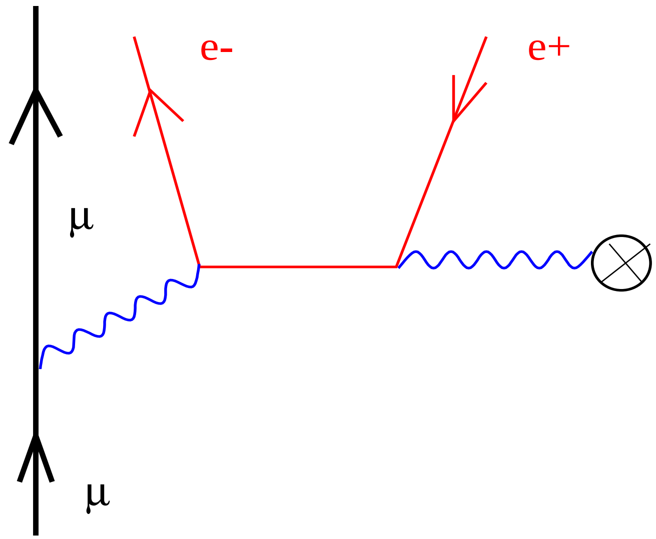
Direct (e^+, e^-) pair creation by muon

Direct (e^+, e^-) pair creation by muon

Creation of a (e^+, e^-) pair by virtual photon in the Coulomb field of the nucleus (for momentum conservation).

$$\mu + \text{nucleus} \longrightarrow \mu + e^+ + e^- + \text{nucleus}$$



It is one of the most important processes of muon interaction.

At TeV muon energies, pair creation cross section exceeds those of other muon interaction processes in a wide region of energy transfers :

$$100 \text{ MeV} \leq \epsilon \leq 0.1 \, E_\mu$$

Average energy loss for pair production increases linearly with muon energy, and in TeV region this process contributes over 50 % to the total energy loss rate.

energy transfers

Main contribution to the total cross section is given by transferred energies:

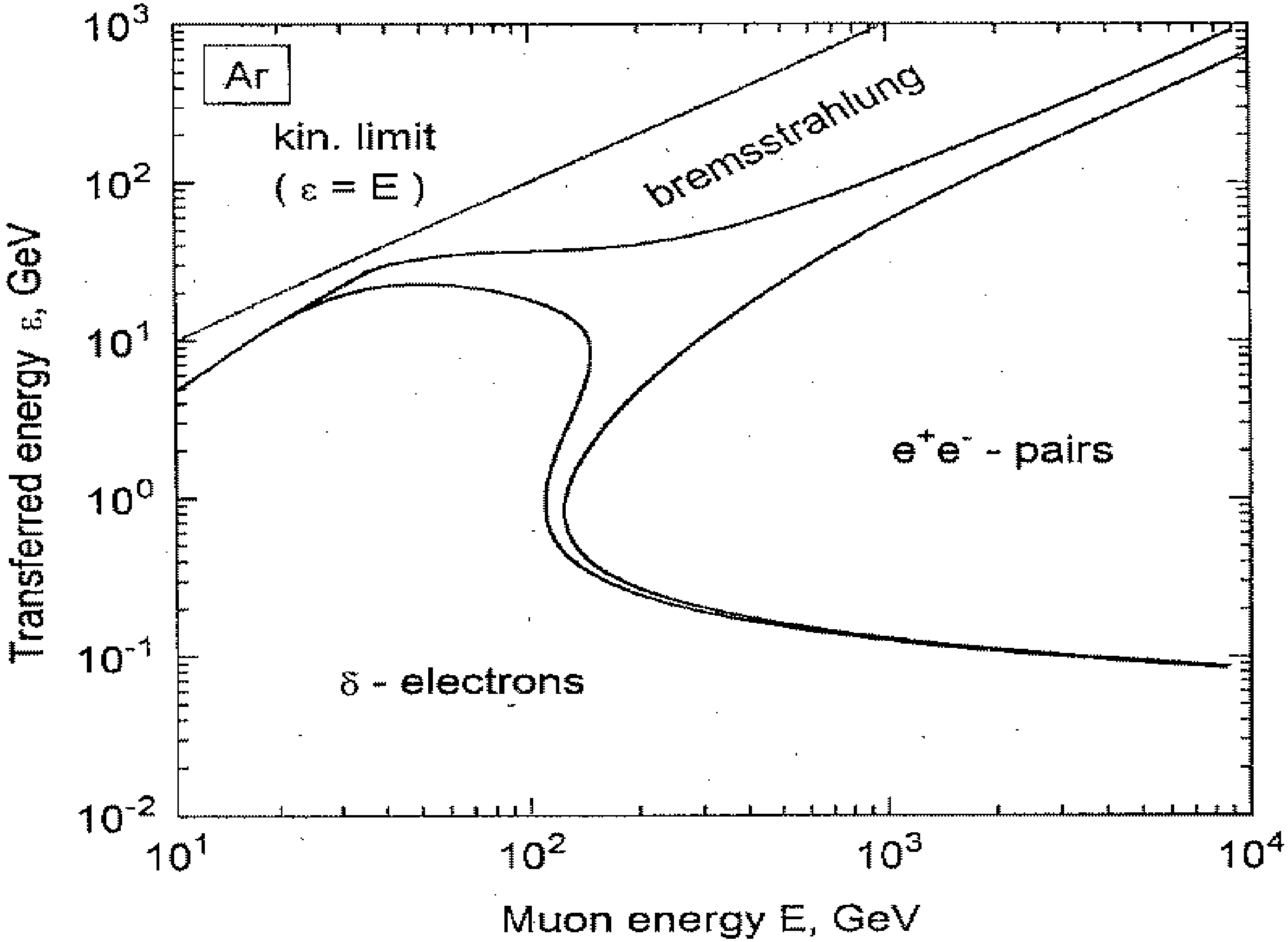
$$5\text{MeV} \leq \epsilon \leq 0.01\ E_\mu$$

