Pre–Equilibrium and Equilibrium decays in Geant4

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Geant4 Hadronic Working Group
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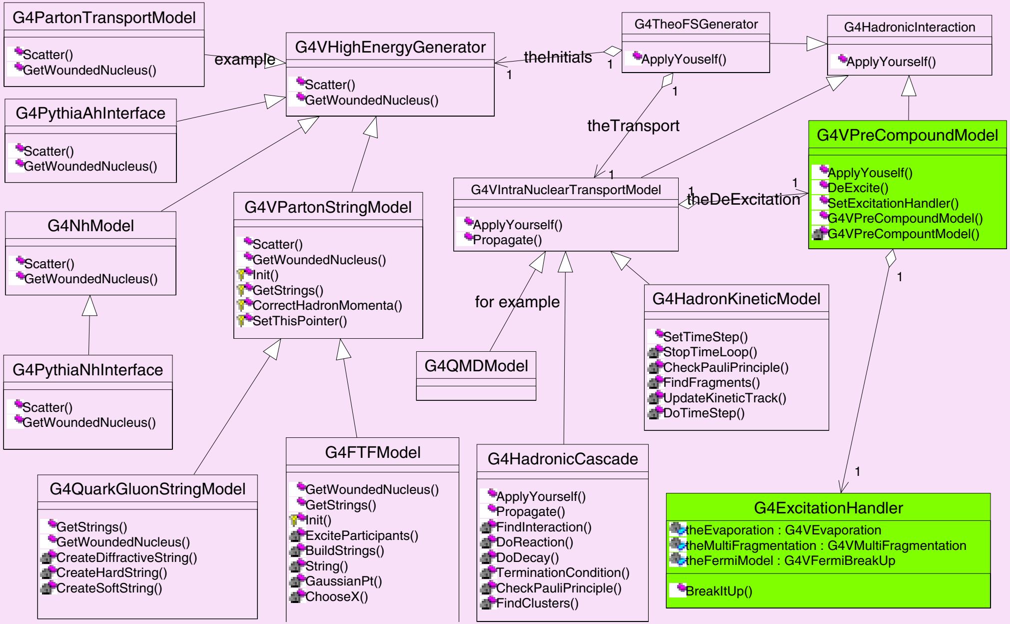
1. Theoretical Driven Hadronic Models



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2. Pre–Equilibirum Decays

- The responsability of the Pre–Equilibrium domain is to decay excited nuclei with excitation energies $O(100\ \text{MeV})$.

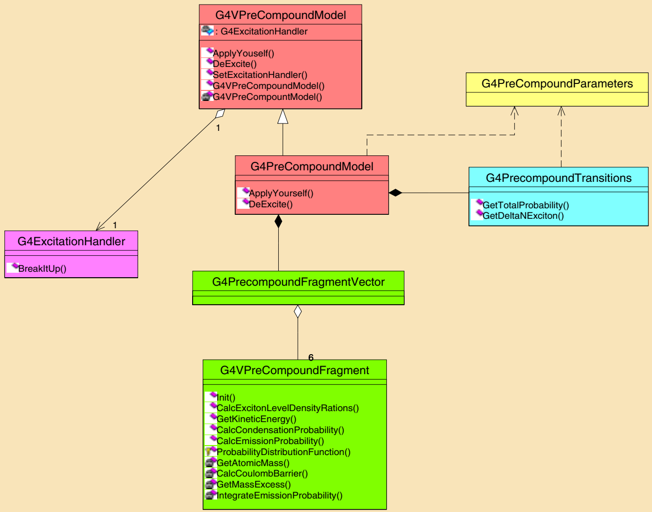
- Pre–Equilibrium fills the gap between the arbitrary cutoff of the Intra–Nuclear cascade and equilibrium decays.

- It models the high energy continuum region of ejectile spectrum.

- The Griffin's semiclassical Exciton Model has been used.

- In the Exciton Model, the composite nucleus states are characterized by the number of excitons n (excited particles p and holes h).

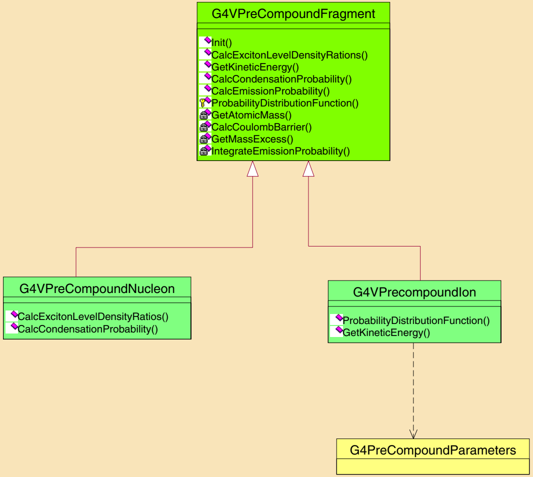
- Succesive two–body interactions give rise to an intranuclear cascade which eventualy leads to a fully equilibrated nucleus.



- At each stage of this equilibrium process there is a competition between two decay modes:
  - Emission of paticles into the continuum.
  - Exciton–Exciton interaction to more complex configurations. Selection rules: $\Delta n = 0, \pm 2$, $\Delta p = 0, \pm 1$, $\Delta h = 0, \pm 1$
- The transition rate between two states n and n' is:

$$\lambda_{nn'} = \frac{2\pi}{\hbar} \overline{|M|^2} \rho_{n'}(E^*)$$

2.1. Pre–Compound Fragments



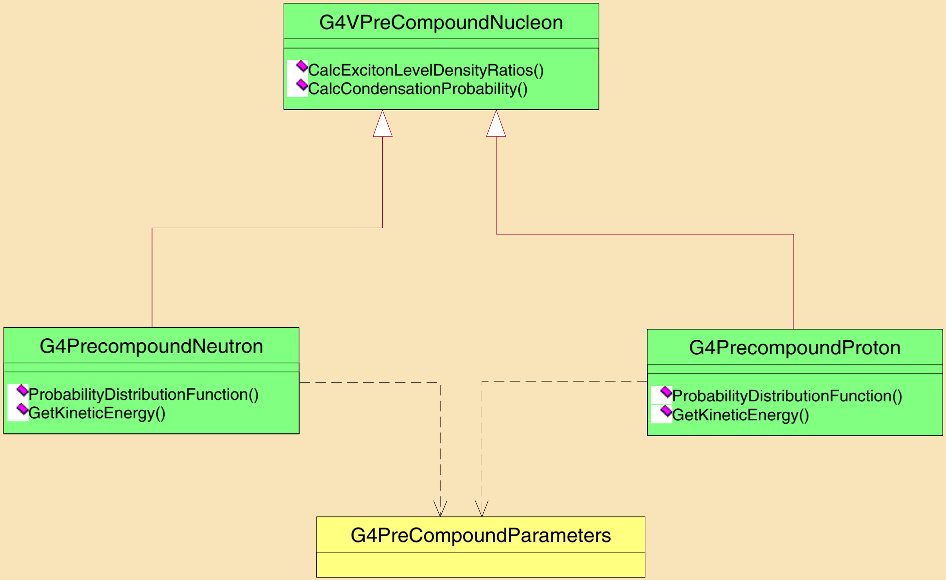
- Assuming equally spaced single–nucleon states with density g, the state density becomes

$$\rho_n(E^*) = \frac{g(gE^*)^{n-1}}{p!h!(n-1)!}$$

- We have to distinguish between simple fragments (nucleons) and more complex fragments (ions).

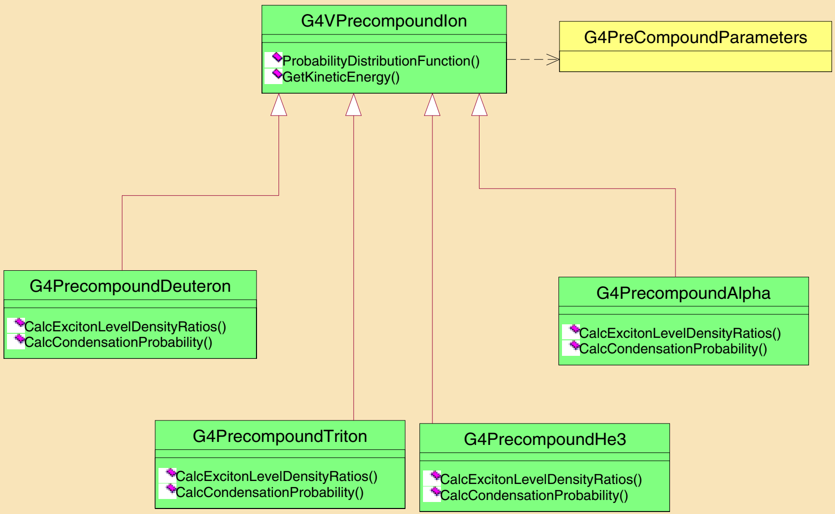
2.1.1. Pre–Compound Nucleons

- In the decay rate we must include a factor that takes into account the condition for the exciton to be a proton or an neutron.

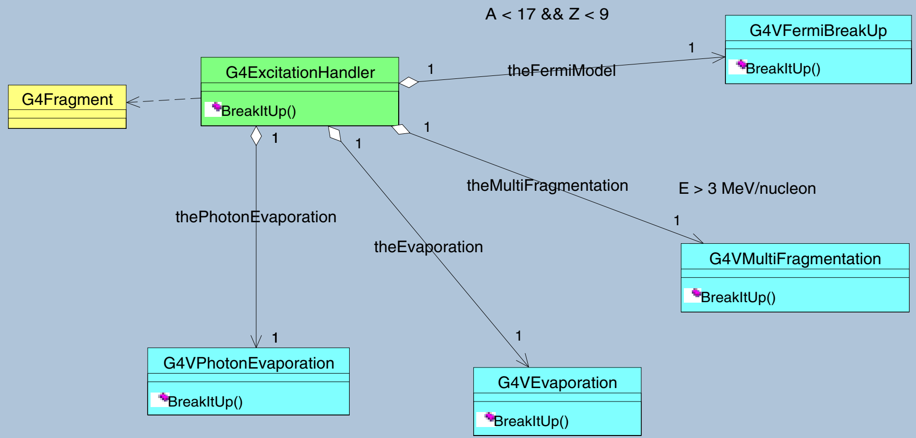


2.1.2. Pre–Compound Ions

- For ions, we have to consider the condensation probability of such fragment.



