Parameterization of e and γ initiated showers in lead-glass calorimeters

Roberto Petti

CERN

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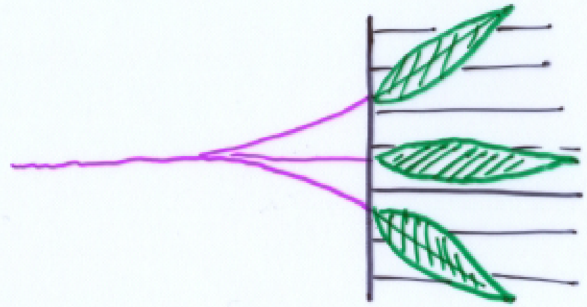
✦ *The NOMAD detector is designed to measure momenta & energies of particles with high precision in order to be sensitive to* $\nu_\mu \rightarrow \nu_\tau$ *oscillations at* $\mathcal{O}(10^{-4})$ *level:*

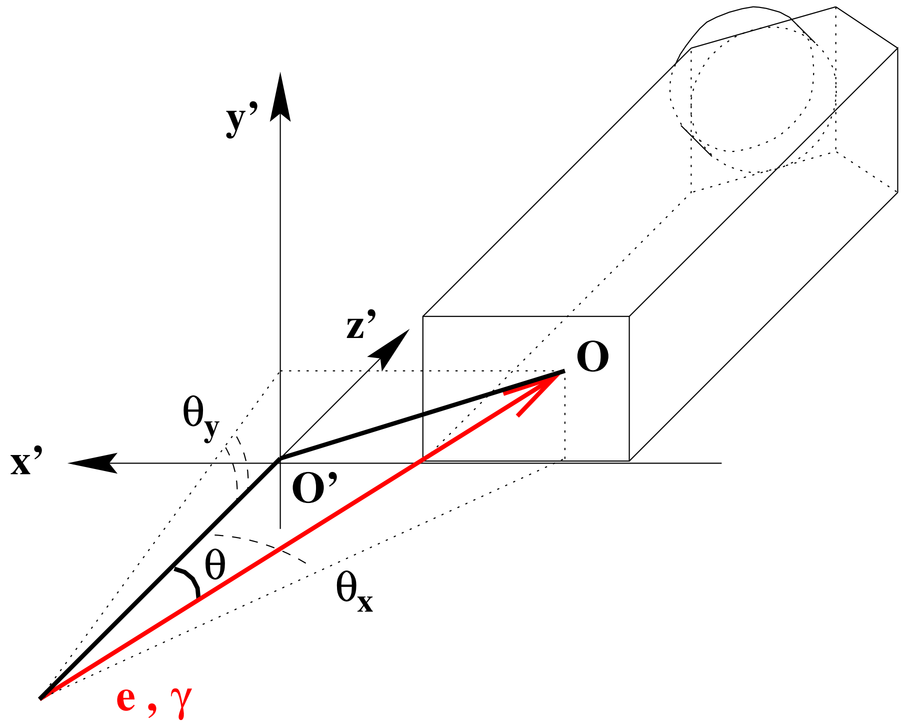


a) *broad energy spectra (WBB);*
b) *particles from* ν *interactions;*
c) *magnetic field.*

$\Longrightarrow$ *wide* E, θ *ranges:*
$0.1 \leq E \leq 100\ GeV$
$0^0 \leq \theta \leq 40^0$

✦ *Impossible to reconstruct electromagnetic showers (compact objects) without a precise knowledge of the shower development as a function of* (θ, ϕ, E).

THE LEAD-GLASS BLOCKS



✦ **Lead-glass counters (35×25):**

- *wrapping: 100 μm diffusive Tyvek;*
- *transmittance: $T(\lambda) \geq 0.99$ at 410 nm;*
- *refraction index: $n = 1.648$ at 589 nm;*
- *density: 3.85 g/cm^3;*
- $X_0 = 2.7$ cm, $R_M = 4.0$ cm;
- *dimensions: $79 \times 112\ mm^2 \times 19 X_0$;*
- *photodetectors: 77 mm diameter tetrodes, mounted at $\phi = 45^0$, $\langle G \rangle = 40$ with $B = 0.4$ T.*

✦ **Energy resolution:**

$\sigma(E)/E = a + b/\sqrt{E} \quad b = \sqrt{\delta_{\rm en}^2 + \delta_{\rm phot}^2}$

$a = 1.0\%$, $b = 3.2\%$, $\delta_{\rm en} = 1.4\%$, $\delta_{\rm phot} = 2.9\%$.