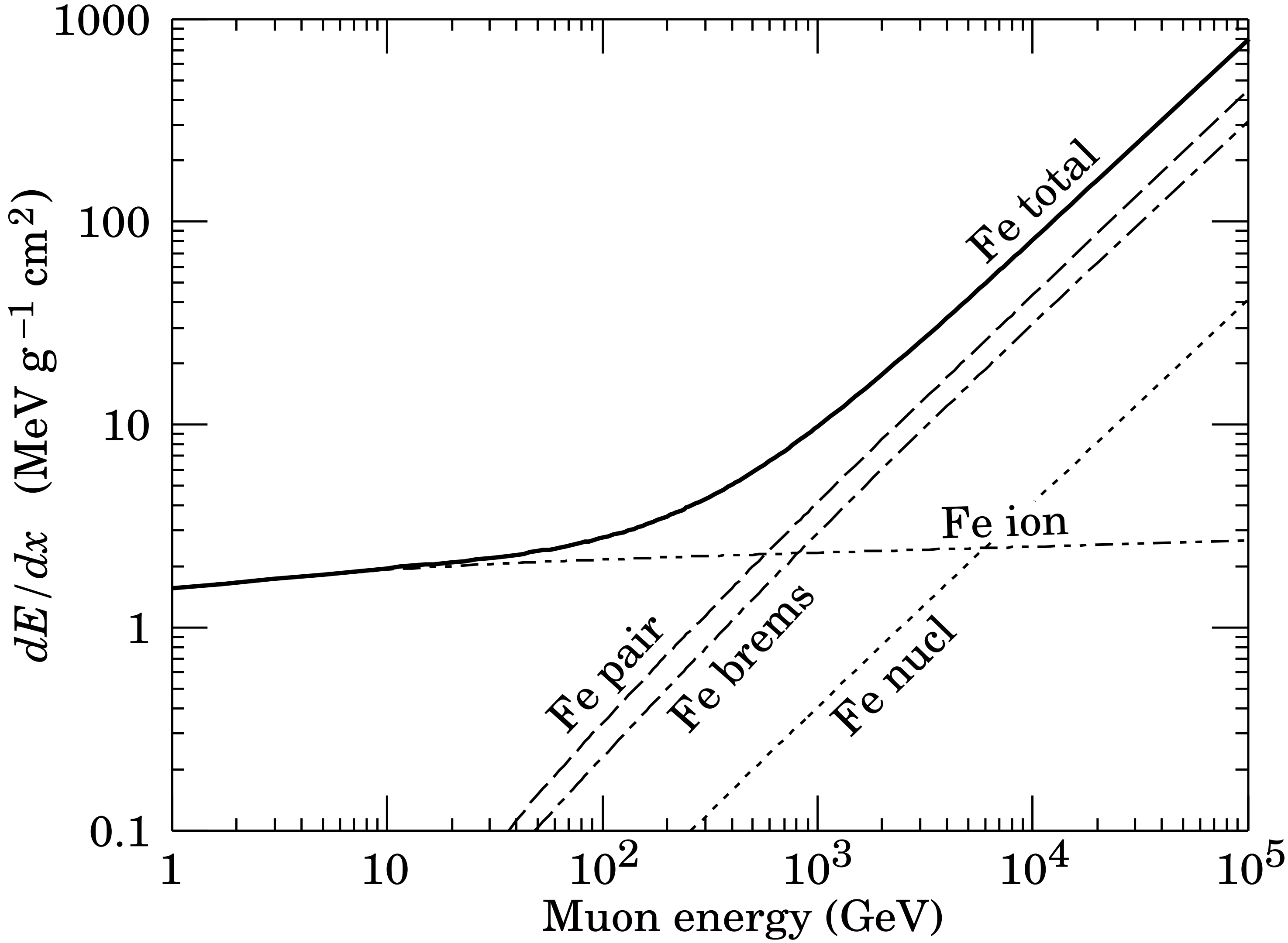
The contribution to average muon energy loss is determined mostly by region:

$$10^{-3}\ E_\mu \leq \epsilon \leq 0.1\ E_\mu$$

Thus, to adequatly describe the number of pairs produced, average energy loss and stochastic energy loss distribution one need to reproduce with a sufficient accuracy the differential cross section behaviour in a wide range of energy transfers:

$$5\text{MeV} \leq \epsilon \leq 0.1\ E_\mu$$


Ar
kin. limit
(ε = E)
bremsstrahlung
e⁺e⁻ - pairs
δ - electrons
Transferred energy ε, GeV
Muon energy E, GeV
10³
10²
10¹
10⁰
10⁻¹
10⁻²
10¹
10²
10³
10⁴


1000
100
10
1
0.1
dE/dx (MeV g^{-1} cm^2)
1
10
10^2
10^3
10^4
10^5
Muon energy (GeV)
Fe total
Fe pair
Fe brems
Fe nucl
Fe ion

differential cross section

The differential cross section is given by Kokoulin et al. [Koko71].
It includes :

- screening of the field of the nucleus
- correction for finite nuclear size
- contribution from the atomic electrons [Keln97]
- ...

See [Koko71] for a complete discussion.

differential cross section

The differential cross section per atom can be written as :

$$\frac{d\sigma}{d\epsilon} = \frac{4\,\alpha^2\,r_e^2}{3\pi}\,\frac{1-v}{\epsilon}\,[Z(Z+\zeta)]\,F(Z,E,\epsilon) \tag{1}$$

with

$$F(Z,E,\epsilon) = \int_0^{\rho_{max}} \left[\Phi_e(v,\rho) + \left(\frac{m_e}{m_\mu}\right)^2\,\Phi_\mu(v,\rho)\right]\,d\rho \tag{2}$$

where

$$\begin{aligned} \epsilon &= \epsilon^+ + \epsilon^- &&= \text{total energy of the created pair;} \\ v &= \epsilon/E && \\ \rho &= (\epsilon^+ - \epsilon^-)/\epsilon &&= \text{asymmetry coefficient;} \end{aligned}$$

The functions Φ_e, Φ_μ, ζ can be found in [Koko00].

limits

$$
\begin{aligned}
\epsilon_{min} &= 4 m_e c^2 \\
\epsilon_{max} &= E - \frac{3\sqrt{e}}{4}\, m_\mu c^2\, Z^{1/3} \\
\rho_{min} &= 0 \\
\rho_{max} &= \left[1 - \frac{6(m_\mu c^2)^2}{E(E-\epsilon)}\right] \sqrt{1 - \frac{\epsilon_{min}}{\epsilon}}
\end{aligned}
$$

Energetic pairs and truncated energy loss rate

One may wish to take into account separately the high-energy pairs emitted above a given threshold ϵ_{cut} (miss detection, explicit simulation ...).