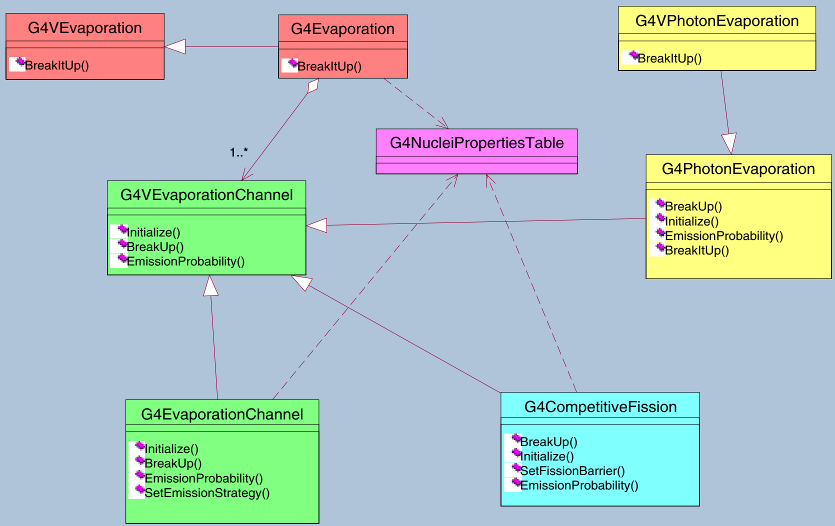
3. Equilibrium Decays



- Compound nuclei are excited nuclei that have reached statistical equilibrium.
- `G4ExcitationHandler` manages 5 de–excitation mechanisms.
  - Evaporation is the main de–excitation mechanism.
  - Fission as an evaporation competitive channel for heavy nuclei.
  - Fermi Break–Up model for light nuclei.
  - Multifragmentation for very excited nuclei.
  - Photon evaporation as competitive channel in evaporation and for residual excitation energies.

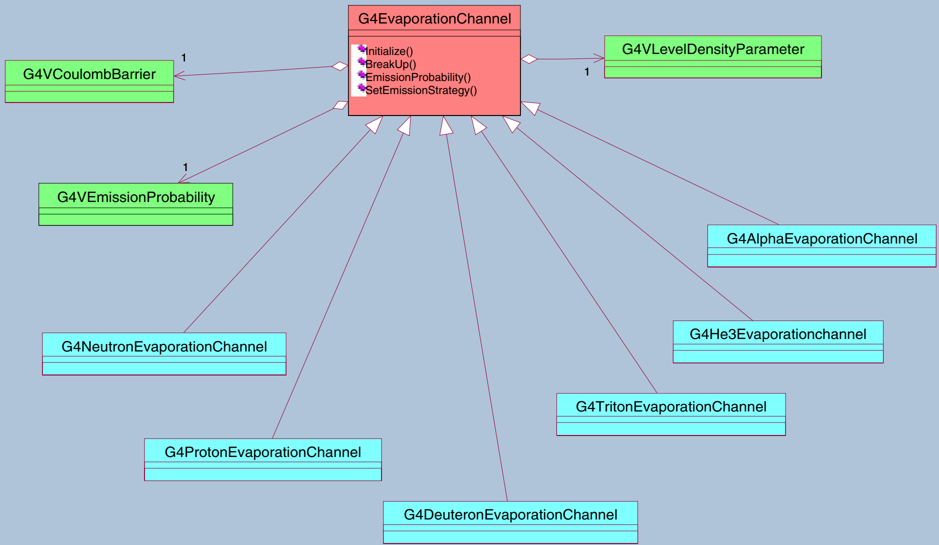
4. Evaporation

- `G4Evaporation` implements the statistical Weisskopf–Ewing model.
- Channels are treated polymorphically through the abstract interface `G4VEvaporationChannel`.
- By default there are 8 evaporation channels:
  - p, n, deuteron, triton, ^{3}He, alpha
  - Photon
  - Fission



4.1. Evaporation Channels

- G4EvaporationChannel implements those channels that always result in emission of nucleons or light ions.
- The most important “ingredients” are abstracted out:
  - Coulomb Barrier
  - Level Density Parameter
  - Evaporation Probability



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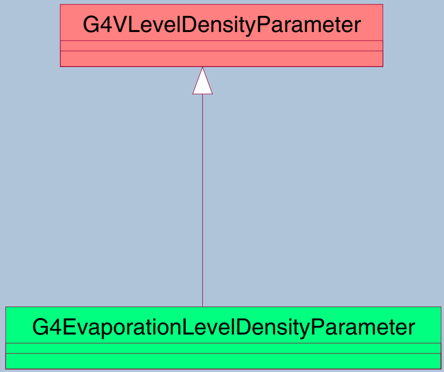
4.2. Level Density Parameters



- This parameter plays a major role in the level density models.
- We use the functional form proposed by Ignatyuk:

$$a(A, Z, U) = a_0(A)\left[1 + \Delta_{\text{shell}}(A, Z)\frac{f(U - \Delta_{\text{pair}})}{U - \Delta_{\text{pair}}}\right]$$

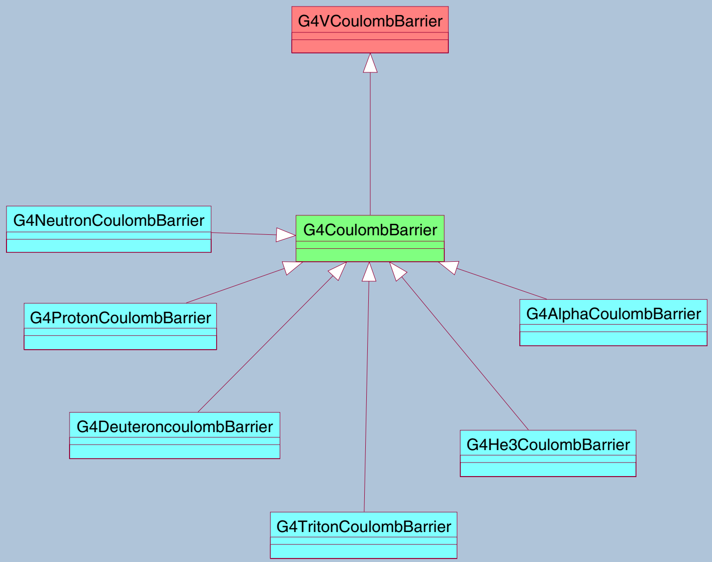
  There are other possible equations.
- $a_0(A) = \alpha A + \beta A^{2/3} B_s$ is the Fermi–gas value of a at high excitation energies.
- There are several choices for the parameters vaules ...

4.3. Coulomb Barriers

- Coulomb barriers are calculated according to:

$$V = K \frac{Z_f Z_{\text{res}} e^2}{R_{\text{comp}} (A_f^{1/3} + A_{\text{res}}^{1/3})}$$

- K is a barrier penetration factor that depends on the kind of particle.

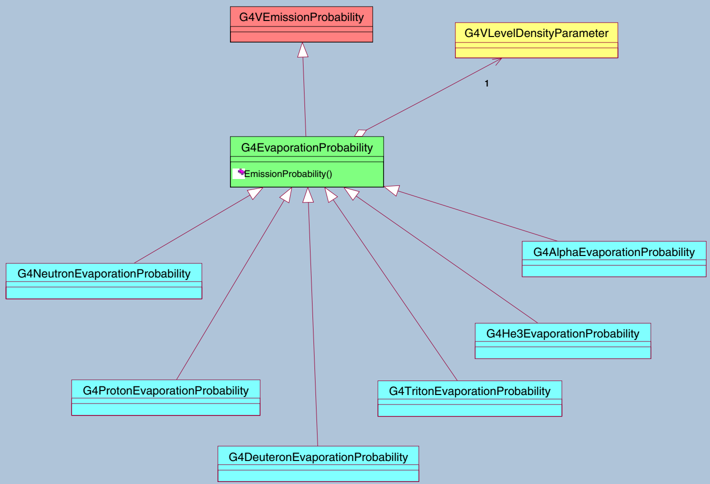


4.4. Emission Probabilities

- Weisskopf's expression for the probability per unit time for the emission of a particle f:

$$W_f = \int_0^{T^{\max}} \frac{(2s_f+1)m_f}{\pi^2\hbar^3}\sigma_f(T)\frac{\rho(U_f - T)}{\rho(U_{\text{res}})}\mathrm{d}T$$

- Empirical equations for Inverse Reaction Cross Sections:
  - For neutrons $\sigma_c(A,T) = (\alpha(A) + \beta(A)/T)\sigma_g$
  - For charged particles $\sigma_c(Z,T) = (1 + C(Z))(1 - V/T)\sigma_g$



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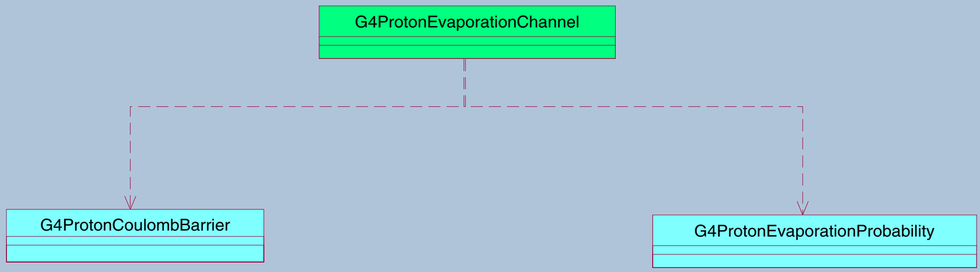
4.5. An example: Proton Evaporation Channel



- Particular classes like, for example, `G4ProtonEvaporationChannel`, are responsible for the proper initialization of channels.
- They provide data: A,Z, ...
- They instantiate the right:
  - Coulomb Barrier: `G4ProtonCoulombBarrier`
  - Evaporation Probability: `G4ProtonEvaporationProbability`

5. Fission

- Fission is an important channel of de–excitation of heavy nuclei (A > 200).

