Electromagnetic interactions of particles with matter

October 9, 2000

Abstract

This document is a brief review to the main mechanisms of electromagnetic interactions of charged particles and photons with matter, pertinent in calorimetry

1. Common to all charged particles
   - ionization $(\sim keV \longrightarrow)$
   - Coulomb scattering from nuclei $(\sim keV \longrightarrow)$
   - Cerenkov effect
   - transition radiation
2. Muons
   - (e+,e-) pair production $(\sim 100GeV \longrightarrow)$
   - bremsstrahlung $(\sim 100GeV \longrightarrow)$
   - nuclear interaction $(\sim 1TeV \longrightarrow)$
3. Electrons and positrons
   - bremsstrahlung $(\sim 10MeV \longrightarrow)$
   - e+ annihilation
4. Photons
   - gamma conversion $(\sim 10MeV \longrightarrow)$
   - incoherent scattering $(\sim 100keV \longrightarrow \sim 10MeV)$
   - photo electric effect $(\longleftarrow \sim 100keV)$
   - coherent scattering $(\longleftarrow \sim 100keV)$

σ	cross section per atom. $(cm^2/atom)$
$n_{at} = \mathcal{N}\rho/A$	number of atoms per unit of volume. $(atoms/cm^3)$
	$n_{at} = n_1 + n_2 + \cdots = \frac{\mathcal{N}\rho w_1}{A_1} + \frac{\mathcal{N}\rho w_2}{A_2} + \cdots$
$\Phi = n_{at}\,\sigma$	number of interactions per unit of length. $(1/cm)$
	Σ : macroscopic cross section
	μ : absorption, attenuation coefficients ..etc..
$\lambda = 1/\Phi$	mean free path, interaction length, etc .. (cm)
$t = x\rho$	mass-thickness, mass/surface .. (g/cm^2)
Φ/ρ	nb of interactions per (mass/surface). $(1/(g/cm^2))$
	μ/ρ : **mass** attenuation coefficient ..etc..
X_0/ρ	radiation length, expressed in mass/surface (g/cm^2)
dE/dt	energy loss per (mass/surface) $(MeV/(g/cm^2))$

Ionization

Ionization

The basic mechanism is an inelastic collision of the moving charged particle with the atomic electrons of the material, ejecting off an electron from the atom :

$$\mu + atom \rightarrow \mu + atom^{+} + e^{-}$$

In each individual collision, the energy transfered to the electron is small. But the total number of collisions is large, and we can well define the average energy loss per (macroscopic) unit path length.

Mean rate of energy loss

The Bethe-Bloch formula gives the mean rate of energy loss by moderately relativistic charged particles (other than electrons).

$$-\frac{dE}{dx}(\beta) = 2\pi r_e^2 mc^2 n_{el} \frac{z_p^2}{\beta^2} \left[\ln\left(\frac{2mc^2\beta^2\gamma^2 T_{max}}{I^2} \right) - 2\beta^2 - \delta(\beta) \right]$$

where

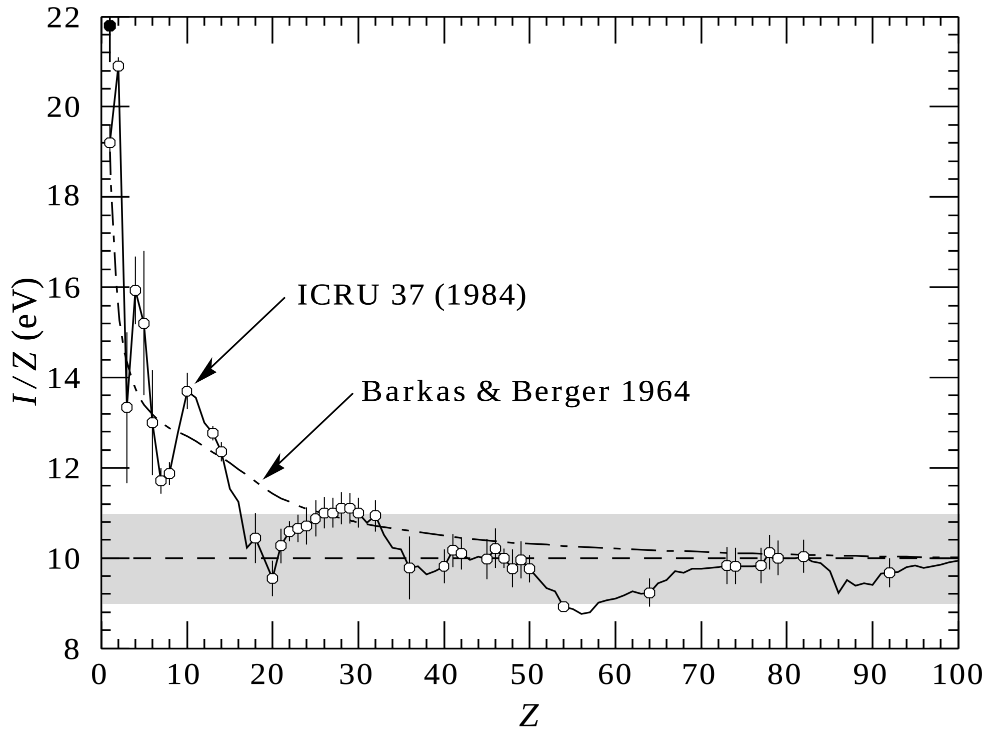
r_e	classical electron radius: $e^2/(4\pi\epsilon_0 mc^2)$
mc^2	energy-mass of electron
n_{el}	electrons density in the material
z_p	charge of the incident particle
T_{max}	maximum kinetic energy transferable to a free electron
I	mean excitation energy in the material
δ	density effect function

$$n_{el} = Z\, n_{at} = Z\, \frac{\mathcal{N}_{av}\rho}{A} \qquad T_{max} = \frac{2mc^2(\gamma^2-1)}{1+2\gamma m/M+(m/M)^2}$$

mean excitation energy

There exists a variety of phenomenological approximations for I, the simplest being $I = 10eV \times Z$

See [ICRU84] or [PDG00] for up to date recommended values.



the density effect

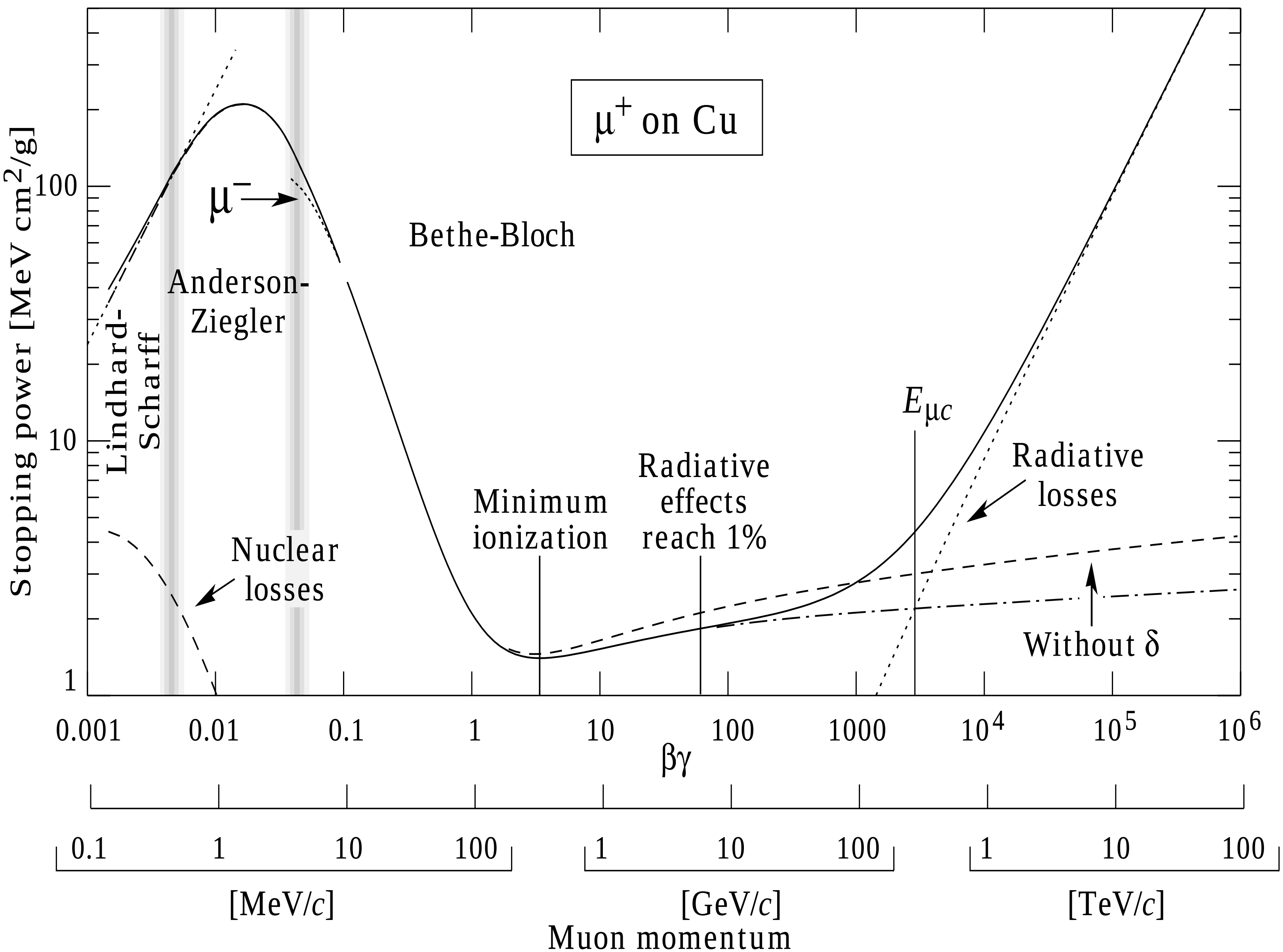
δ is a correction term which takes into account of the reduction in energy loss due to the so-called density effect. This becomes important at high energy because media have a tendency to become polarised as the incident particle velocity increases. As a consequence, the atoms in a medium can no longer be considered as isolated. To correct for this effect the formulation of Sternheimer [Ster71] is generally used.