✦ **Photostatistics:**

$\langle\ \rangle = 1430$ *photoelectrons/GeV.*

INTEGRAL APPROACH

- *Two different methods can be pursued to obtain a parameterization of the lead-glass response to electromagnetic showers:*

  I. Differential approach:

  (r, z, E) *parameterization of shower* $\longrightarrow$ *integration over block size*

  II. Integral approach:

  (r, θ, ϕ, E) *parameterization of block response* $\longrightarrow$ *angular dependence*

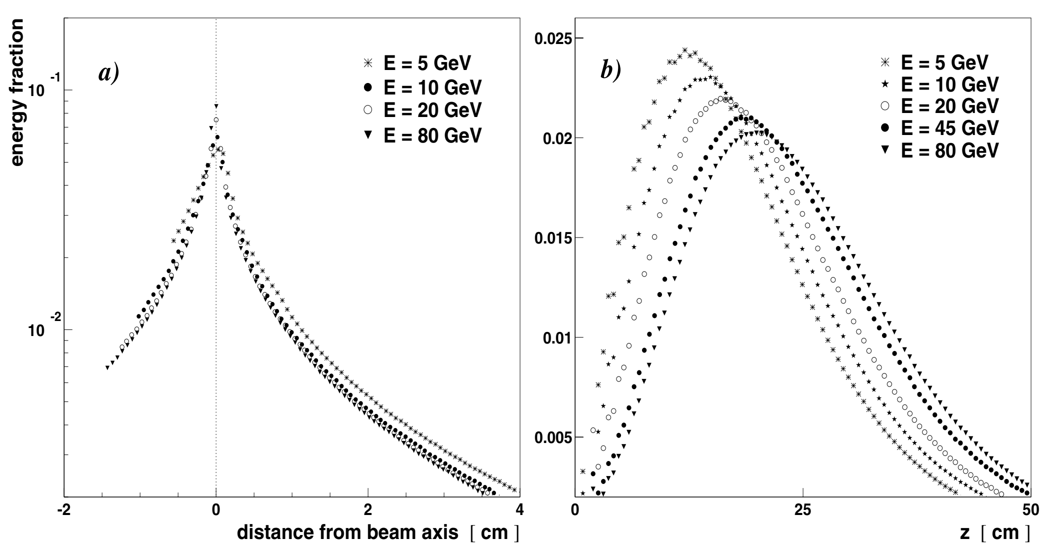
- *An algorithm optimized for cluster reconstruction requires precise and fast (analytical) predictions to be used in recursive computations*

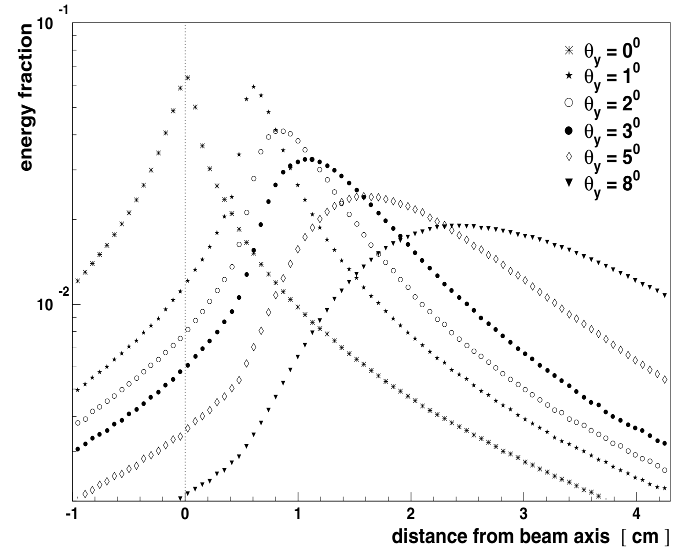
  $\Longrightarrow$ *integral approach preferred*

- *Integral parameterization from MC (GEANT) simulations and test-beam data.*

DIFFERENTIAL PROFILES

$\theta = 0^0$ $\qquad\qquad$ $\theta \neq 0^0$





TRANSVERSE $\qquad$ **LONGITUDINAL** $\qquad$ **TRANSVERSE**

- ✦ *Transverse profile for* $\theta = 0^0$: $\qquad \rho(x,y) \; = \; \sum_i a_i e^{-r/b_i} \qquad r = \sqrt{x^2+y^2}$
- ✦ *Longitudinal profile for* $\theta = 0^0$: $\qquad \xi(z) \; = \; A z^{\alpha} e^{-z/\lambda} \qquad \boldsymbol{\alpha \propto \log(E)}$