Those pairs must be excluded from the mean energy loss count.

$$-\frac{dE}{dx}\bigg]_{\epsilon<\epsilon_{cut}} = n_{at}\int_{\epsilon_{min}}^{\epsilon_{cut}} \epsilon\,\frac{d\sigma}{d\epsilon}\,d\epsilon$$

n_{at} is the number of atoms per volume.

Then, the truncated total cross-section for emitting 'hard' pairs is:

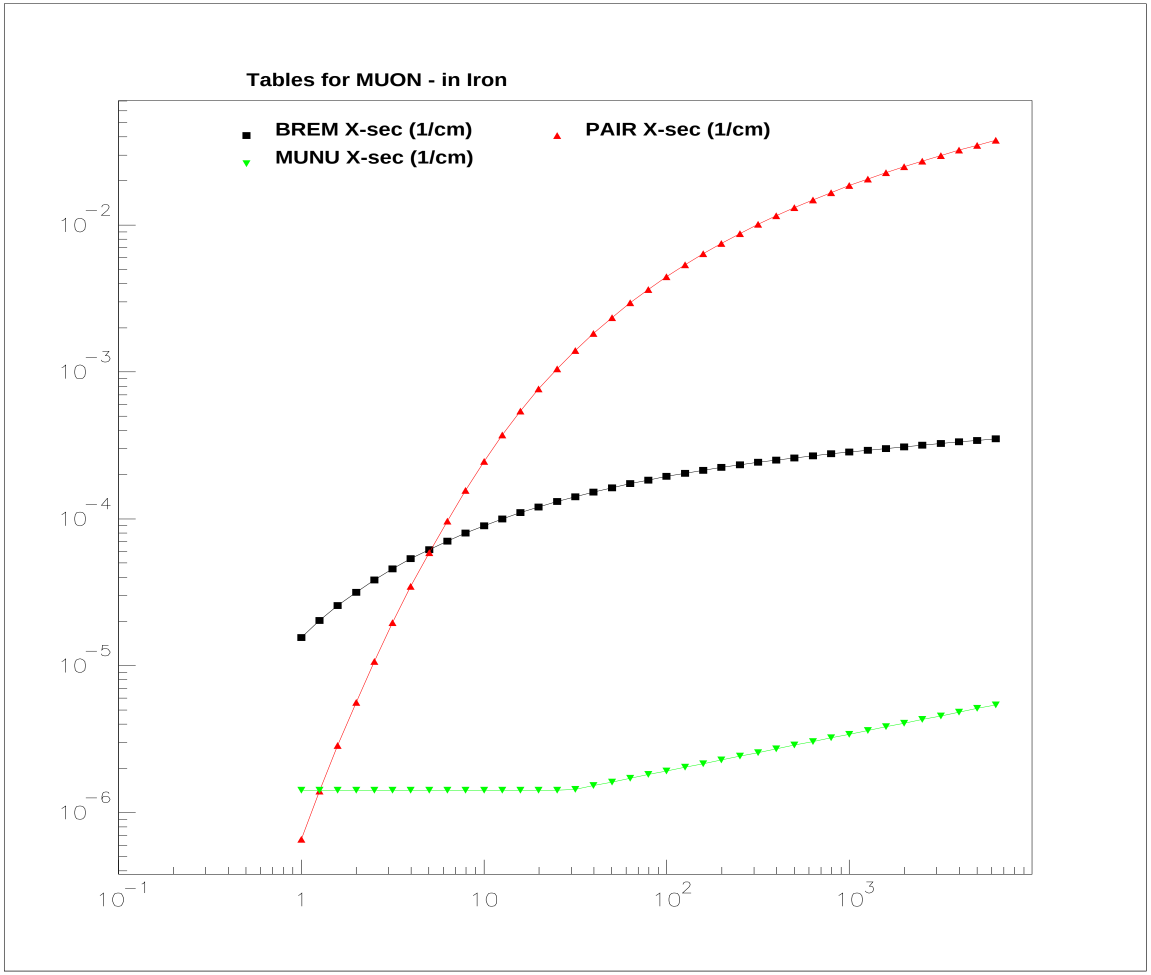
$$\sigma(E, \epsilon_{cut} \le \epsilon \le \epsilon_{max}) = \int_{\epsilon_{cut}}^{\epsilon_{max}} \frac{d\sigma}{d\epsilon}\,d\epsilon$$

