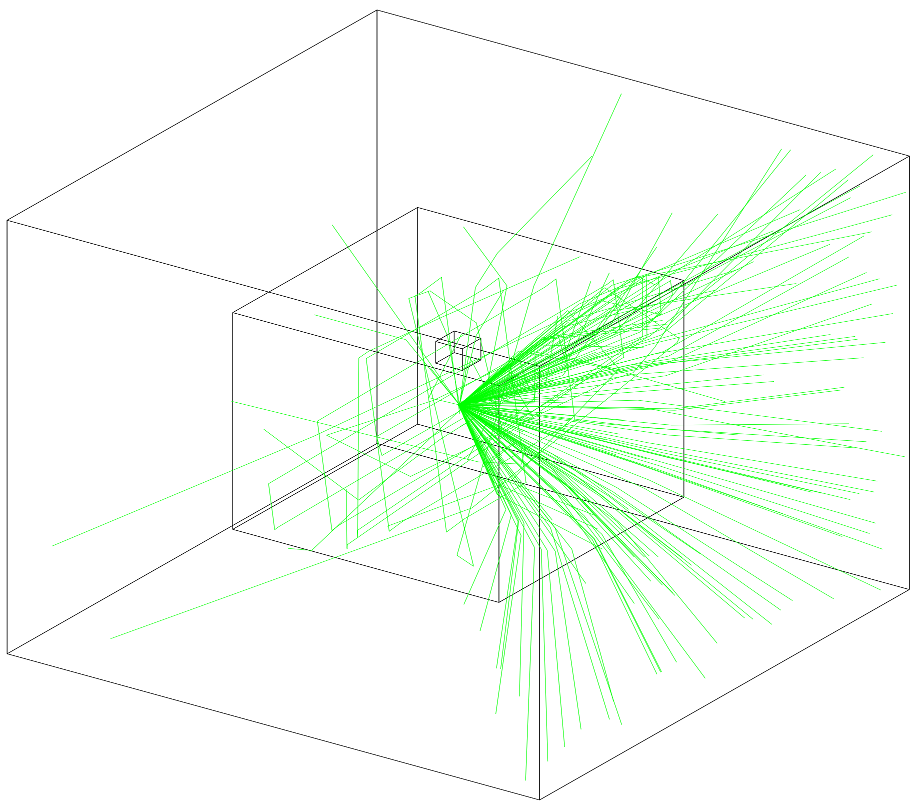
The generated photons are uniformly distribued along the track.

The energy distribution of the photon is sampled from the density function:

$$f(\epsilon) = \left[1 - \frac{1}{n^2(\epsilon)\beta^2}\right]$$

The Cerenkov radiation is an example of pure AlongStep process.

Cerenkov emission of optical photons by 15 MeV e^+ in water



The energy lost by the charged particle due to Čerenkov emission is small compared to collision loss, even in gas :

$$\sim 10^{-1} \text{ to } 10^{-3} \text{ MeV/(g/cm}^2\text{)}$$

References

[Jackson98] J.D.Jackson, Classical Electrodynamics, John Wiley and Sons (1998)

[Grupen96] C. Grupen, Particle Detectors, Cambridge University Press (1996)

[PDG] D.E. Groom et al. Particle Data Group . Rev. of Particle Properties. Eur. Phys. J. C15,1 (2000)
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