low energies

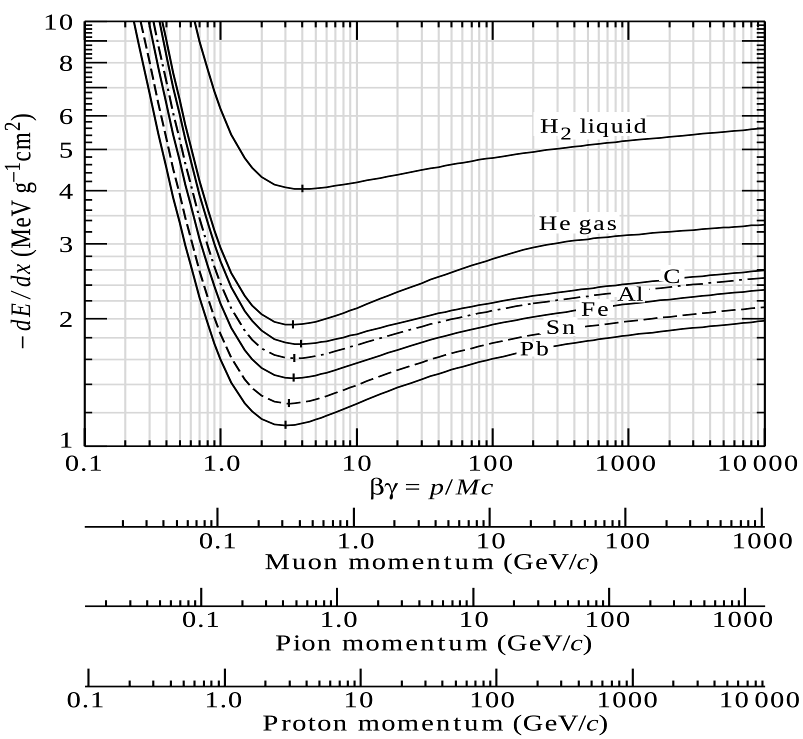
The mean energy loss can be described by the Bethe-Bloch formula only if the projectile velocity is larger than that of orbital electrons. In the low-energy region where this is not verified, a different kind of parameterisation should be used.
For instance:

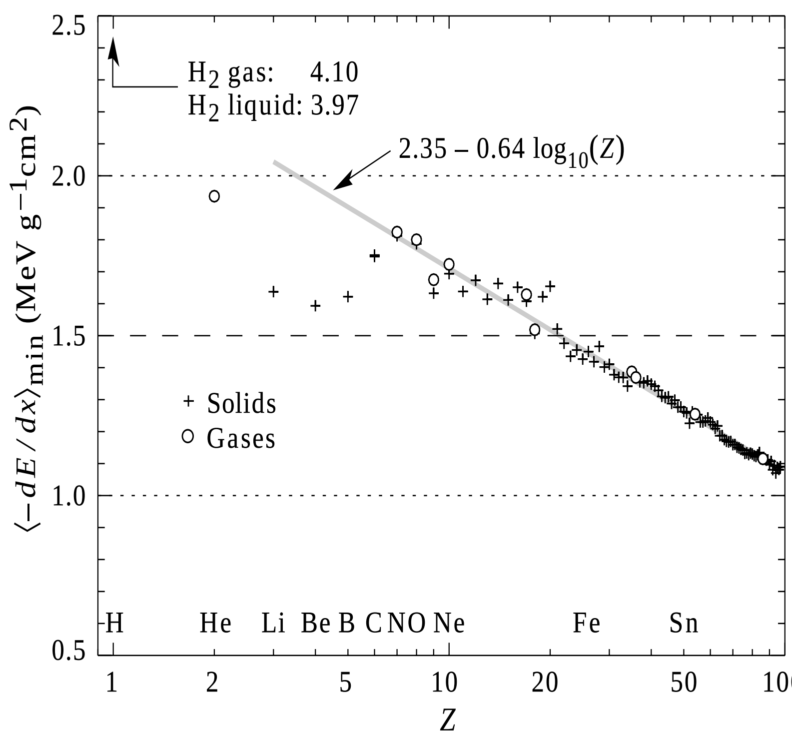
- Andersen and Ziegler [Ziegl77] for $0.01 < \beta < 0.05$
- Lindhard [Lind63] for $\beta < 0.01$

See ICRU Report 49 [ICRU93] for a detailed discussion of low-energy corrections.


μ+ on Cu
μ−
Bethe-Bloch
Anderson-
Ziegler
Lindhard-
Scharff
Nuclear
losses
Minimum
ionization
Radiative
effects
reach 1%
$E_{\mu c}$
Radiative
losses
Without δ
Stopping power [MeV cm2/g]
1
10
100
0.001
0.01
0.1
1
10
100
1000
10^4
10^5
10^6
βγ
0.1
1
10
100
1
10
100
1
10
100
[MeV/c]
[GeV/c]
[TeV/c]
Muon momentum

minimum ionization



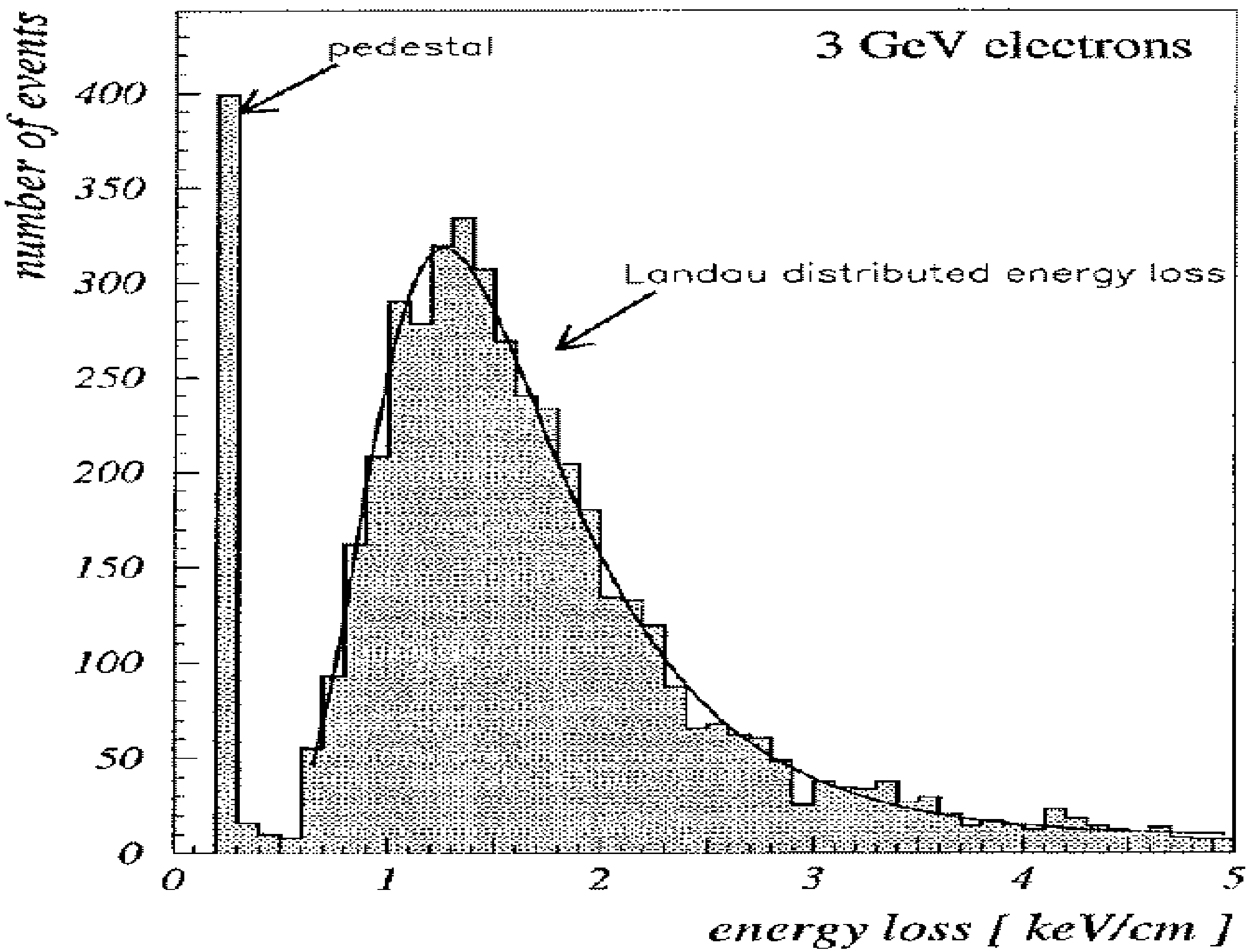


Fluctuations in energy loss

$\langle \Delta E \rangle = (dE/dx).\Delta x$ gives only the average energy loss by ionization. There are fluctuations. Depending of the amount of matter in Δx the distribution of ΔE can be strongly asymmetric ($\rightarrow$ the Landau tail).