$\Longrightarrow$ *transverse size of the shower defined by the Molière radius* R_M

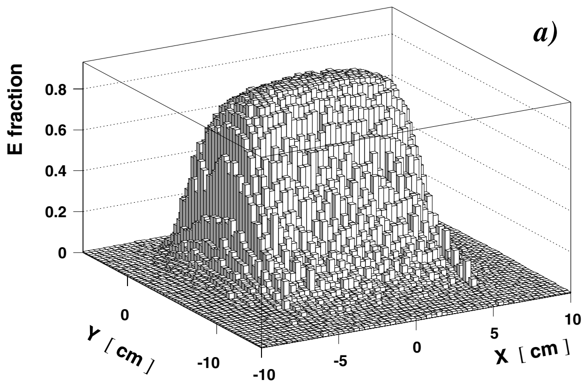
$\Longrightarrow$ *logarithmic energy dependence associated to the longitudinal profile*

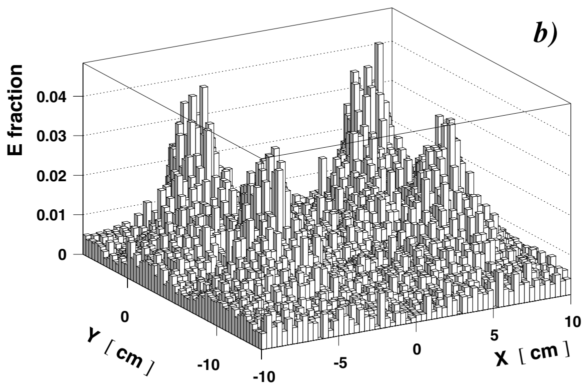
INTEGRAL RESPONSE

✦ NORMALIZATION *to the total energy deposition, on an event-by-event basis:*

$$F(x,y,\theta,\phi,E) \;=\; N\int_{\Delta x} dx \int_{\Delta y} dy\; \rho(x,y,E)$$

$1/N =$ *total energy deposition in a cell matrix*





✦ FACTORIZATION *in X-Y (small leakage):*

$$F(x,y,\theta,\phi,E) \;\simeq\; F_x(x,\theta,\phi,E) \times F_y(y,\theta,\phi,E)$$

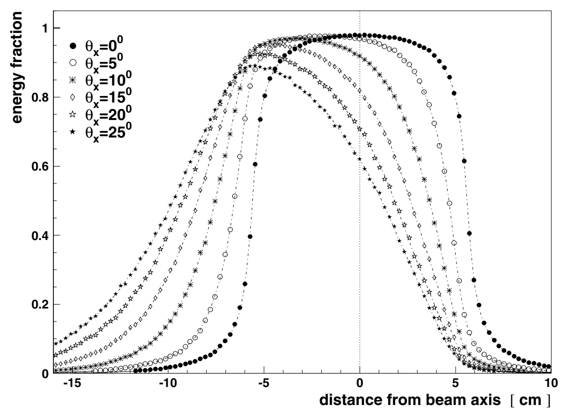
$$F_x(x,\theta,\phi,E) \;=\; \int_{\Delta x} dx \int_{-\infty}^{+\infty} dy\; \rho(x,y,E)$$

✦ *General parameterization of the form:*

$$F_x(x,\theta,\phi,E) \;=\; \frac{1}{\pi}\sum_i a_i \times \arctan\left(x/b_i\right) \;+\; \sum_j \frac{1}{c_j\sqrt{2\pi}} \exp\left[-\left(x-d_j\right)^2/2c_j^2\right]$$

where a_i, b_i, c_j, d_j *are functions of* (θ,ϕ,E).