The muon deflection angle is of the order of:

$$\theta = \frac{mc^2}{E}$$

Above ∼1000 TeV the LPM suppression mechanism may have an effect.

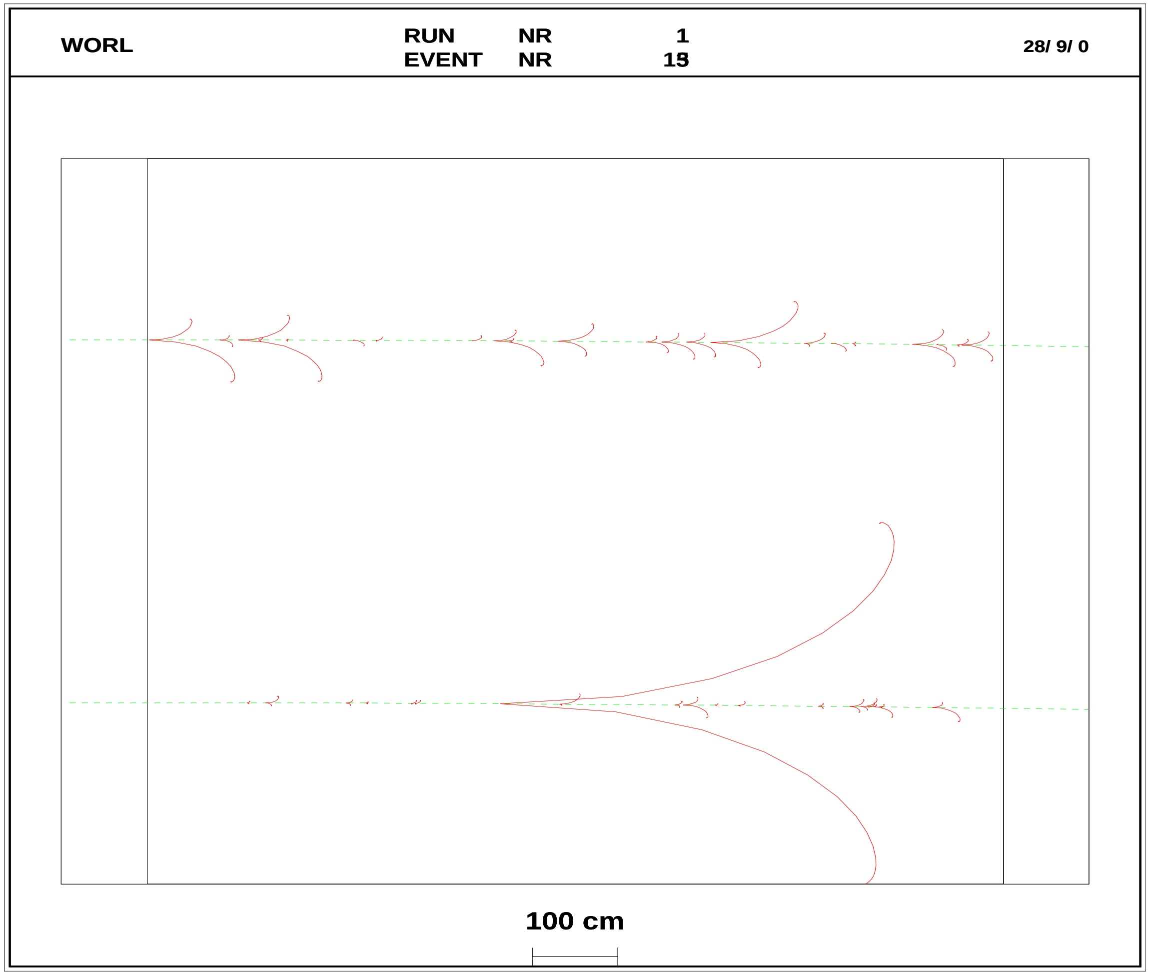
number of interactions per cm in Iron. (cut 100 MeV)



muon kinetic energy (GeV)