The large fluctuations are due to a small number of collisions with large energy transfers.

The figure shows the energy loss distribution of 3 GeV electrons in 5 mm of an Ar/CH4 gas mixture [Affh98].



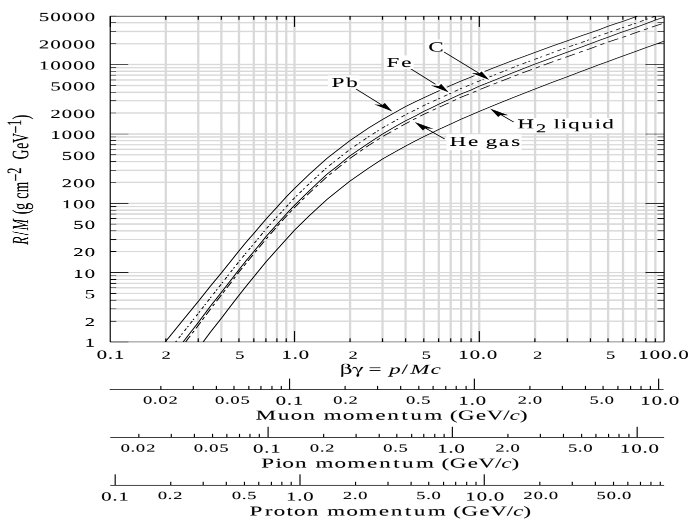
Only when $(dE/dx).\Delta x \gg T_{max}$ the distribution becomes nearly Gaussian:

$$
\begin{array}{lll}
((dE/dx).\Delta x)/T_{max} & \leq 0.01 & \text{Landau distribution} \\
 & < 10 & \text{Vavilov distribution} \\
 & \geq 10 & \text{Gaussian limit}
\end{array}
$$

Energy-Range relation

Mean total (zigzag) pathlength of an ionizing particle of kinetic energy T:

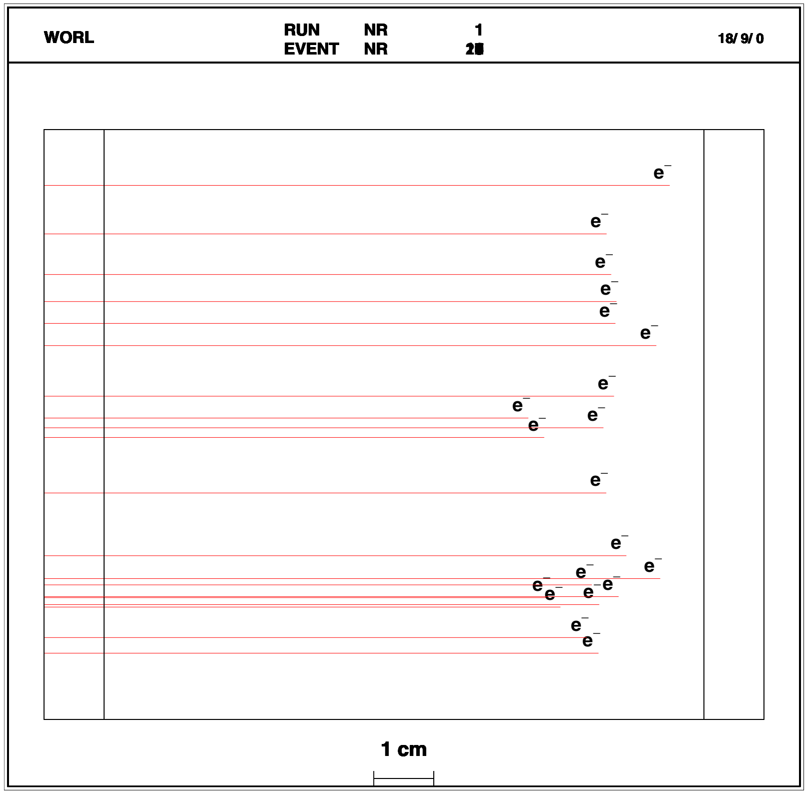
$$R(T) = \int_{\epsilon=Tlow=0}^{\epsilon=T} \frac{1}{(d\epsilon/dx)}\, d\epsilon$$

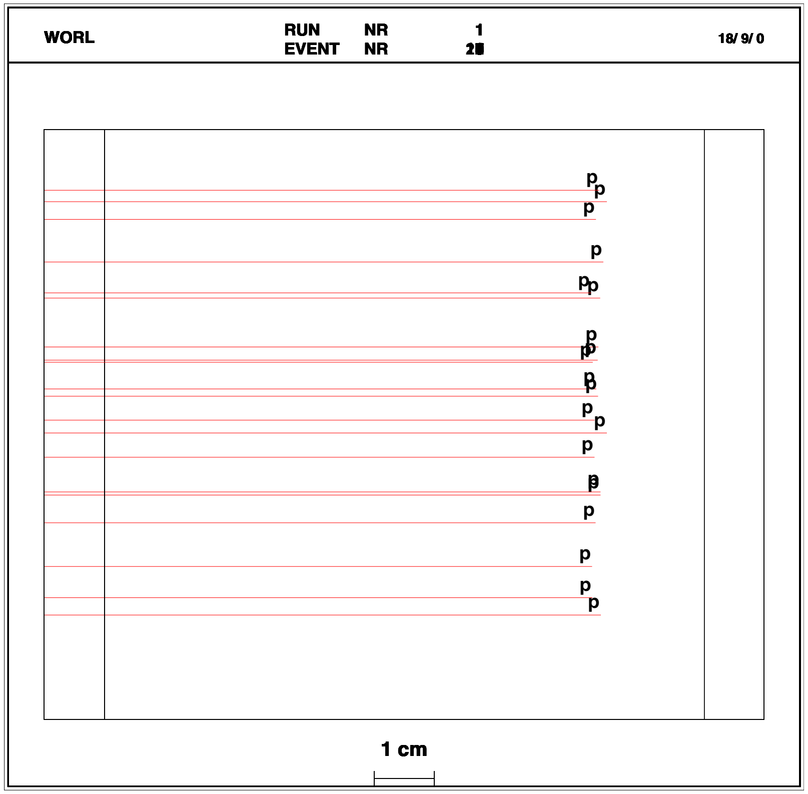


Because of multiple Coulomb scatterings, the mean range, defined as the straight-line thickness, is smaller than R(T).

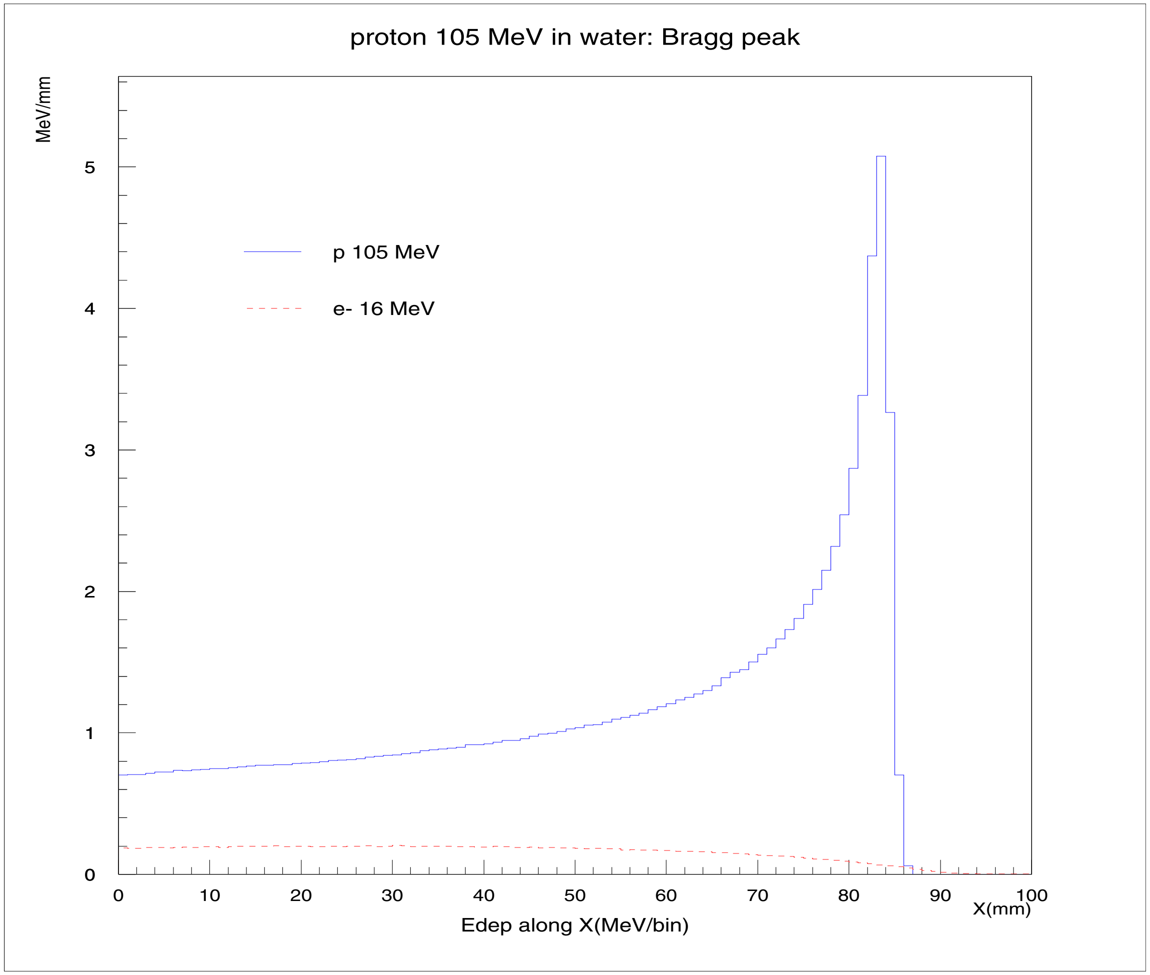
Fluctuations on ΔE lead to fluctuations on the actual range (straggling).

penetration of e^- (16 MeV) and proton (105 MeV) in 10 cm of water.