ANGULAR FACTORIZATION



- *Any rotation decomposed in two rotations* θ_x AND θ_y *around each of the two axes.*

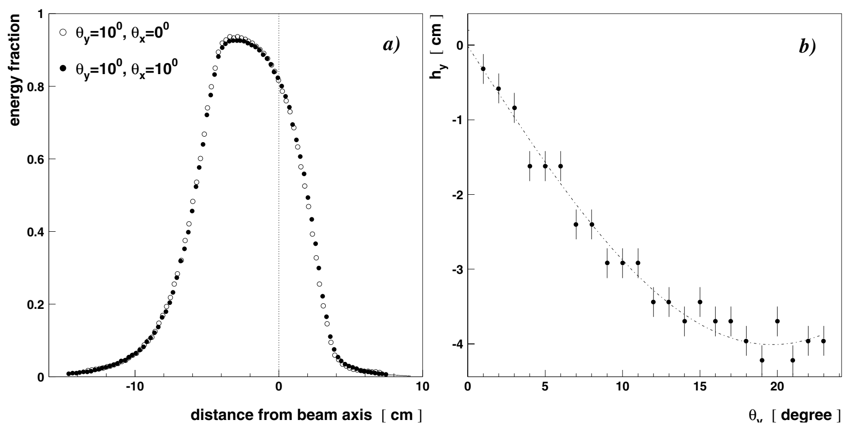
- *The projected profile F_x depends only from the corresponding angle θ_x:*

$$F_x \neq F_x(\theta_y)$$
$$F_x(x, \theta, \phi, E) = F_x(x, \theta_x, E)$$

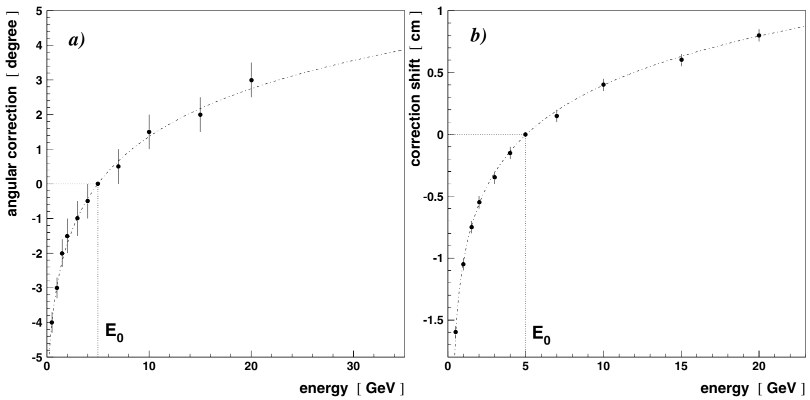
- ANGULAR SHIFT $h_x(\theta_x, E)$ *in the position of the maximum of F_x:*

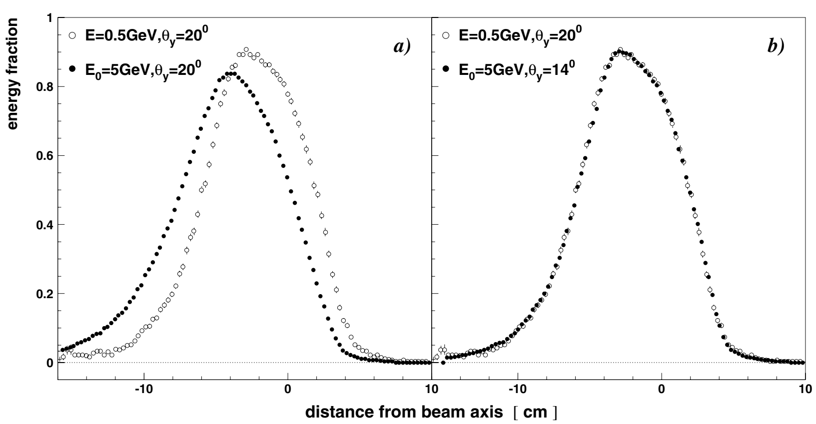
$$h_x = \frac{\Delta x}{2} \sin\left(k_x \theta_x\right)$$

where, for small angles, $\frac{\Delta x}{2} \times k_x = D$ and D is the longitudinal position of the shower centre.



ENERGY SCALING





✦ *Since the energy dependence is due to the longitudinal component, the energy "visible" from the transverse profile increases with the angle:*

$$F_x(x, \theta_x, E) \;\equiv\; F_x(x', \theta'_x, \boxed{E_0})$$

✦ **SCALING LAW** *through three corrections:*

I. *Angular correction (actual rescaling):*

$$\theta_x \longrightarrow \theta_x + f(\theta_x, E, E_0) \;\equiv\; \boxed{\theta'_x}$$