- `G4CompetitiveChannel` follows the Bohr–Wheeler statistical approach.

- Fission probability is proportional to the level density at the saddle point:

$$W_{\rm fis} = \frac{1}{2\pi\rho_{\rm comp}(U)} \int_0^{U-B_{\rm fis}} \rho_{\rm sp}(E^* - \Delta_{\rm fis} - B_{\rm fis} - T){\rm d}T$$

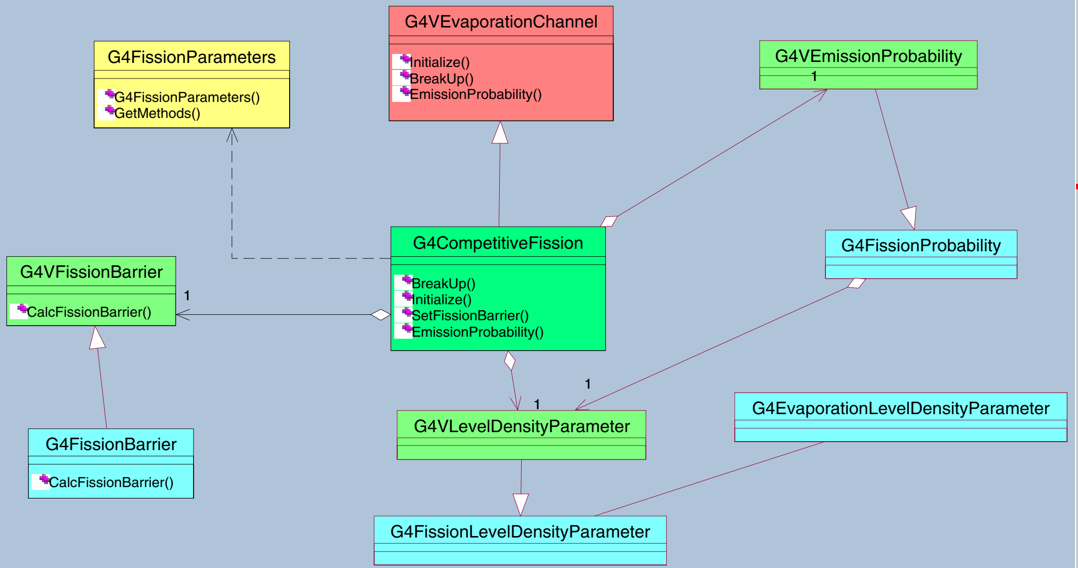
- The height of fission barrier is defined as the difference between the saddle point and ground state masses. It is approximated as

$$B_{\rm fis} = B^0_{\rm fis} + \Delta_{\rm shell} + \Delta_{\rm sp}$$

- Fission fragments mass distribution consists of a symmetric and an asymmetric components:

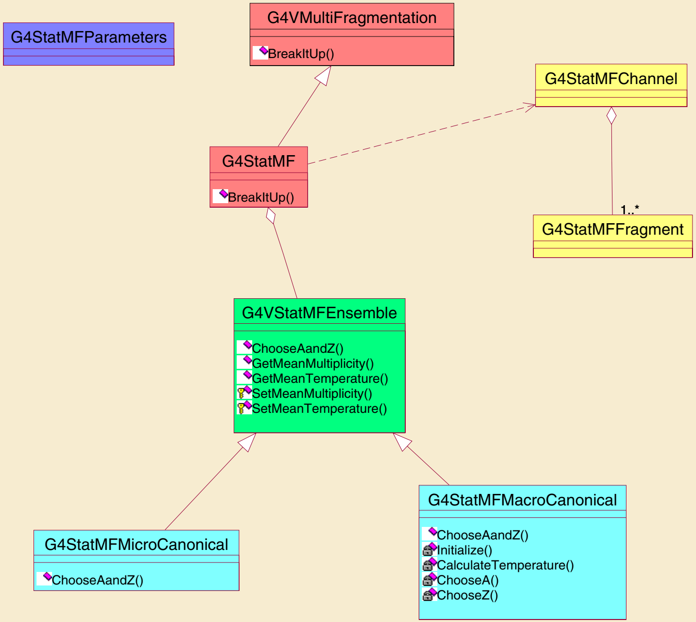
$$F(A_{\rm fis}) = F_{\rm sym}(A_{\rm fis}) + w(U, A, Z)F_{\rm asym}(A_{\rm fis})$$

and $w(U, A, Z)$ is the relative contribution of each component.

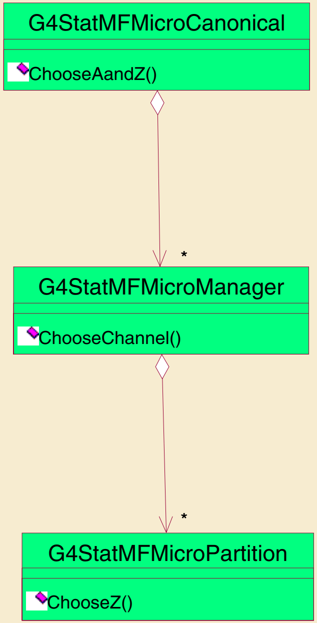

G4FissionParameters
G4FissionParameters()
GetMethods()
G4VEvaporationChannel
Initialize()
BreakUp()
EmissionProbability()
G4VEmissionProbability
1
G4CompetitiveFission
BreakUp()
Initialize()
SetFissionBarrier()
EmissionProbability()
G4FissionProbability
G4VFissionBarrier
1
CalcFissionBarrier()
1
1
G4VLevelDensityParameter
G4EvaporationLevelDensityParameter
G4FissionBarrier
CalcFissionBarrier()
G4FissionLevelDensityParameter

6. Multifragmentation

- At very high excitation energies (> 3 MeV/nucleon) we have an explosion–like de–excitation process.
- `G4StatMF` implements an statistical mechanism based on the Copenhagen Model.
- Due to the huge number of open channels, we have to use two approaches.



6.1. Microcanonical Ensemble

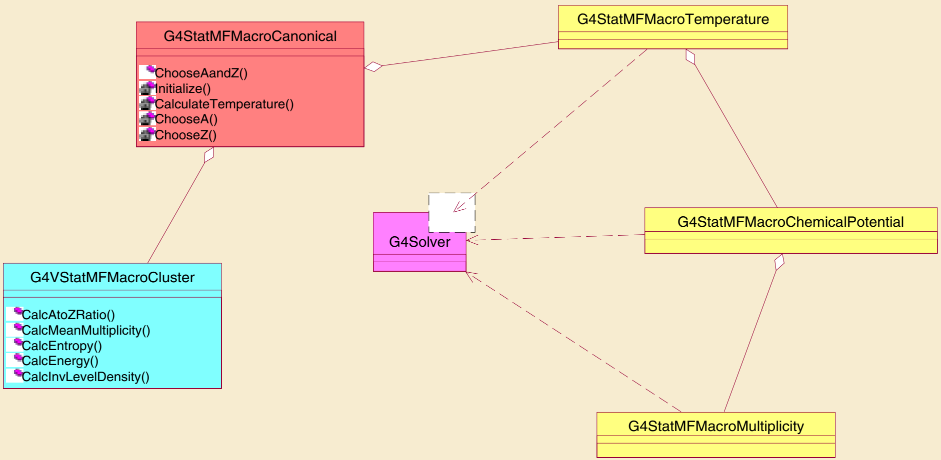


- In the microcanonical ensemble all microscopic states of the system obey strictly the conservation laws.
- The statistical weights of a break–up partition are determined by its entropy.

$$W_f^{\mathrm{mic}} \sim \exp(S_f(E_0, V, A_0, Z_0))$$

- We calculate all possible partitions with multiplicity less than M_0.
- We calculate the mean multiplicity $\langle M \rangle$.
- If $\langle M \rangle < M_1$ one of these partitions is ramdomly selected according with their statistical weights.

6.2. Macrocanonical Ensemble



- In the macrocanonical ensemble, we have only constraints on the average mass and charge of the system.
- The distribution of partition probabilities in the macrocanonical approximation is given by

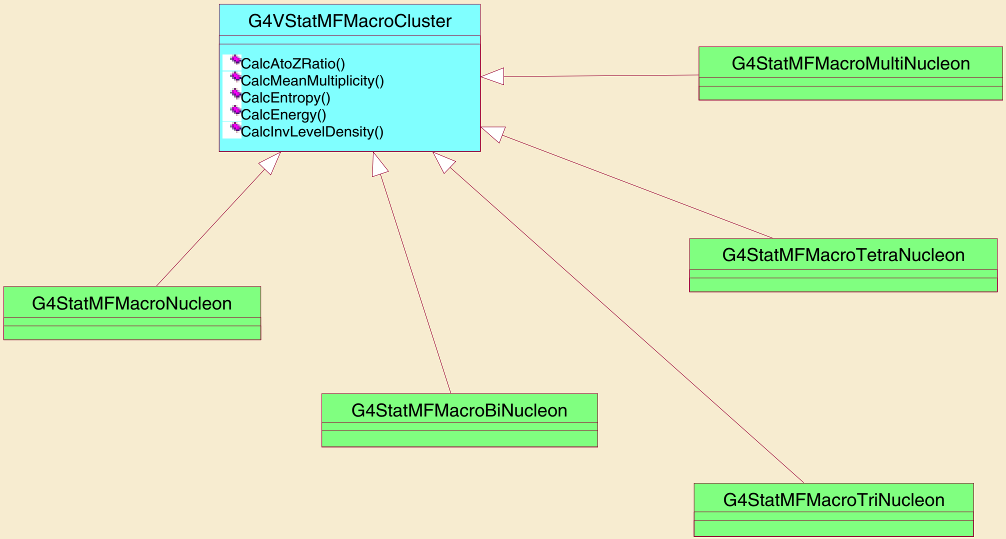
$$W_f^{\mathrm{mac}} \sim \exp(-\Omega_f(T, V, \mu, \nu)/T)$$

- We have to solve for T, μ, ν, in order to find out $\langle N_{\mathrm{AZ}}\rangle$, $\langle N_{\mathrm{A}}\rangle$ and $N(\mathrm{Z})$.