Bragg curve. More energy per unit length are deposit towards the end of trajectory rather at its beginning.



Energetic δ rays and truncated energy loss rate

One may wish to take into account separately the high-energy knock-on electrons produced above a given threshold T_{cut} (miss detection, explicit simulation ...).

The differential cross-section for producing an electron of kinetic energy T, with $I \ll T_{cut} \leq T \leq T_{max}$, can be written:

$$\frac{d\sigma}{dT} = 2\pi r_e^2 mc^2 Z \frac{z_p^2}{\beta^2} \frac{1}{T^2} \left[1 - \beta^2 \frac{T}{T_{max}} + \frac{T^2}{2E^2}\right]$$

(the last term only for particle of spin 1/2)

Then, the truncated total cross-section for emitting such electrons is:

$$\sigma(E, T_{cut} \le T \le T_{max}) = \int_{T=T_{cut}}^{T=T_{max}} \frac{d\sigma}{dT}\, dT$$

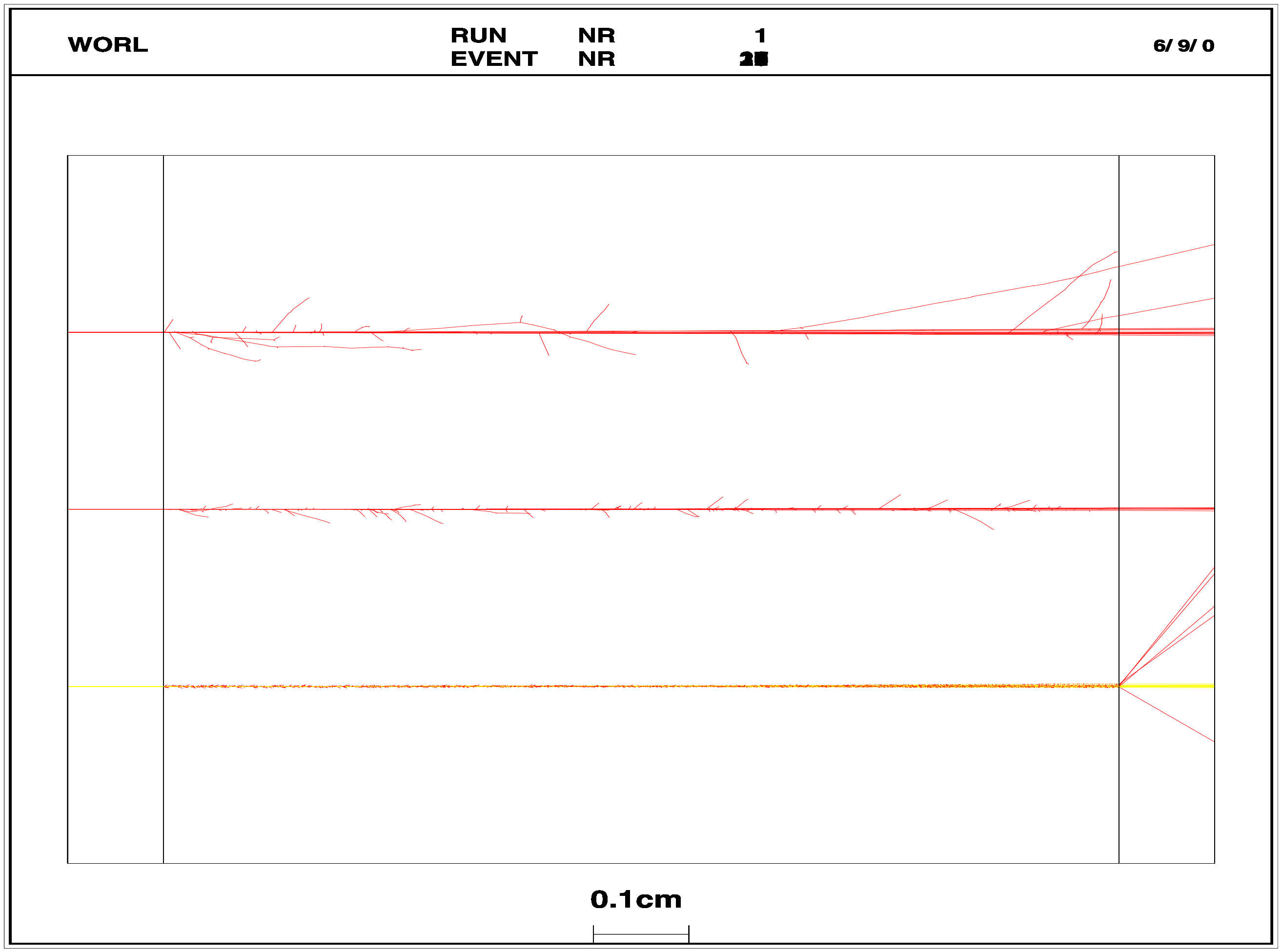
Those electrons must be excluded from the mean energy loss count. The truncated energy loss rate is:

$$-\frac{dE}{dx}\bigg]_{T<T_{cut}} = 2\pi r_e^2 mc^2 n_{el} \frac{z_p^2}{\beta^2} \times \left[\ln\left(\frac{2mc^2\beta^2\gamma^2 T_{cut}}{I^2}\right) - \beta^2\left(1 + \frac{T_{cut}}{T_{max}}\right) - \delta\right]$$

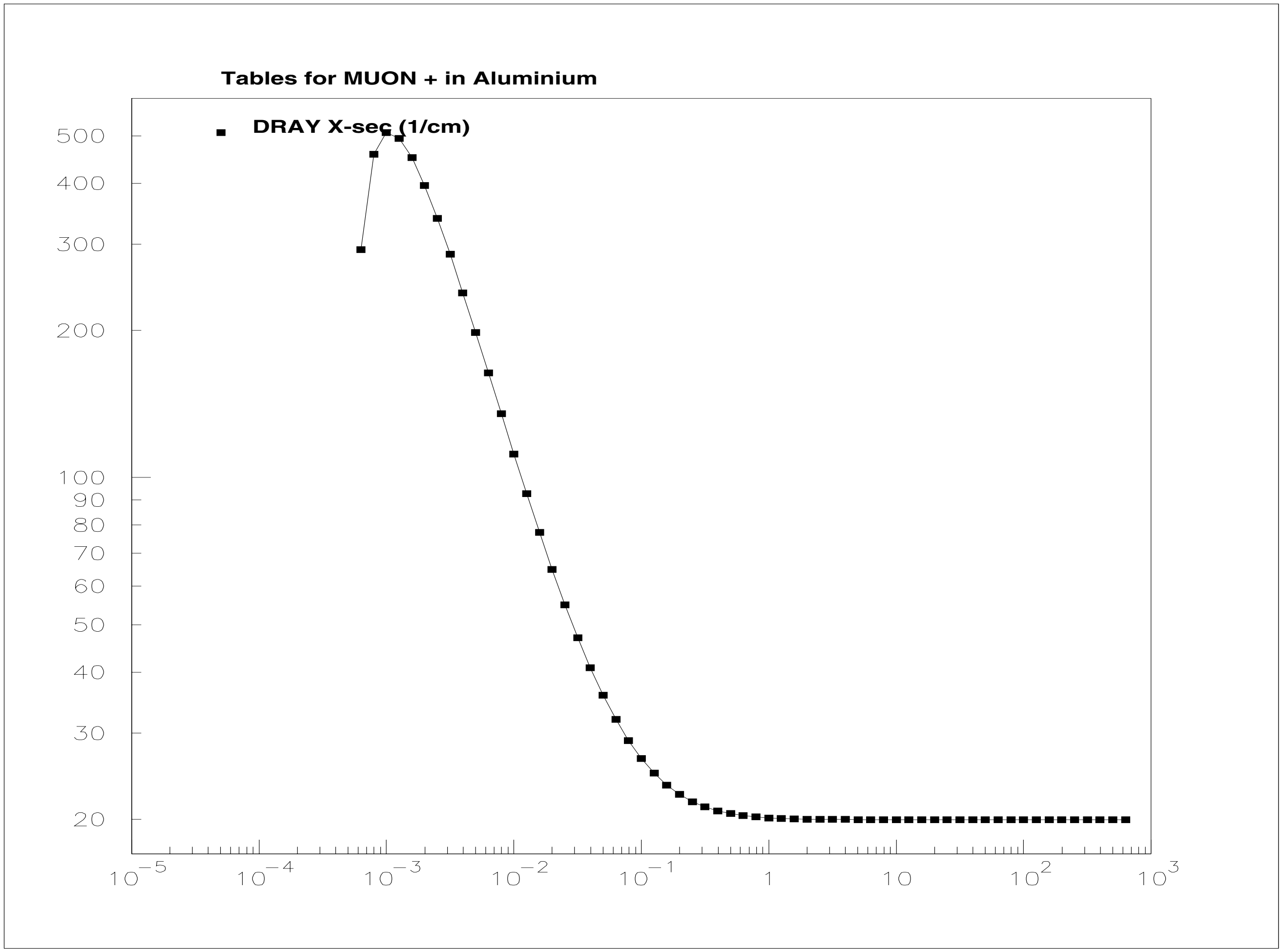
The fluctuations on the truncated energy loss are smaller, since the large energy transfers are excluded.