II. *Position shift accounting for the different systematic offset h_x for the angles θ_x and θ'_x:*

$$x \longrightarrow x + g_1(\theta'_x, \theta_x, E_0) \;\equiv\; x_1$$

III. *Position shift accounting for the actual energy dependence in the longitudinal profile:*

$$x_1 \longrightarrow x_1 + g_2(\theta_x, E, E_0) \;\equiv\; \boxed{x'}$$

$\Longrightarrow$ *due to the longitudinal energy dependence f, g_2 are logarithmic functions of E.*

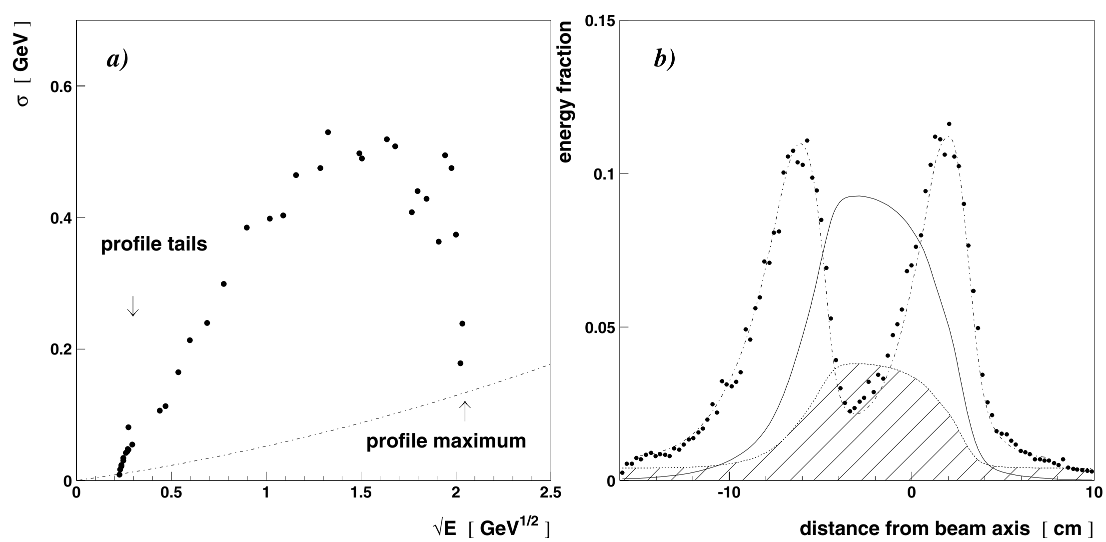
SHOWER FLUCTUATIONS

- *The event-by-event normalization cancels out fluctuations on the total measured energy from the F functions.*

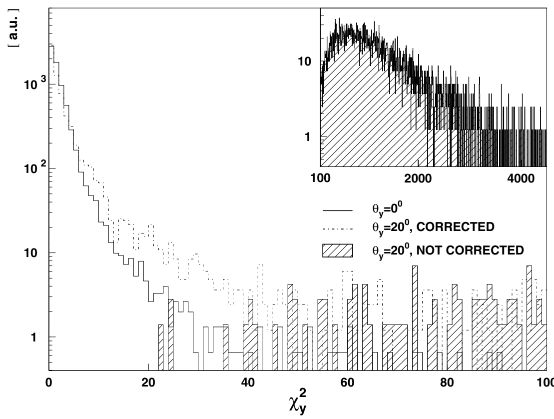
- *Therefore, the F functions describe the RELATIVE energy deposition inside the cluster. The corresponding rms, σ's, are then related only to fluctuations due to the incomplete* CONTAINMENT *of charged tracks of the shower:*

$$\begin{aligned}\sigma_x(x,\theta_x,E) &= s(\sqrt{E}) \times \sigma'_x(x,\theta_x,E) \\ \sigma'_x(x,\theta_x,E) &= F_x(x,\theta_x,E) \times [\,1 - F_x(x,\theta_x,E)\,] \oplus \alpha\end{aligned}$$

- *The overall fluctuations on the total energy are recovered back through an event-by-event ABSOLUTE normalization using the measured energy deposition.*