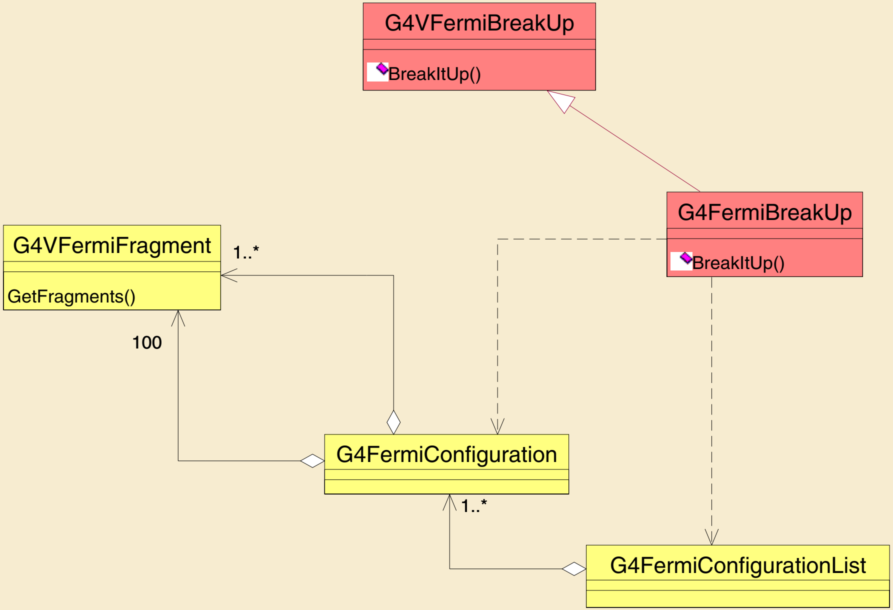
6.2.1. Macrocanonical Clusters

- For clusters with more than 4 nucleons we can use the liquid drop model.

- But this approach is not valid for light clusters.



7. Fermi Break–Up



- For light nuclei (A $\leq$ 16) even samall excitation energies are comparable to their total binding energies.
- The Fermi model is analogous to the statistical multifragmentation.
- Due to the small size of nuclei, we can use only the microcanonical approach.

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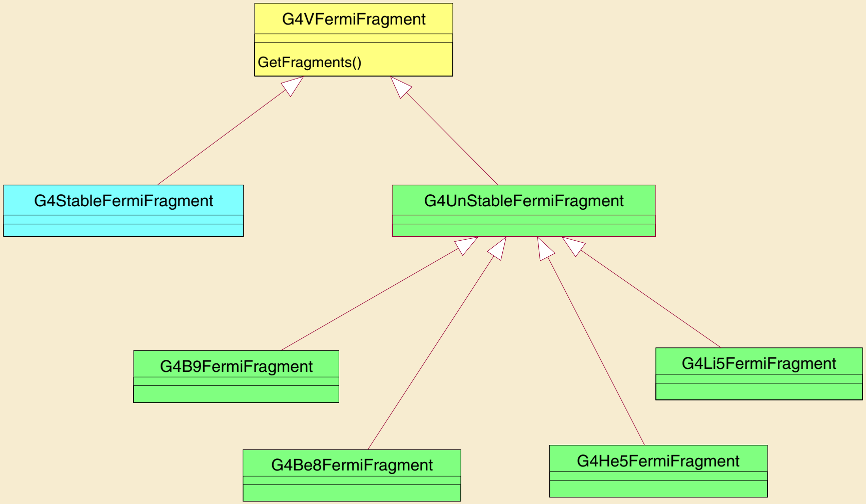
7.1. Fermi Breal–Up Channels

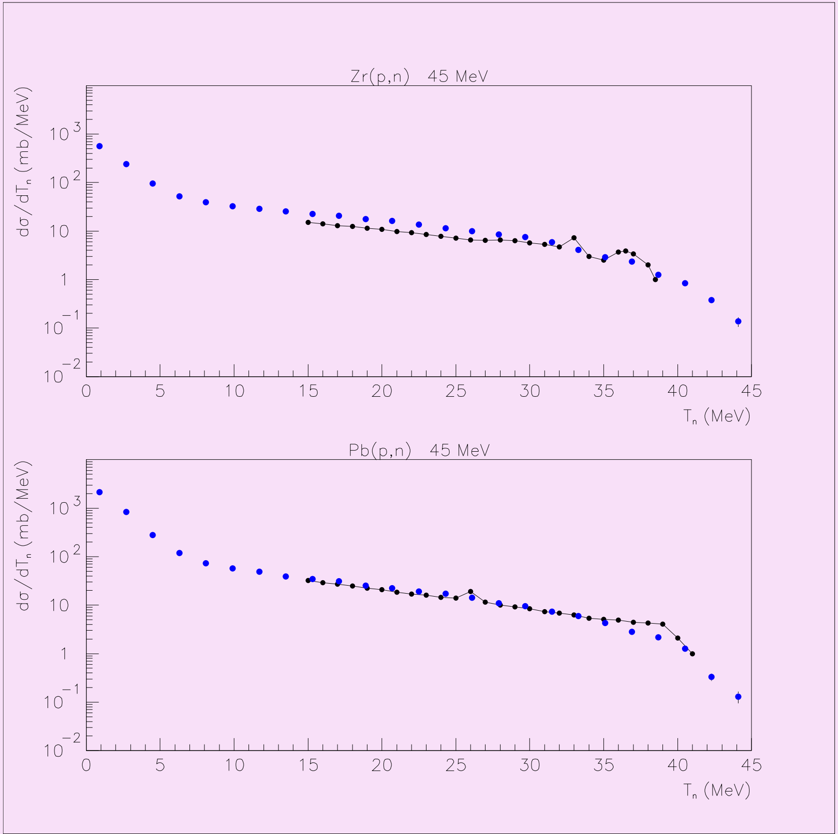


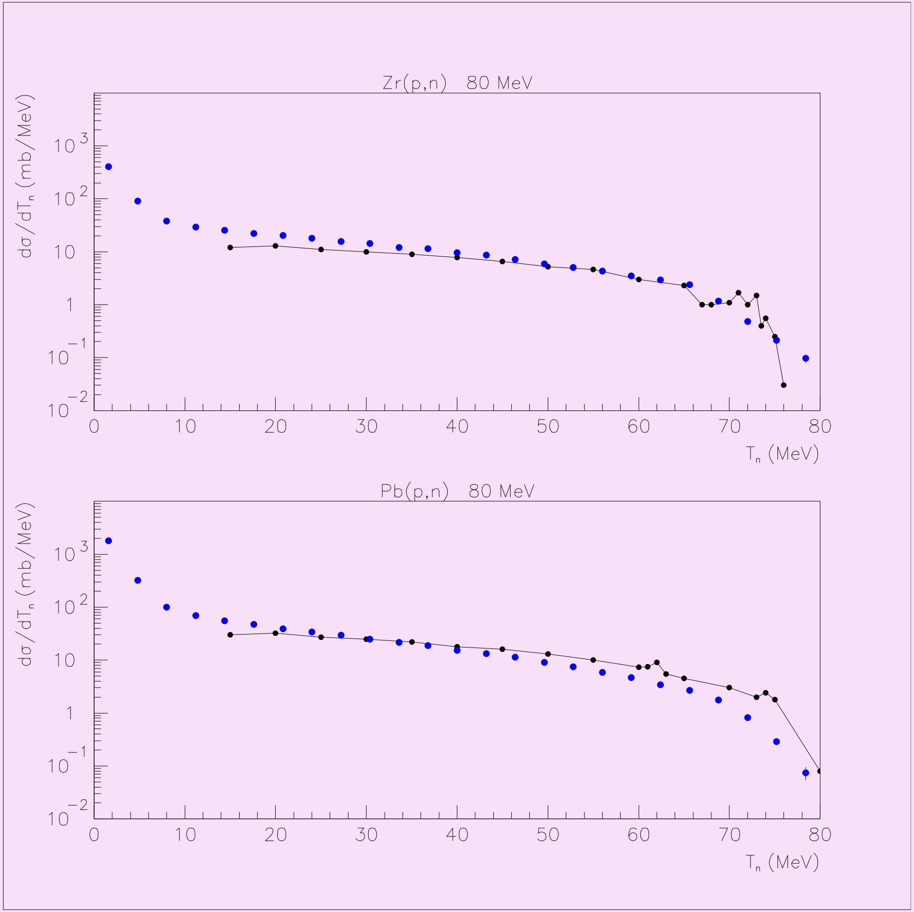
- Coulomb expansion is not considered explicitly, but momentum distributions are obtained sampling over the accesible phase space.

- Long–lived unstable nuclei will decay at the end of the expansion.