delta rays

200 MeV electrons, protons, alphas in 1 cm of Aluminium



muon : number of δ-rays per cm in Aluminium



muon kinetic energy (GeV)

Incident electrons and positrons

For incident $e^{-/+}$ the Bethe Bloch formula must be modified because of the mass and identity of particles (for e^-).

One use the Moller or Bhabha cross sections [Mess70] and the Berger-Seltzer dE/dx formula [ICRU84, Selt84].

truncated Berger-Seltzer dE/dx formula

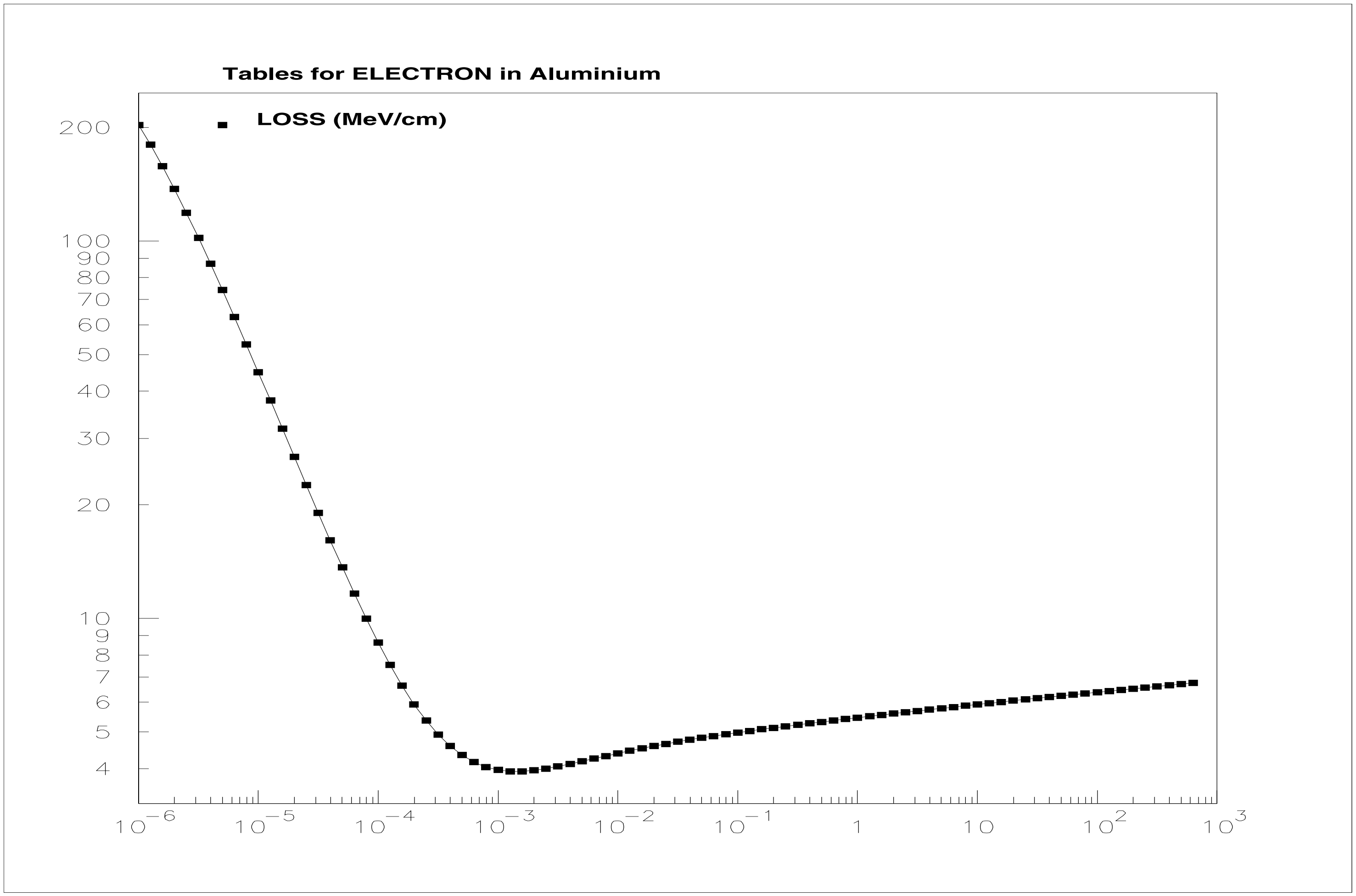
$$-\frac{dE}{dx}\Big]_{T<T_{cut}} = 2\pi r_e^2 mc^2 n_{el} \frac{1}{\beta^2} \times \left[\ln\left(\frac{2mc^2(\gamma+1)}{I^2}\right) + F^{\pm}(\gamma-1, \tau_{up}) - \delta\right]$$

where

T_{cut}	energy cut for $\delta - ray$
τ_c	T_{cut}/mc^2
τ_{max}	maximum energy transfer: $\gamma - 1$ for e^+, $(\gamma - 1)/2$ for e^-
τ_{up}	$\min(\tau_c, \tau_{max})$

The functions $F^{\pm}$ are given in [Selt84].

de/dx due to ionization (Berger-Seltzer formula)



electron kinetic energy (GeV)

differential cross section

For the electron-electron (Möller) scattering we have:

$$\frac{d\sigma}{d\epsilon} = \frac{2\pi r_e^2 Z}{\beta^2(\gamma-1)} \times \left[\frac{(\gamma-1)^2}{\gamma^2} + \frac{1}{\epsilon}\left(\frac{1}{\epsilon} - \frac{2\gamma-1}{\gamma^2}\right) + \frac{1}{1-\epsilon}\left(\frac{1}{1-\epsilon} - \frac{2\gamma-1}{\gamma^2}\right)\right]$$

and for the positron-electron (Bhabha) scattering:

$$\frac{d\sigma}{d\epsilon} = \frac{2\pi r_e^2 Z}{(\gamma-1)}\left[\frac{1}{\beta^2\epsilon^2} - \frac{B_1}{\epsilon} + B_2 - B_3\epsilon + B_4\epsilon^2\right]$$

where

$$
\begin{aligned}
E &= \text{energy of the incident particle} \\
\gamma &= E/mc^2 & y &= 1/(\gamma+1) \\
B_1 &= 2 - y^2 & B_2 &= (1-2y)(3+y^2) \\
B_3 &= (1-2y)^2 + (1-2y)^3 & B_4 &= (1-2y)^3 \\
\epsilon &= T/(E - mc^2)
\end{aligned}
$$

with T the kinematic energy of the scattered electron.

The kinematical limits for the variable ϵ are:

$$\epsilon_0 = \frac{T_{cut}}{E - mc^2} \leq \epsilon \leq \frac{1}{2} \quad \text{for } e^-e^- \qquad \epsilon_0 \leq \epsilon \leq 1 \quad \text{for } e^+e^-$$

References

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[Selt84] S.M. Seltzer and M.J. Berger, Int J. of Applied Rad. 35, 665 (1984)

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[ICRU93] ICRU Report No. 49 (1993)

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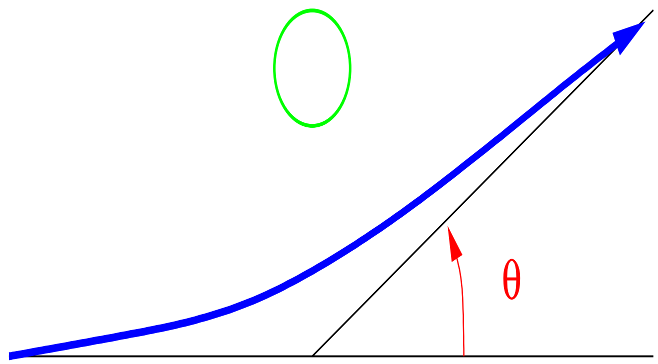
Multiple Coulomb scattering

Single Coulomb scattering

Single Coulomb deflection of a charged particle by a fixed nuclear target.