1 TeV muon in 10 meter of Fe (field 5 tesla).

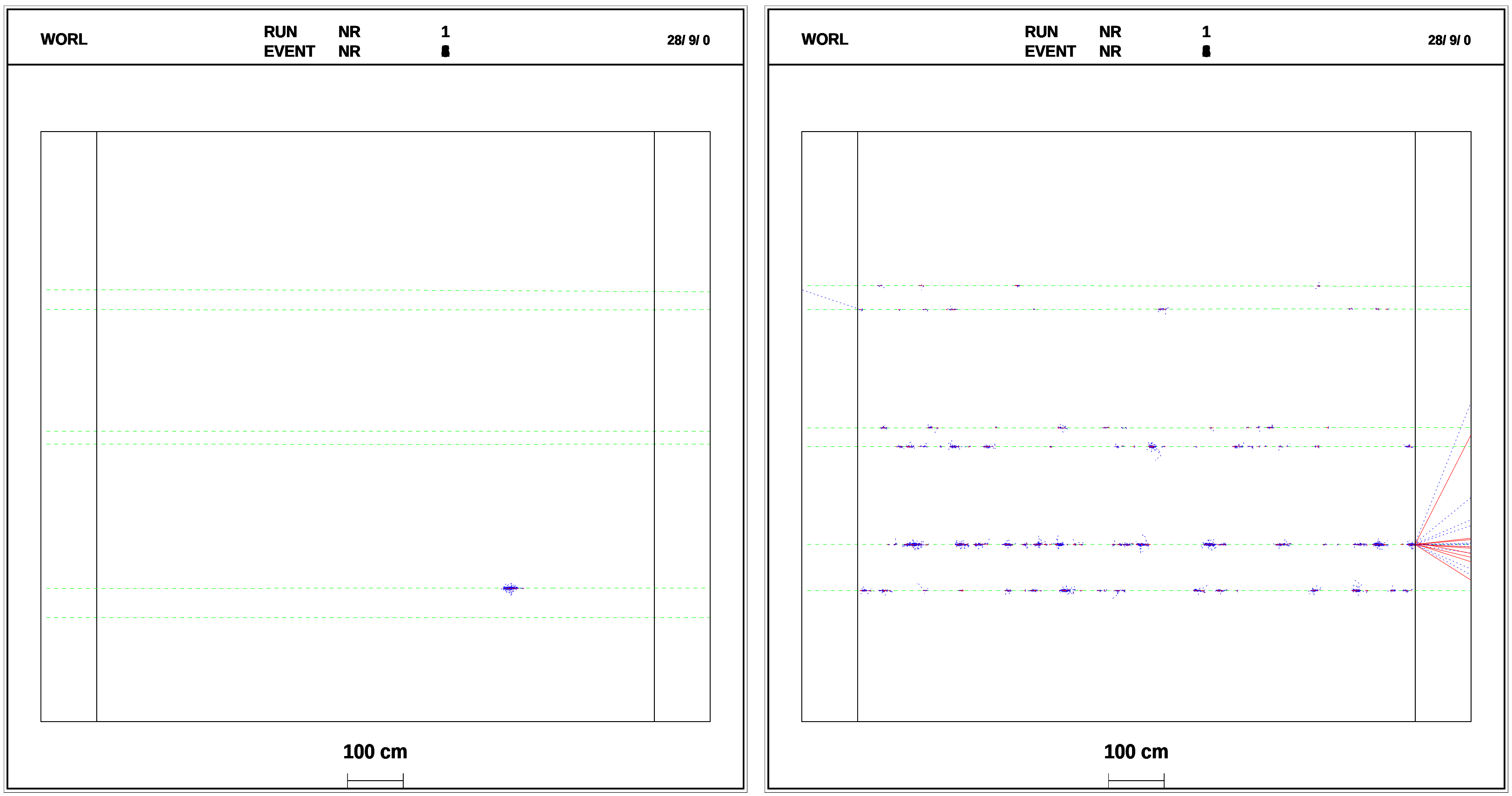