8. Results

8.1. Differential cross sections.





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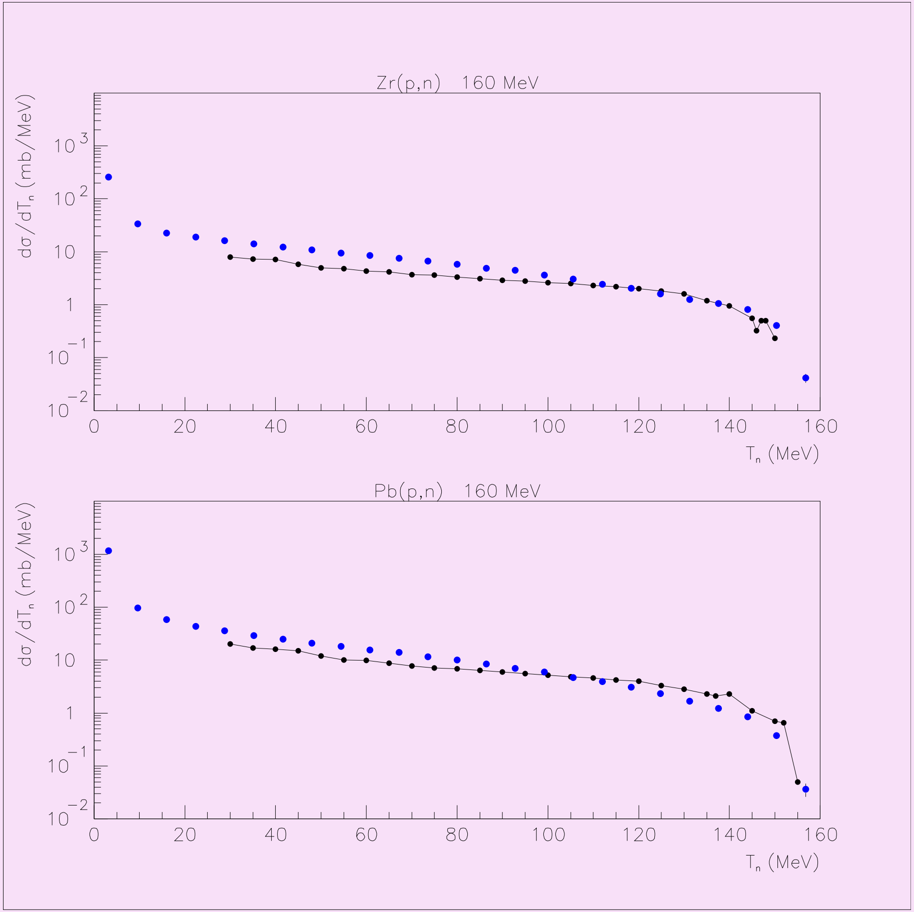

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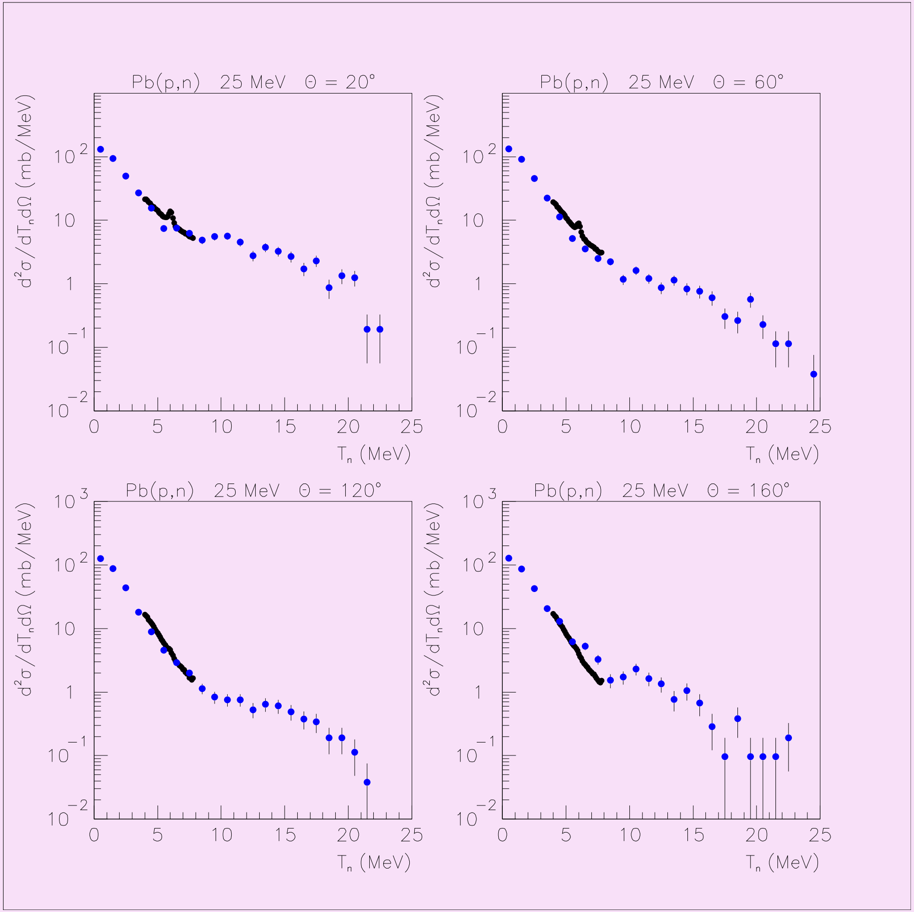
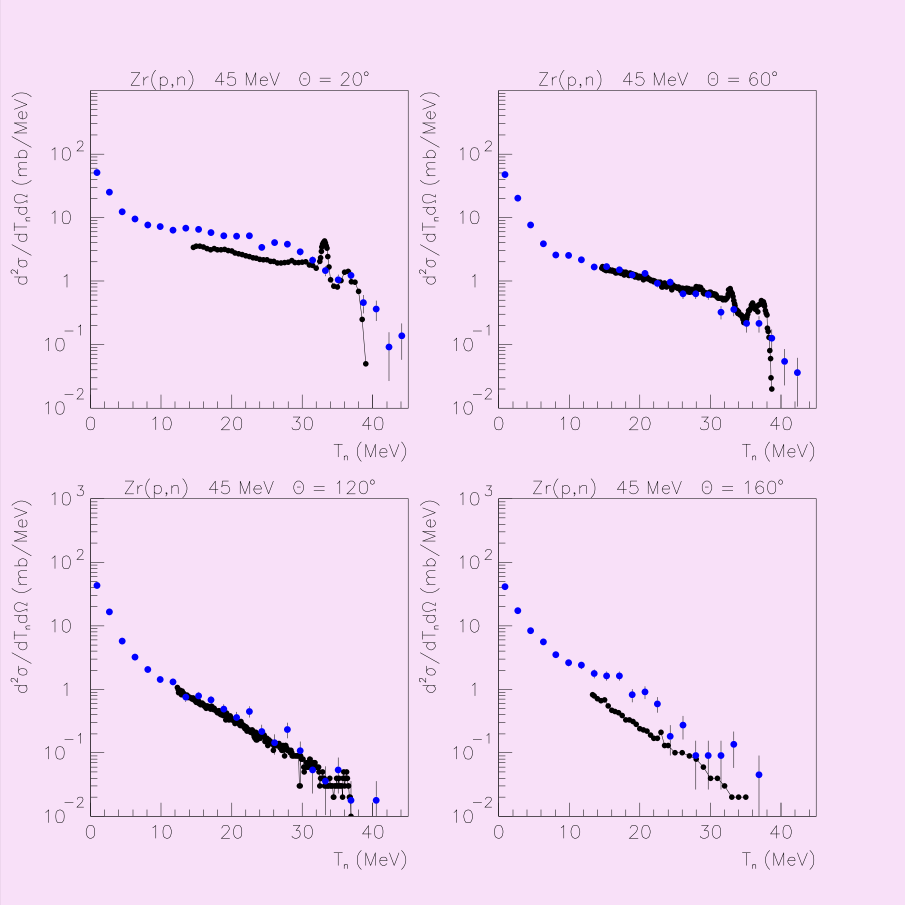
8.2. Angular distributions.