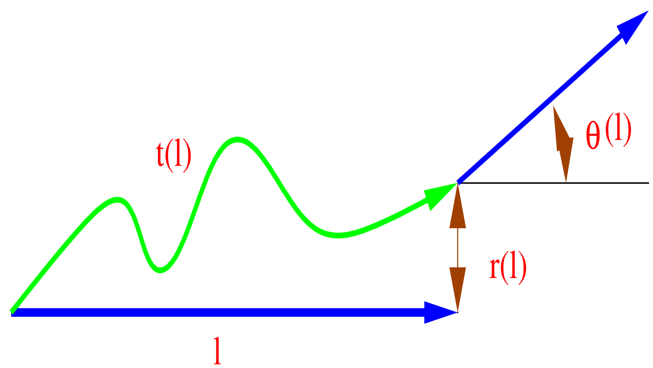


The cross section is given by the Rutherford formula

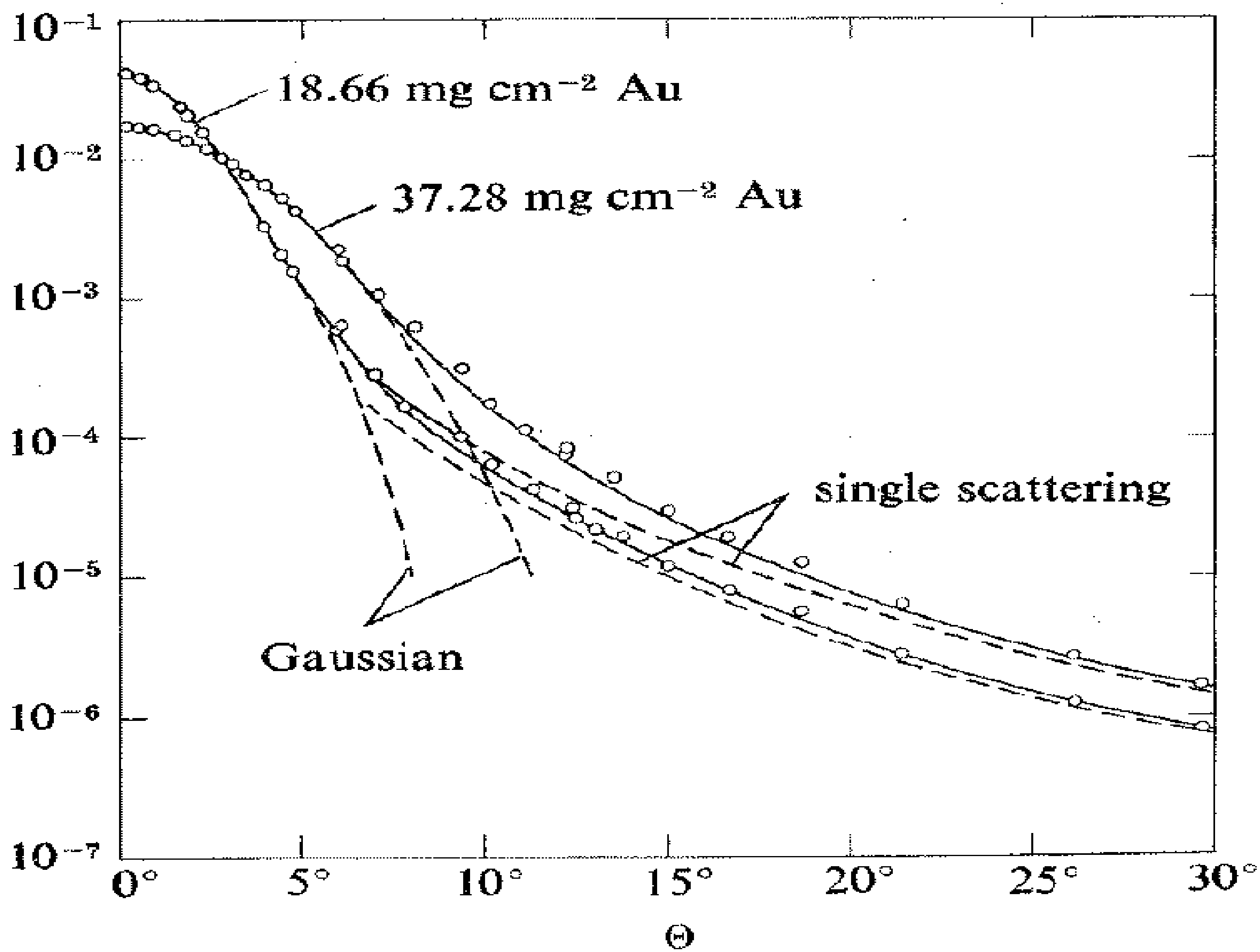
$$\frac{d\sigma}{d\Omega} = \frac{r_e^2 z_p^2 Z^2}{4} \left(\frac{mc}{\beta p}\right) \frac{1}{\sin^4 \theta/2}$$

Multiple Coulomb scattering

Charged particles traversing a finite thickness of matter suffer repeated elastic Coulomb scattering. The cumulative effect of these small angle scatterings is a net deflection from the original particle direction.



If the number of individual collisions is enough (> 20) the multiple Coulomb scattering angular distribution is gaussian at small angles and like Rutherford scattering at large angles.
The Molière theory reproduces rather well this distribution.
[Mol48, Bethe53]


18.66 mg cm^{-2} Au
37.28 mg cm^{-2} Au
single scattering
Gaussian
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0°
5°
10°
15°
20°
25°
30°
Θ

Gaussian approximation

The central part of the spatial angular distribution is approximately

$$P(\theta)\, d\Omega = \frac{1}{2\pi\theta_0^2}\ \exp\left[-\frac{\theta^2}{2\theta_0^2}\right]\ d\Omega$$

with

$$\theta_0 = \frac{13.6\ \mathrm{MeV}}{\beta\, pc} z \sqrt{\frac{l}{X_0}} \left[1 + 0.038 \ln\left(\frac{l}{X_0}\right)\right]$$

where l/X_0 is the thickness of the medium measured in radiation lengths X_0.

This formula of θ_0 is from a fit to Molière distribution. It is accurate to $\leq 10\%$ for $10^{-3} < l/X_0 < 10^2$

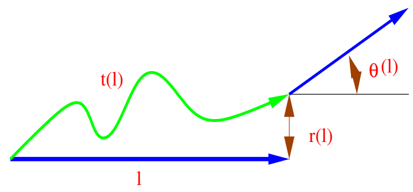
note: the appearance of X_0 in the formula is only for convenience.

Others formulas for θ_0 have been developed, starting from the Molière theory. [Lynch91]

related quantities

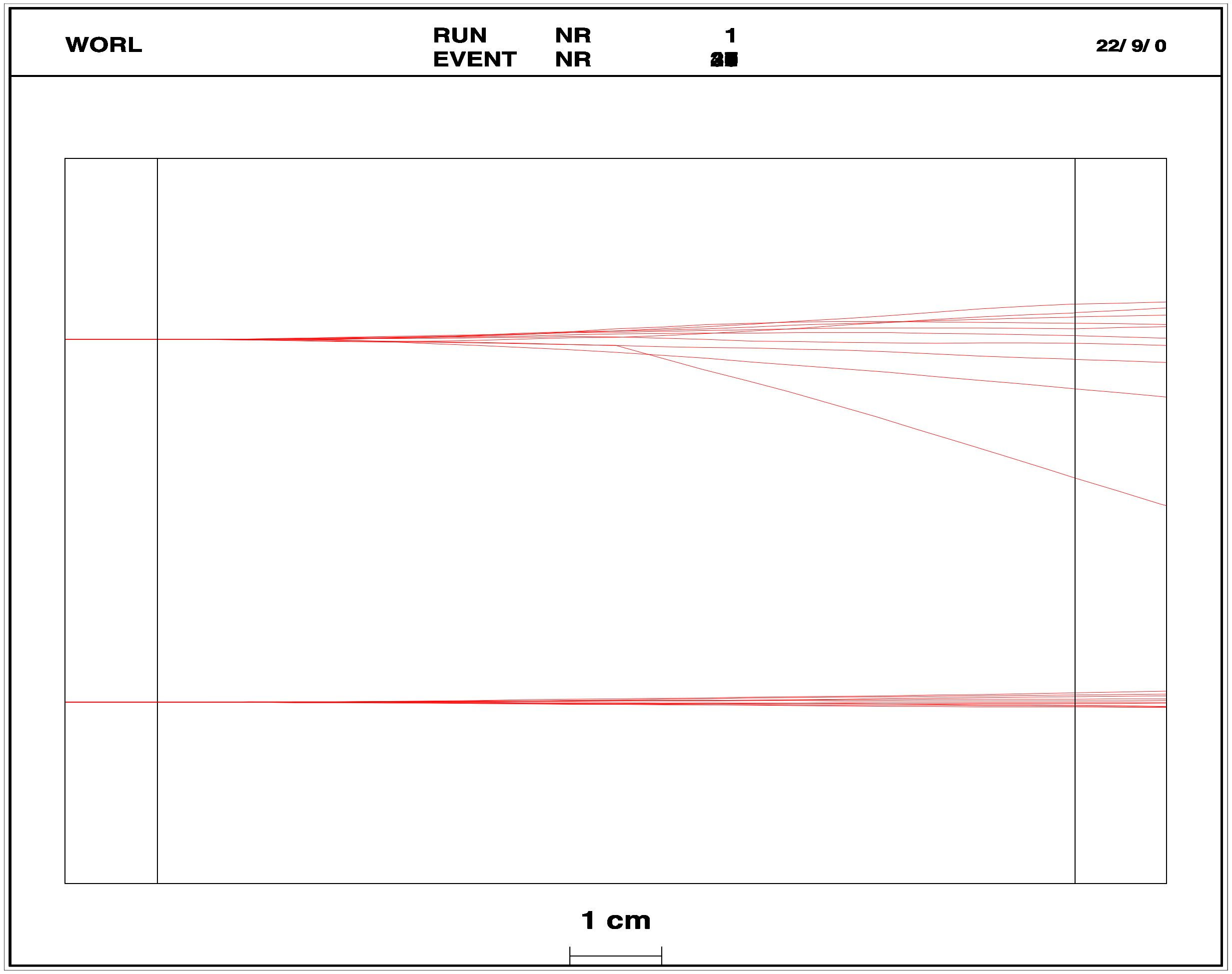
- lateral displacement $r(l)$
- true (or corrected) path length $t(l)$
- projected angular deflection $\theta_{proj}(l)$

they are correlated random variables, for instance needed in Monte Carlo simulation.