CLUSTER RECONSTRUCTION



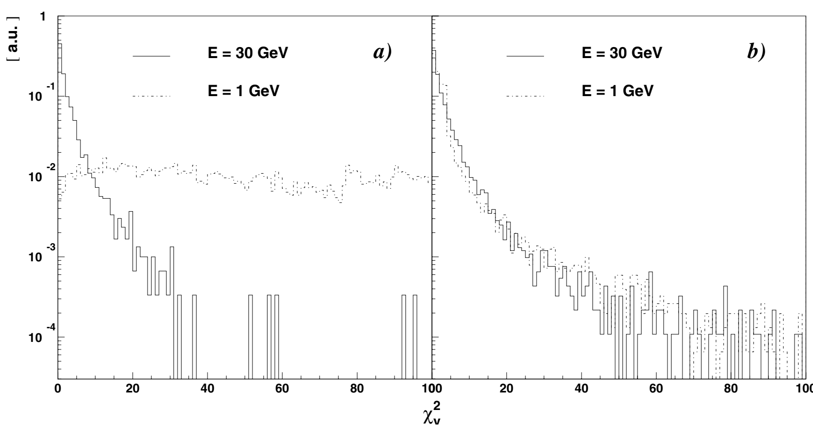
- *A 3×3 matrix (nonet) is built around the predicted XY position of the shower maximum.*

- χ^2-BASED *fitting procedure in XY plane:*

$$\chi^2_x = \sum_{i=1}^{3} \frac{\left[\ \sum_{j=1}^{3} E_{ij}^{exp} - F_x(x_i, \theta_x, E) \times E_n\ \right]^2}{[\ \sigma_x(x_i, \theta_x, E) \times E_n\ ]^2}$$

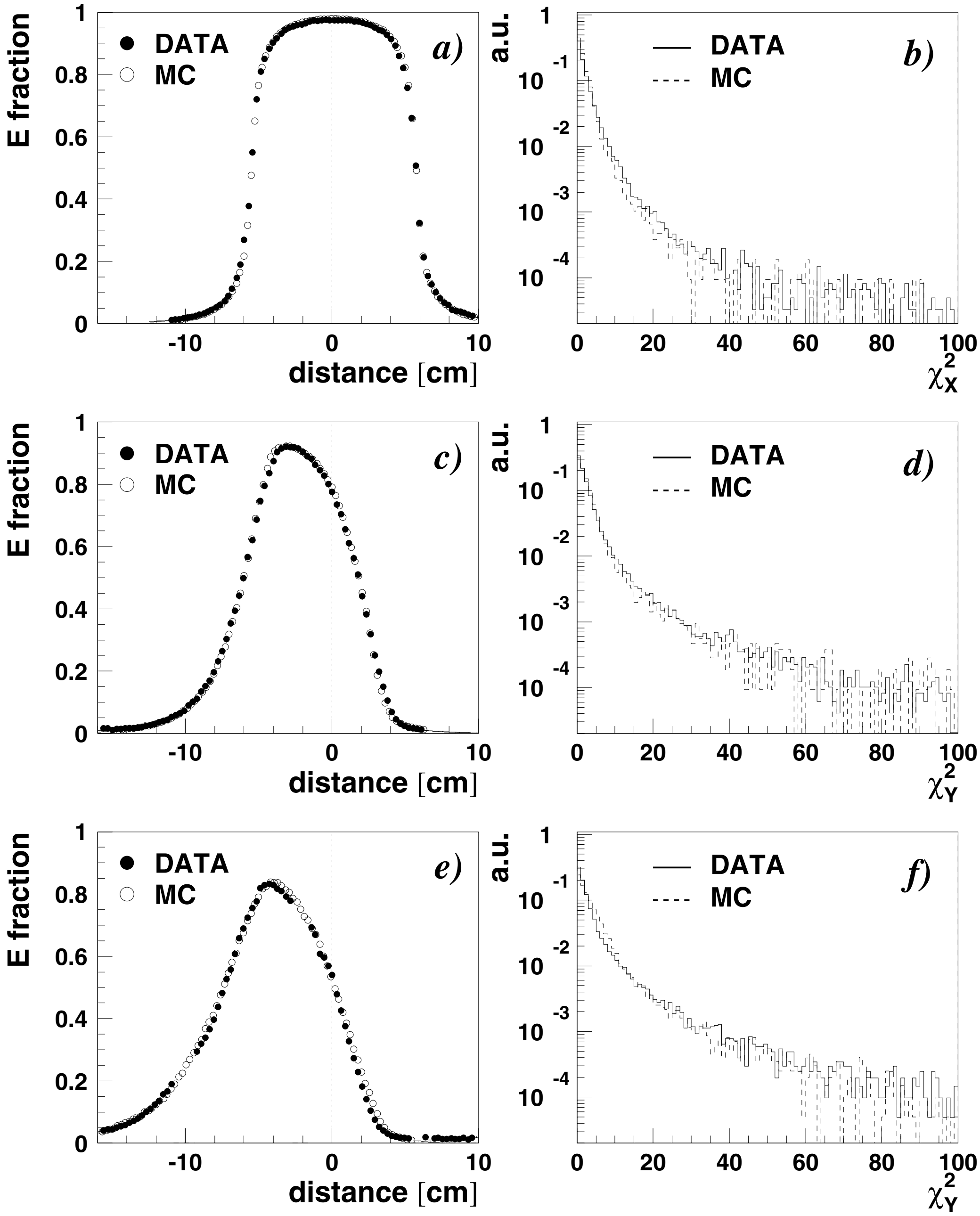
$$\chi^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{\left[\ E_{ij}^{exp} - F(x_i, y_j, \theta_x, \theta_y, E) \times E_n\ \right]^2}{[\ \sigma(x_i, y_j, \theta_x, \theta_y, E) \times E_n\ ]^2}$$