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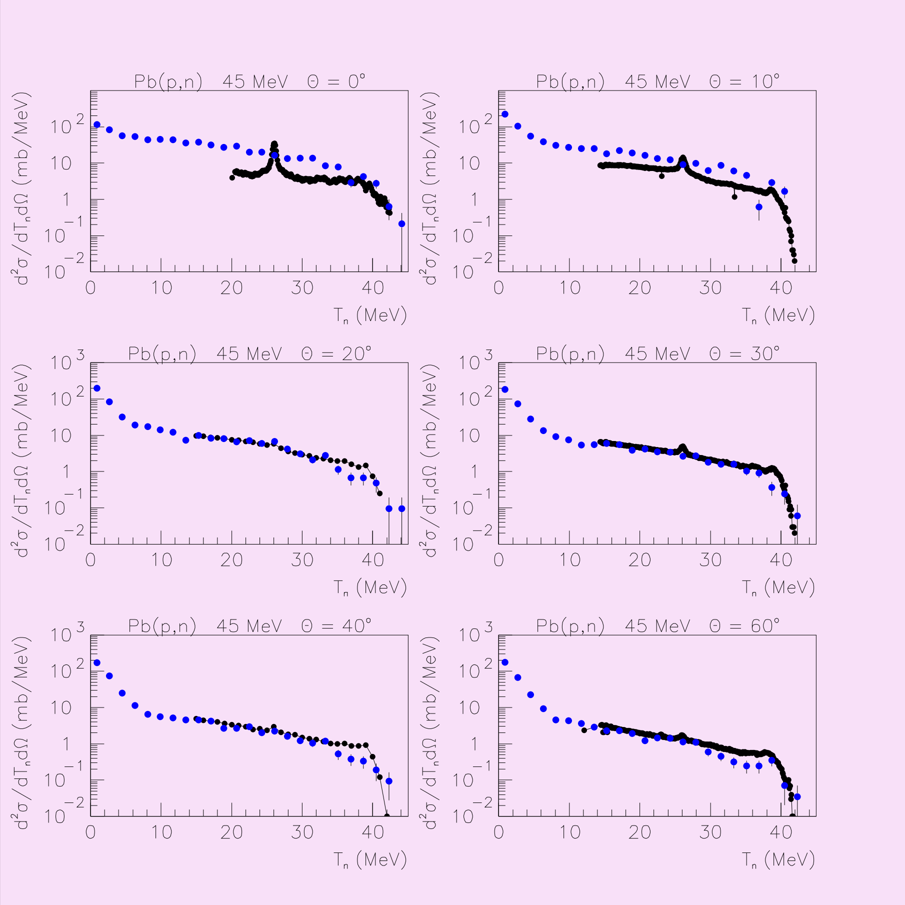
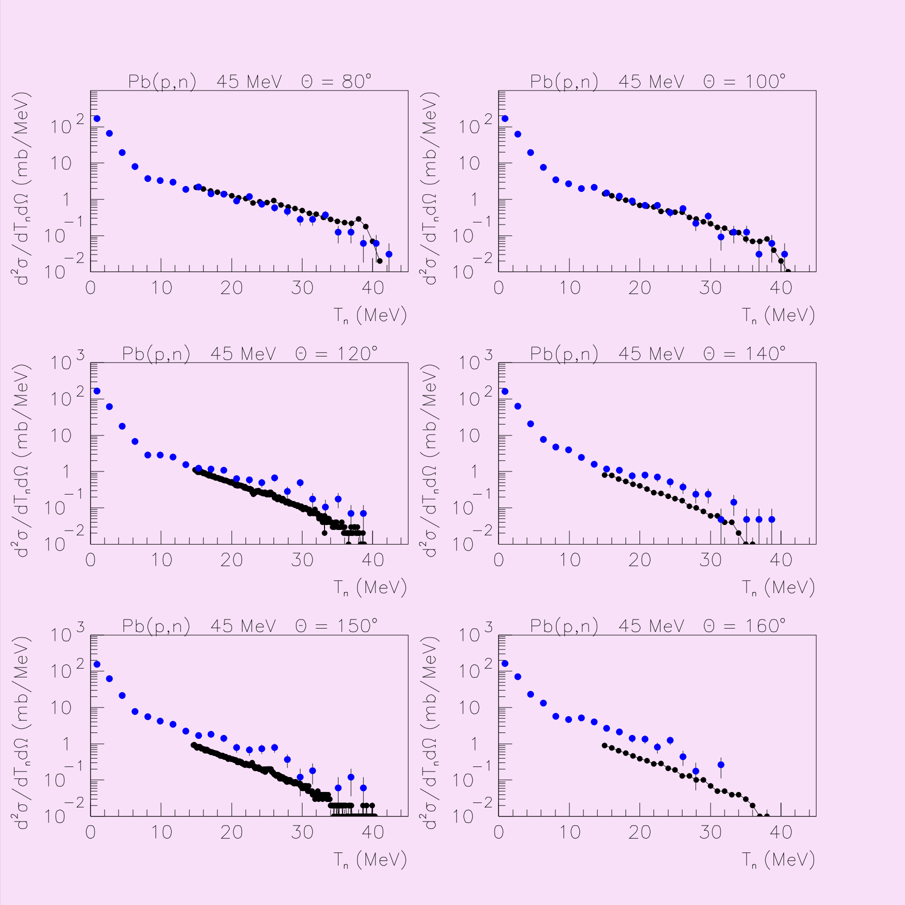

Pb(p,n) 45 MeV Θ = 0°
Pb(p,n) 45 MeV Θ = 10°
Pb(p,n) 45 MeV Θ = 20°
Pb(p,n) 45 MeV Θ = 30°
Pb(p,n) 45 MeV Θ = 40°
Pb(p,n) 45 MeV Θ = 60°
d²σ/dTₙdΩ (mb/MeV)
Tₙ (MeV)



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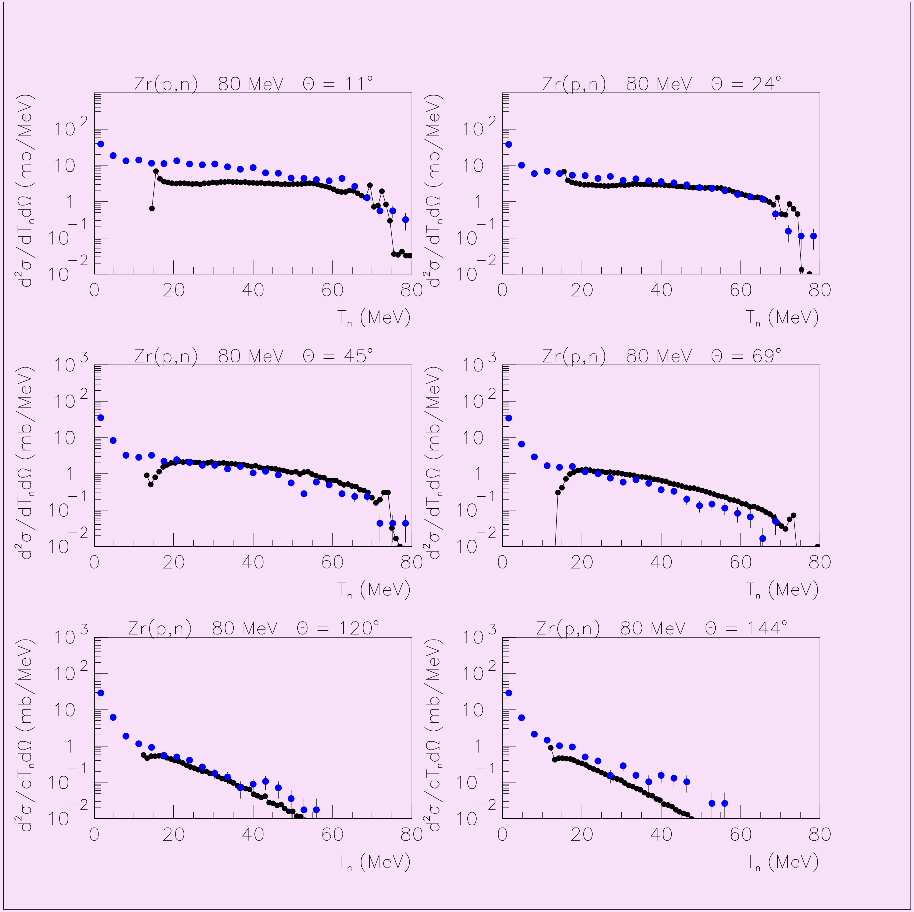

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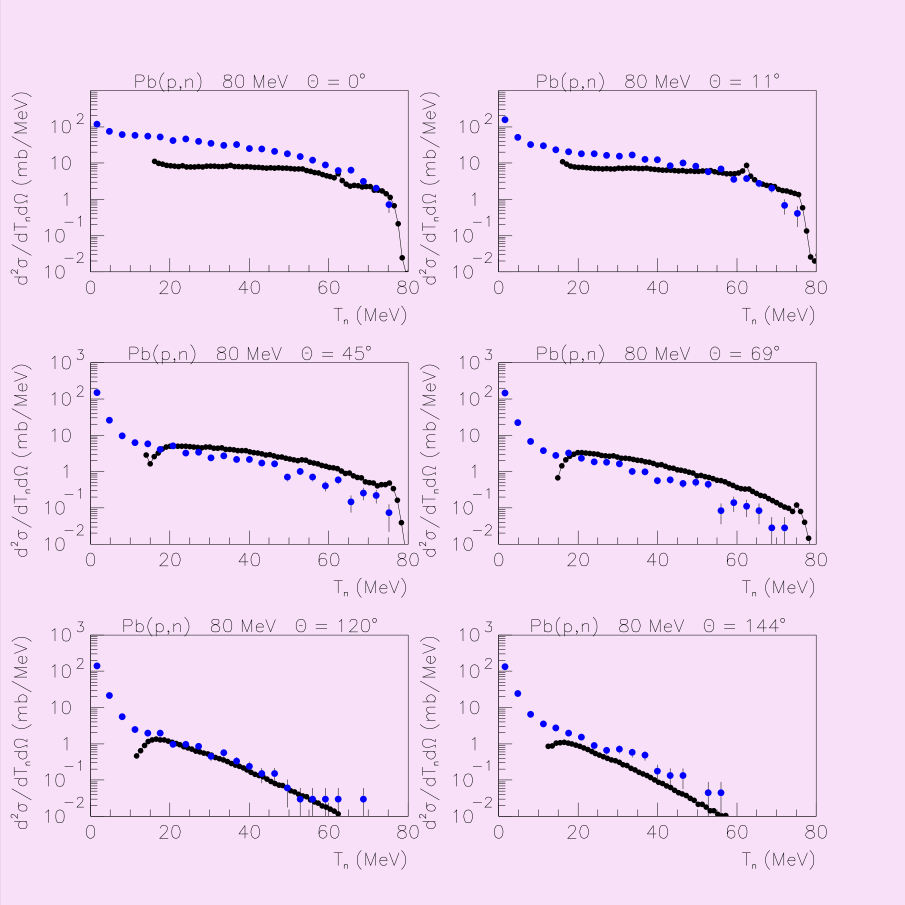
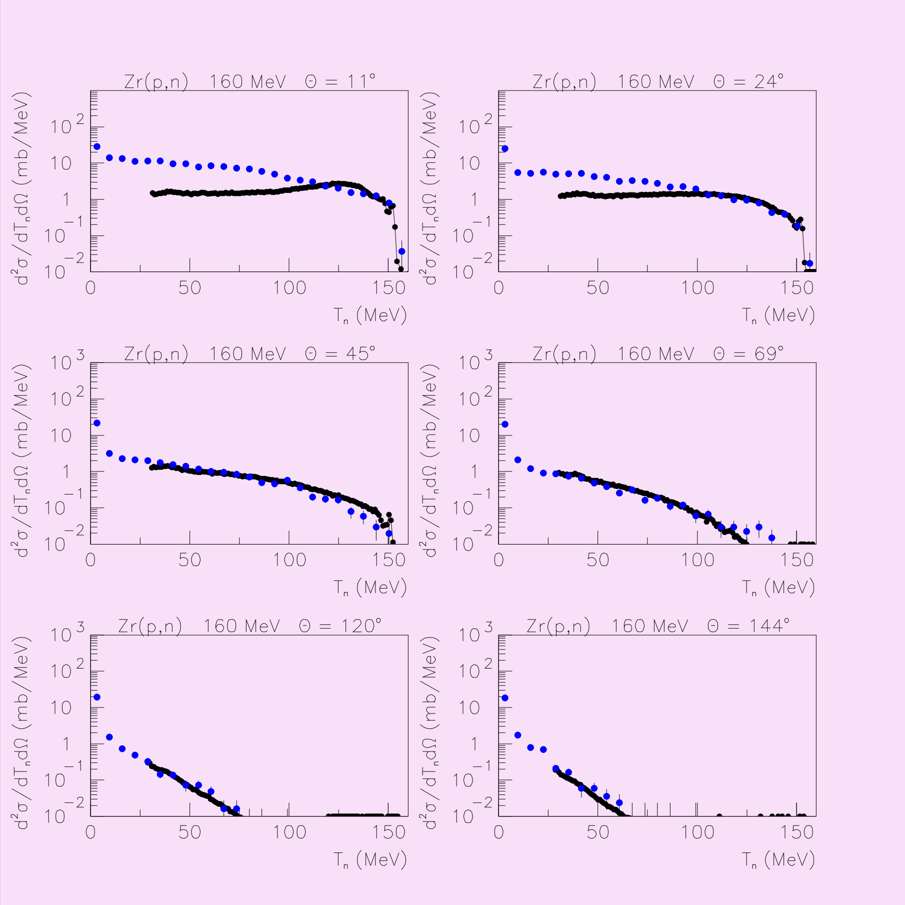