Energy dependence

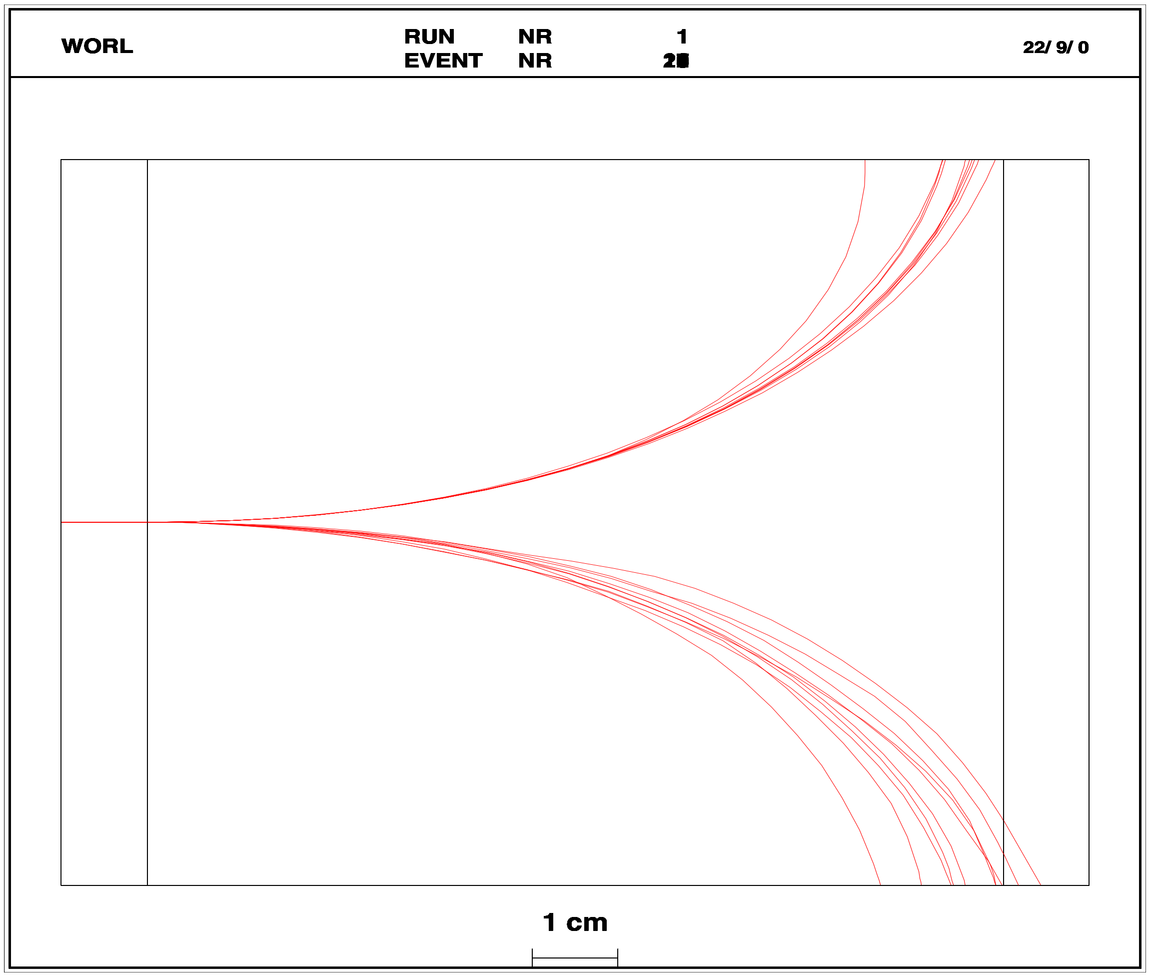
10 π^+ of 200 MeV and 1 GeV crossing 10 cm of Aluminium.



10 cm of Aluminium. Field 5 tesla.

top: 10 e^- (300 MeV): energy loss fluctuations only (no muls)

bottom: 10 e^+ (300 MeV): multiple scattering only (no eloss fluct)



Others models for simulation

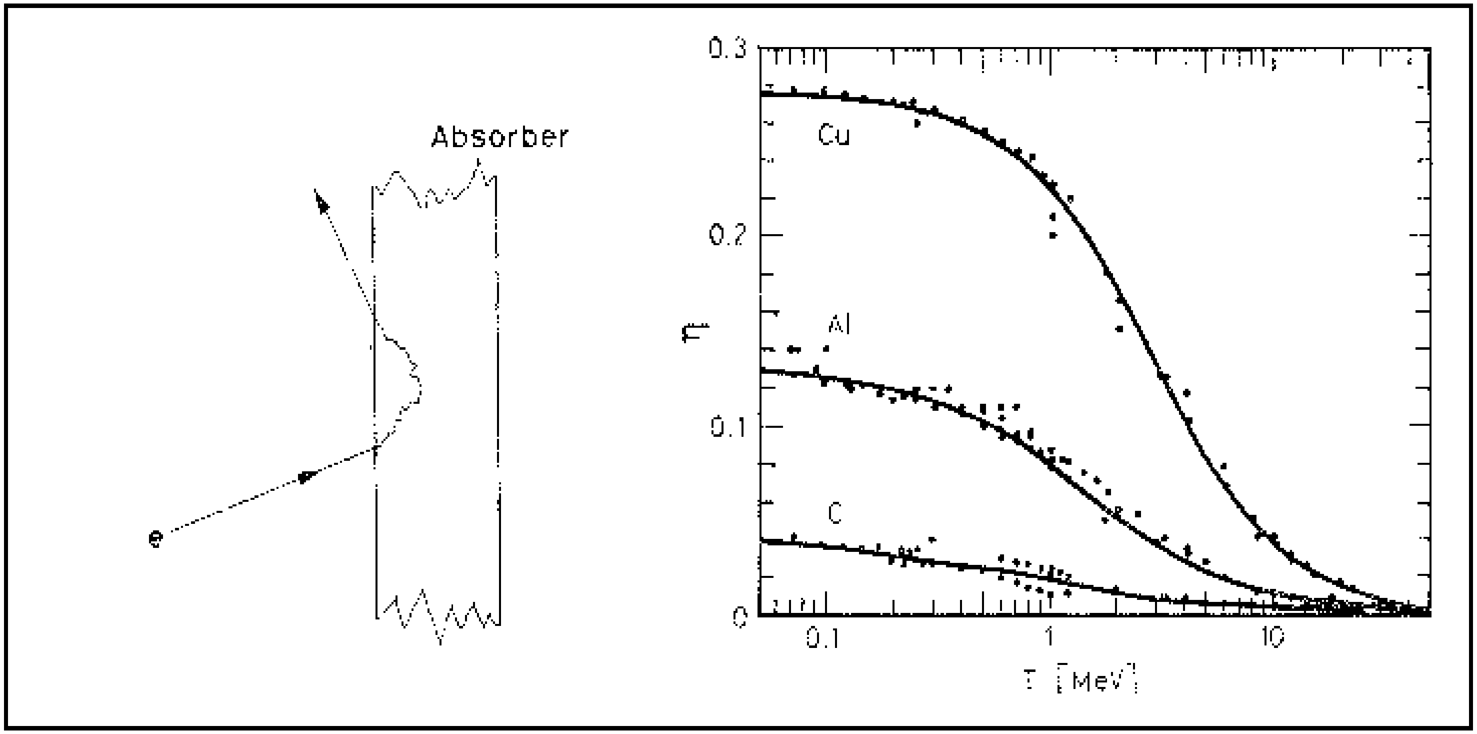
Several models of multiple Coulomb scattering simulation algorithms have been proposed, not necessarily based on the Molière theory. (See the references in [PDG00])
For instance :