with E_{ij}^{exp} energy of the block in the i-th column and j-th row and E_n overall normalization.
$\Longrightarrow$ *approximate χ^2 due to non-gaussian effects*



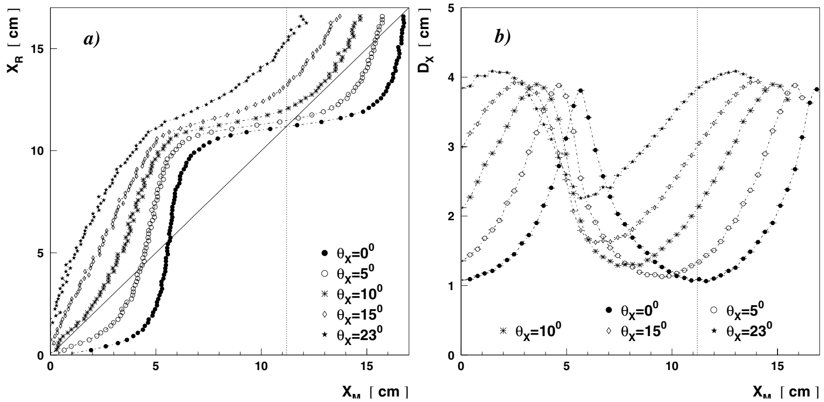
- *The normalization of the F functions to the total energy deposition inside the nonet provides CORRELATIONS between columns (rows).*

✦ Parameterization reliability:



$\Longrightarrow$ **good agreement** *is found between MC predictions and test-beam electron data.*

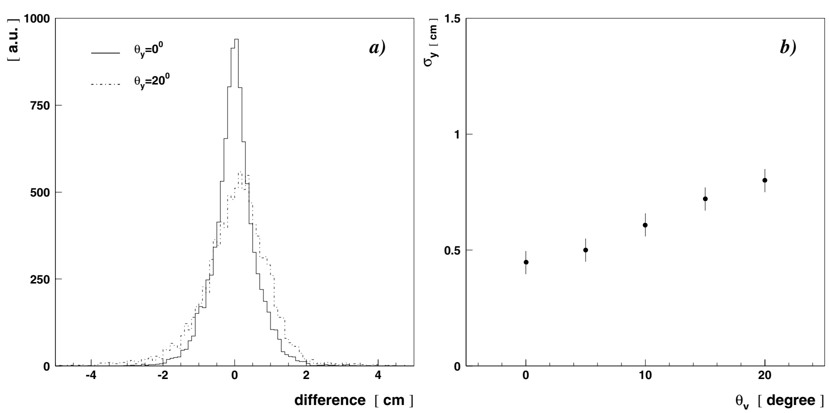
✦ Coordinate measurement:



- **MOMENTA** of the F_x distribution can be directly reconstructed:

  M_x^1 centre of gravity
  D_x^2 cluster radius (w.r.t. M_x^1)

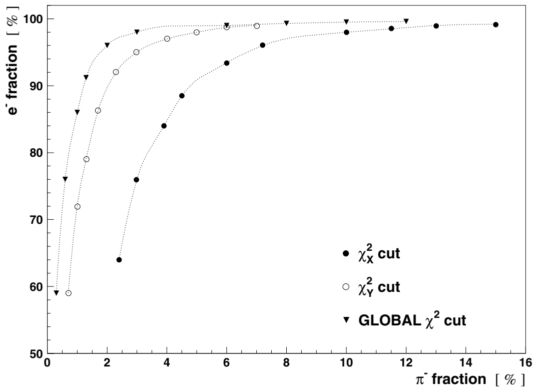
  $\Longrightarrow$ *analytical computation.*