direct pair creation only



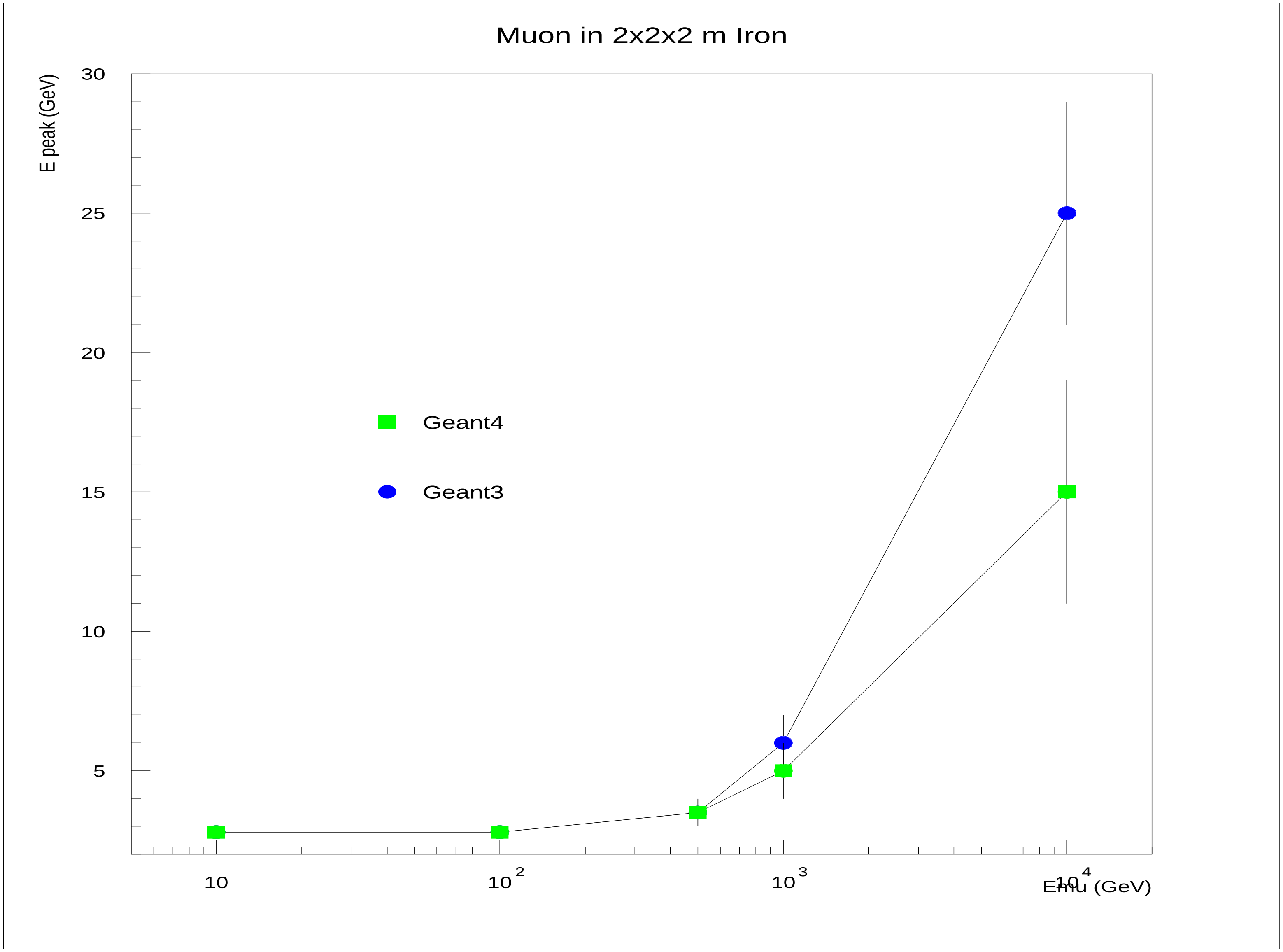
10 meter of Fe : muons 100 GeV, 1 TeV, 5 TeV.