Pb(p,n) 80 MeV Θ = 0°
Pb(p,n) 80 MeV Θ = 11°
Pb(p,n) 80 MeV Θ = 45°
Pb(p,n) 80 MeV Θ = 69°
Pb(p,n) 80 MeV Θ = 120°
Pb(p,n) 80 MeV Θ = 144°
d²σ/dTₙdΩ (mb/MeV)
Tₙ (MeV)



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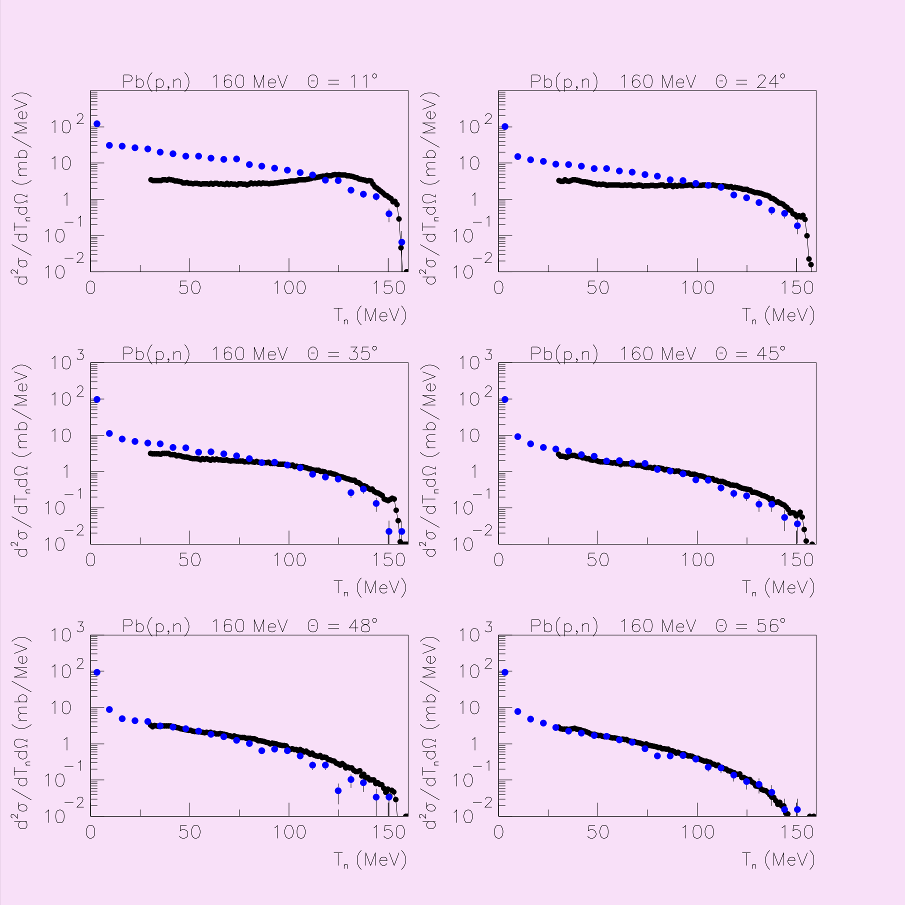

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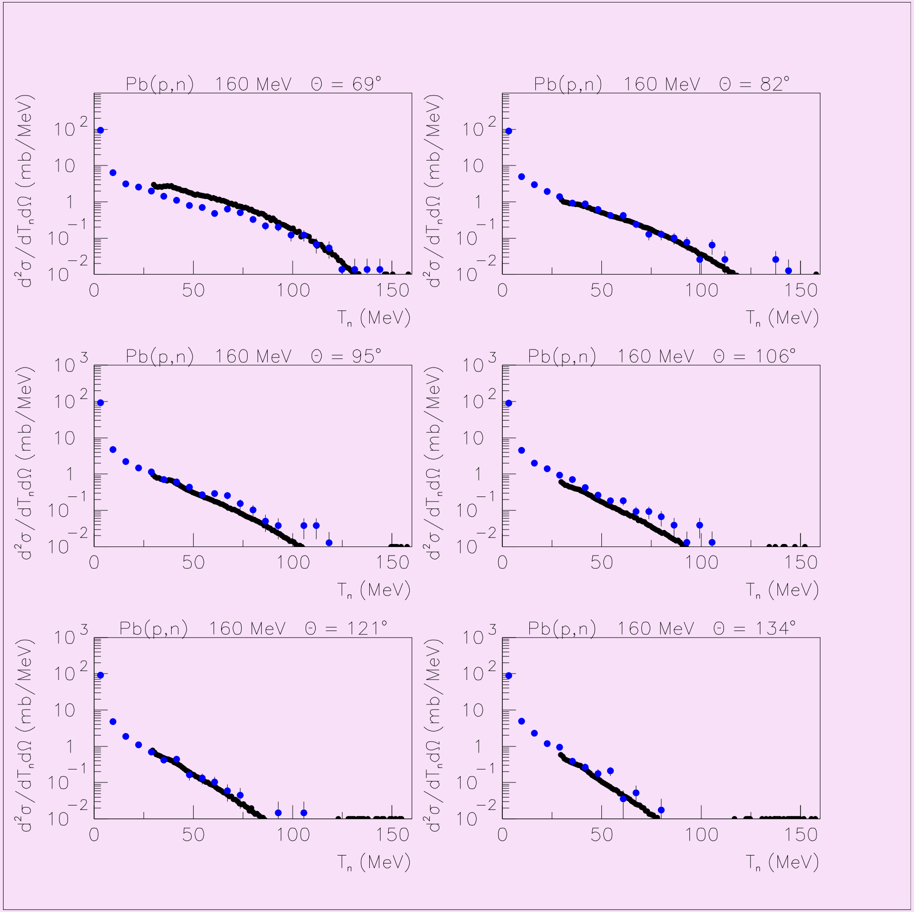
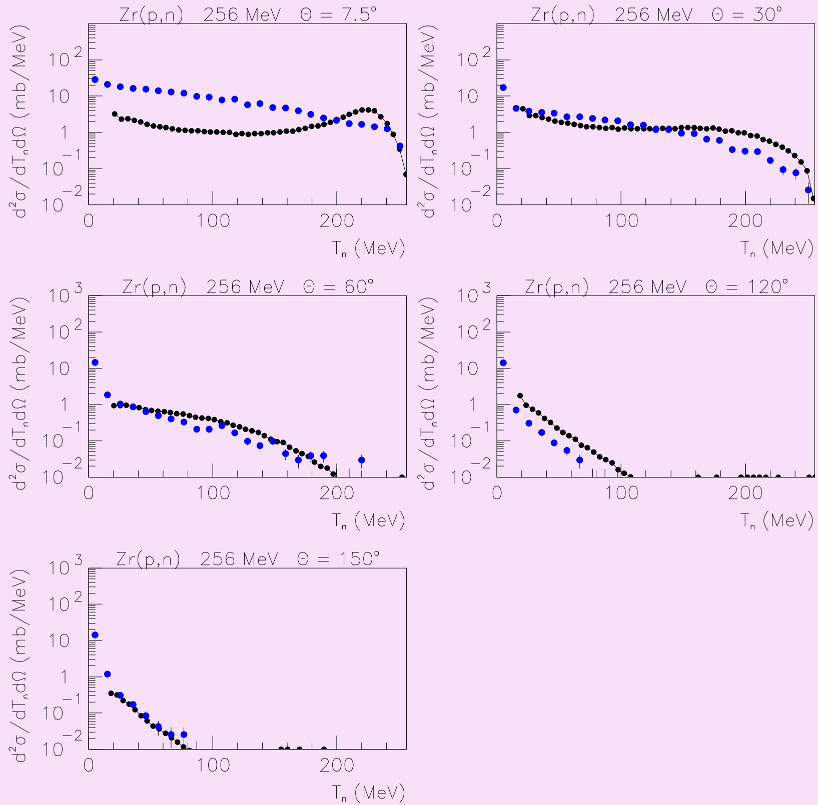
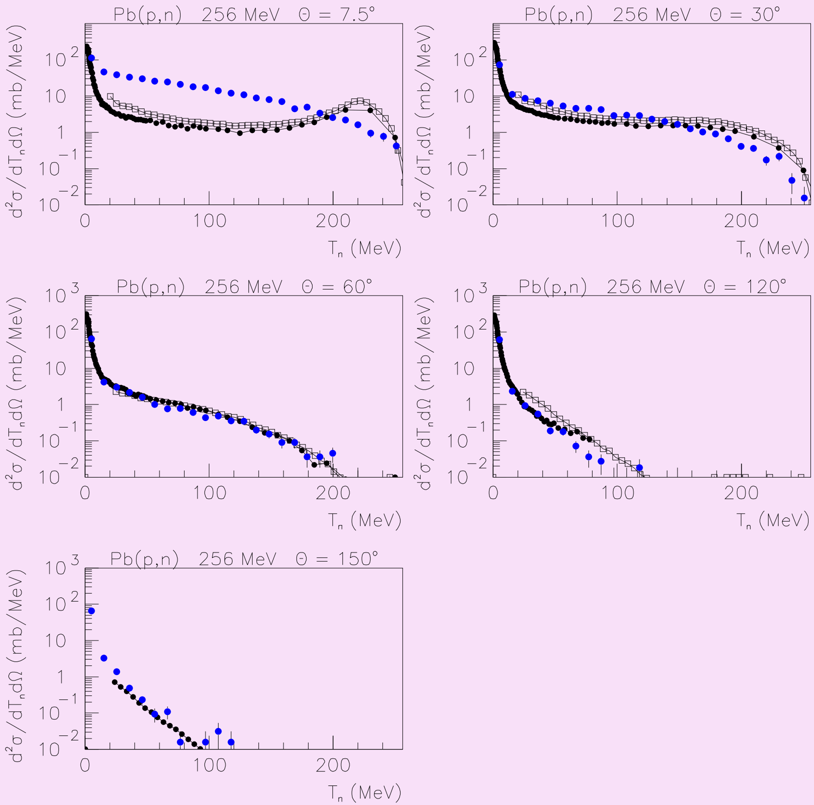