- *Alternatively,* a χ^2 minimization *procedure can be used for the estimation*

  $\Longrightarrow$ *more powerful since the information from shower fluctuations is exploited*

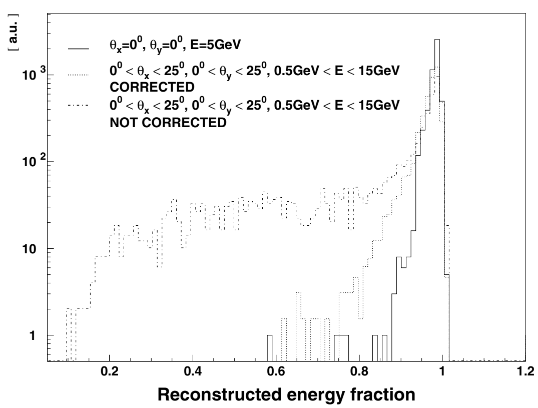
✦ Particle identification:



- e/π SEPARATION from χ^2_x, χ^2_y or χ^2 values, which are independent from kinematic parameters.
- $\overline{\chi}^2 = (\chi^2_x + w\chi^2_y)/(1+w)$, with w fixed weight, can be optimized according to the physical analysis.

$\Longrightarrow$ *rejection power function of E (not angles).*

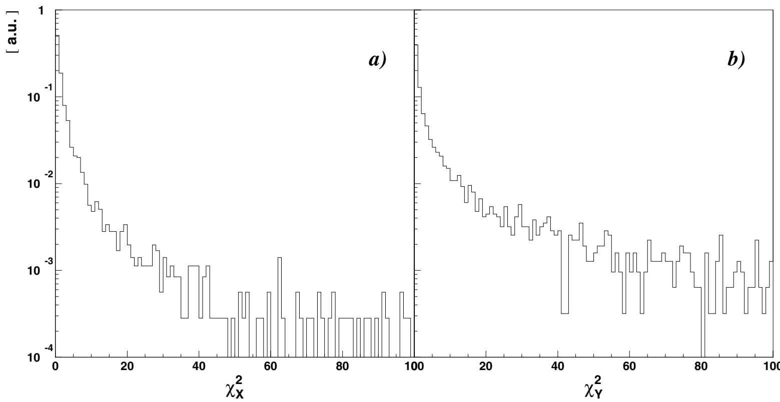
✦ Energy reconstruction:



- *Cluster extraction from shower shape predictions.*
- *A χ^2-based procedure to unfold overlapping showers.*

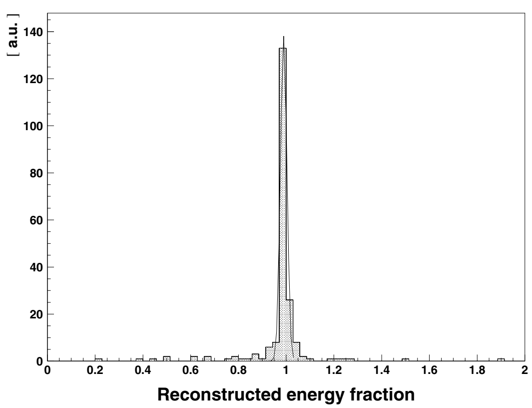
$\Longrightarrow$ *resolution on the rec. energy fraction $\sim 1.6\%$*

✦ Application to ν_e CC events in NOMAD:



- χ^2_x *distribution in good agreement with test-beam (data & MC) results.*
- χ^2_y *distribution distorted by overlaps from bremsstrahlung emission along* Y *(bending).*

$\Longrightarrow$ *different* χ^2*'s are necessary.*



- *The reconstruction of the cluster energy through a recursive* χ^2 *minimization procedure gives an average resolution on the reconstructed energy fraction of* $\sim 1.5\%$ *($E > 2$ GeV, all angles).*

CONCLUSIONS

- *A complete parameterization of the integral response of the NOMAD lead-glass calorimeter to e and γ has been obtained as a function of angles and energy. (details in R. Petti, NOMAD internal note 97-018, NIM A425 (1999) 188-209.)*

- *Both the average distributions and the corresponding fluctuations are parameterized.*

- *Good agreement is found between data and MC predictions.*

- *The algorithm is the basis of the reconstruction of e.m. clusters in NOMAD.*