left : brems only

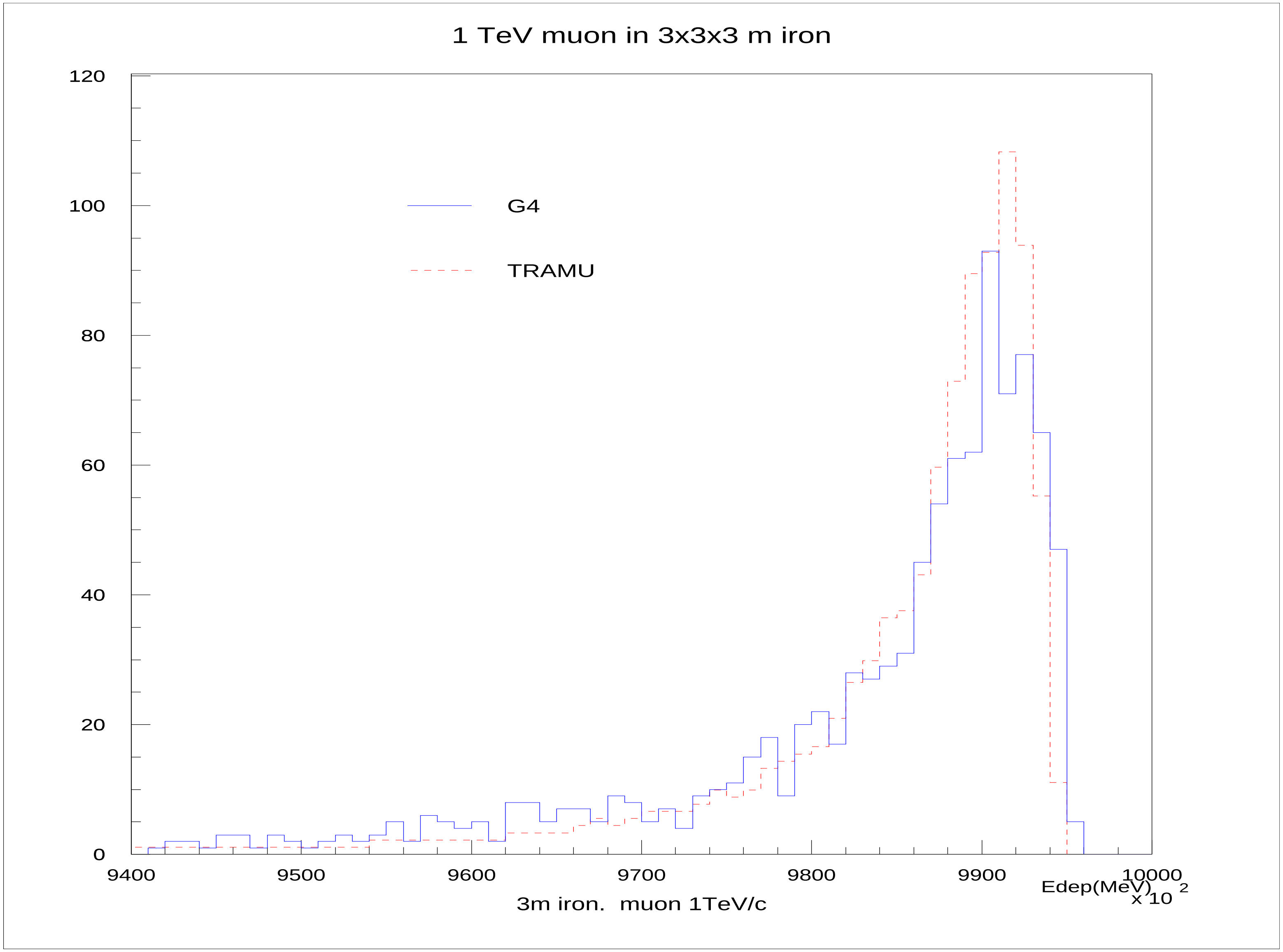
right : brems + direct pair creation



energy deposited by high energy muons in a bloc of Iron



energy spectrum of 1 TeV muons after 3 m of iron



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