Pb(p,n) 160 MeV Θ = 69°
Pb(p,n) 160 MeV Θ = 82°
Pb(p,n) 160 MeV Θ = 95°
Pb(p,n) 160 MeV Θ = 106°
Pb(p,n) 160 MeV Θ = 121°
Pb(p,n) 160 MeV Θ = 134°
d²σ/dTₙdΩ (mb/MeV)
Tₙ (MeV)
0 50 100 150
10³ 10² 10 1 10⁻¹ 10⁻²


Zr(p,n) 256 MeV Θ = 7.5°
Zr(p,n) 256 MeV Θ = 30°
Zr(p,n) 256 MeV Θ = 60°
Zr(p,n) 256 MeV Θ = 120°
Zr(p,n) 256 MeV Θ = 150°
d²σ/dT_ndΩ (mb/MeV)
T_n (MeV)
0
100
200
10^3
10^2
10
1
10^-1
10^-2



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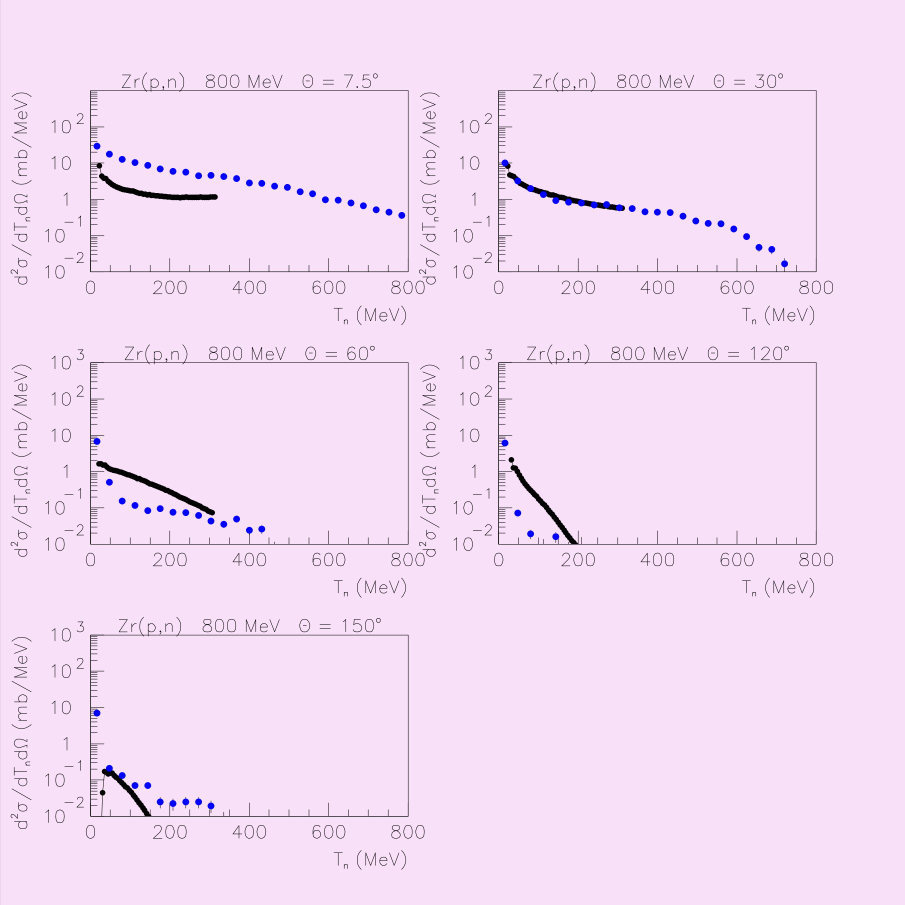
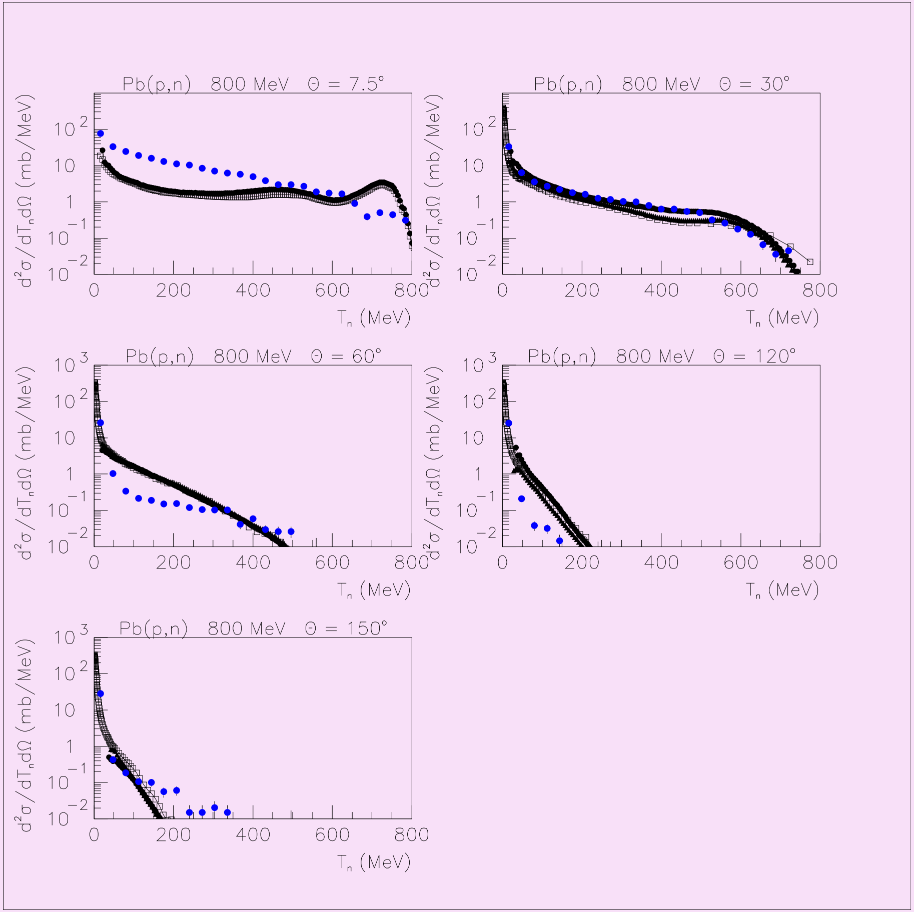


Zr(p,n) 800 MeV Θ = 7.5°
Zr(p,n) 800 MeV Θ = 30°
Zr(p,n) 800 MeV Θ = 60°
Zr(p,n) 800 MeV Θ = 120°
Zr(p,n) 800 MeV Θ = 150°
d²σ/dT_ndΩ (mb/MeV)
T_n (MeV)



Pb(p,n) 800 MeV Θ = 7.5°
Pb(p,n) 800 MeV Θ = 30°
Pb(p,n) 800 MeV Θ = 60°
Pb(p,n) 800 MeV Θ = 120°
Pb(p,n) 800 MeV Θ = 150°
d²σ/dT_n dΩ (mb/MeV)
T_n (MeV)
0
200
400
600
800
10^-2
10^-1
1
10
10^2
10^3