- J.M. Fernandez-Verea et al. : a "mixed" (detailed + condensed) model. [Fer93]
- L.Urban : a condensed model based on Lewis theory.[Urb00]

backscattering of low energy electrons

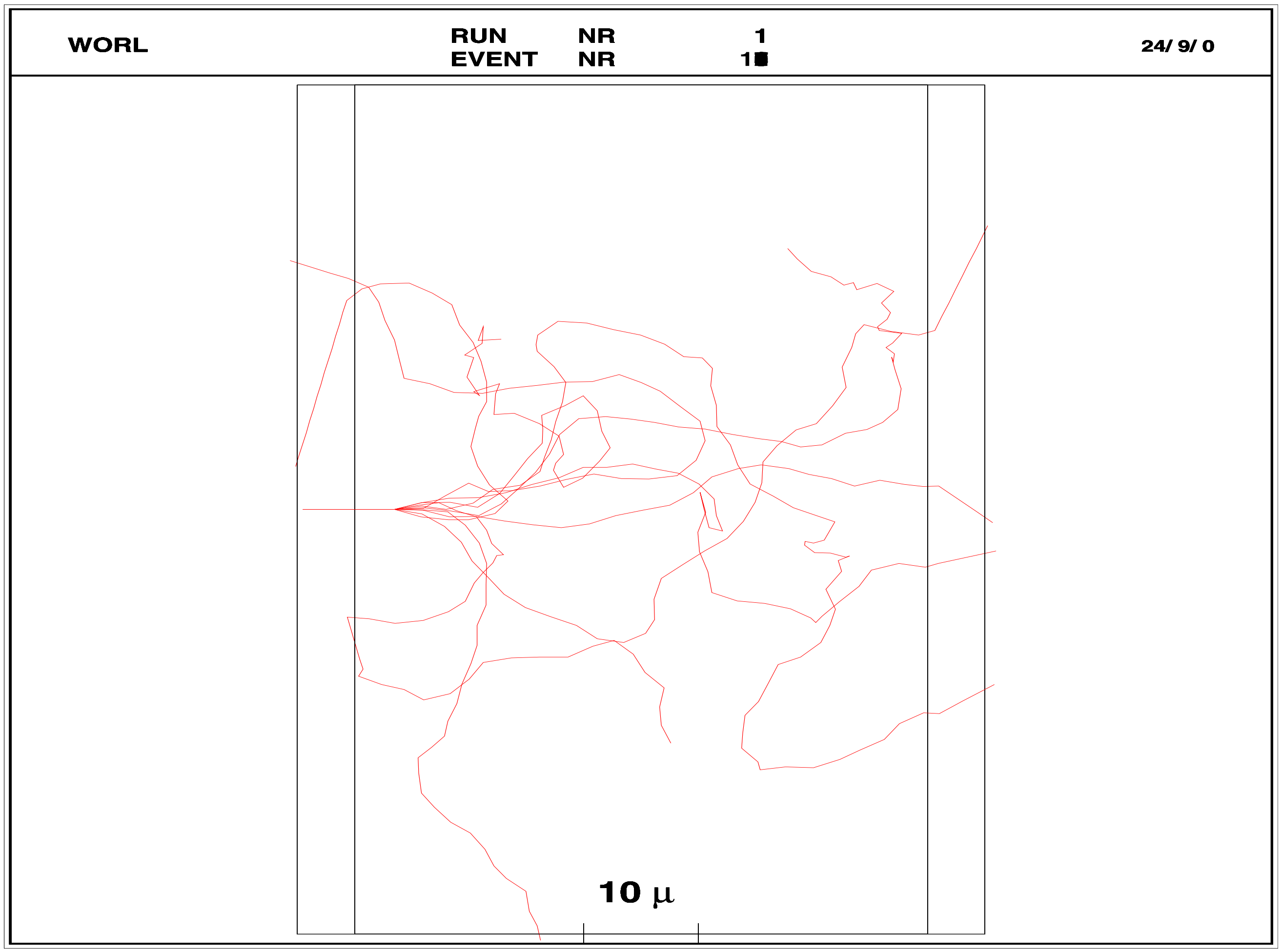
Because of its small mass, electron can have large deflection by scattering from nuclei.

For low energy incident electron beam, the ratio of electrons which are backscattered out of the detector may be important (*albedo*).



albedo : The incident beam is 10 electrons of 600 keV entering in 50 μm of Tungsten.

4 electrons are transmitted, 2 are backscattered.



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Čerenkov radiation

Čerenkov radiation

In a material with refractive index n, a charged particle emits photons if its velocity is greater than the local phase velocity of light.

The charged particle polarizes the atoms along its trajectory. These time dependent dipoles emit electromagnetic radiations.

If $v < c/n$ the dipole distribution is symmetric around the particle position, and the sum of all dipoles vanishes.

If $v > c/n$ the distribution is asymmetric and the total time dependent dipole is non nul, thus radiates.

mechanism of the Čerenkov radiation [Grupen96].

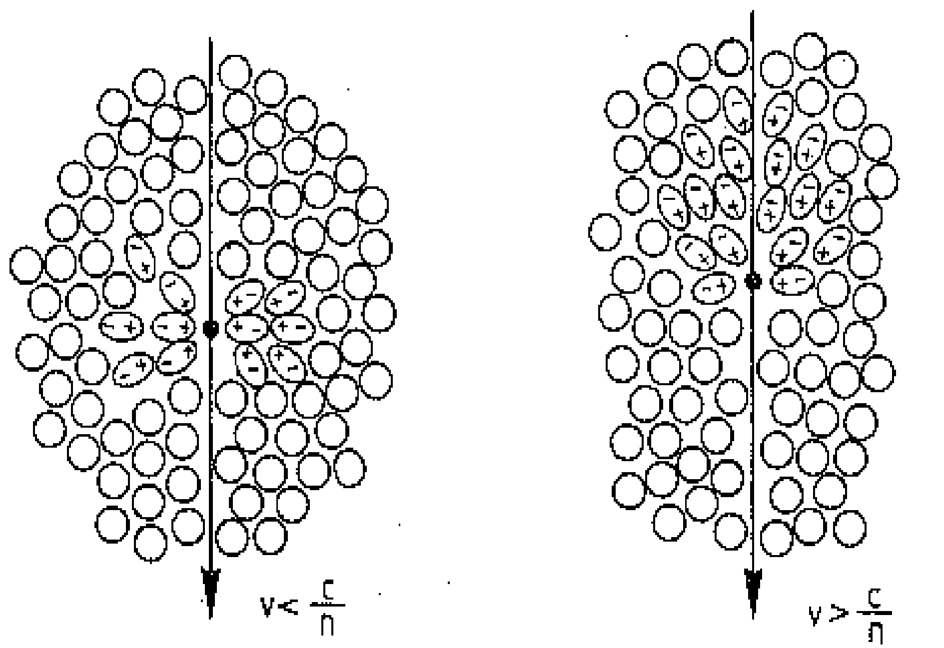
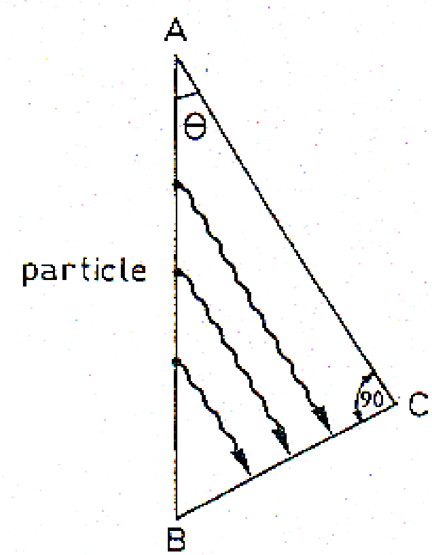


Fig. 6.7. Illustration of the Cherenkov effect [68].



The Huyghens construction gives immediately :

$$\cos\theta = \frac{1}{\beta n}$$

Thus :

$$\frac{1}{n} \leq \beta < 1 \Longrightarrow 0 \leq \theta < \arccos\frac{1}{n}$$

The number of photons produced per unit path length and per energy interval of the photons is

$$\frac{d^2N}{d\epsilon\, dx} = \frac{\alpha z^2}{\hbar c}\ \sin^2\theta = \frac{(\alpha z)^2}{r_e\ mc^2}\ \left[1 - \frac{1}{\beta^2\ n^2(\epsilon)}\right]$$

in which

$$\beta\ n(\epsilon) > 1$$

In the X-ray region $n(\epsilon) \approx 1$. There is no X-ray Čerenkov emission.

The energy lost by the charged particle due to Čerenkov emission is small compared to collision loss, even in gas :

$$\sim 10^{-1} \text{ to } 10^{-3} \text{ MeV/(g/cm}^2\text{)}$$

References

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http://pdg.lbl.gov/

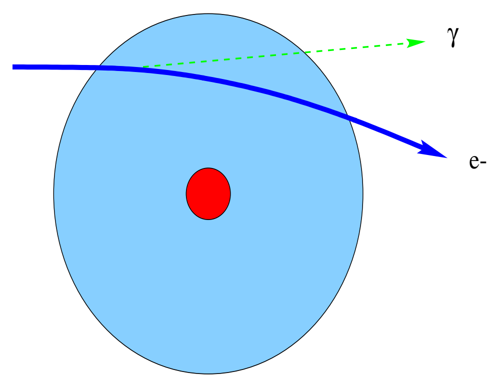
Bremsstrahlung

Bremsstrahlung

A fast moving charged particle is decelerated in the Coulomb field of atoms. A fraction of its kinetic energy is emitted in form of real photons.

The probability of this process is $\propto 1/M^2$ (M: masse of the particle) and $\propto Z^2$ (atomic number of the matter).

Above a few tens MeV, bremsstrahlung is the dominant process for e- and e+ in most materials. It becomes important for muons (and pions) at few hundred GeV.



differential cross section

The differential cross section is given by the Bethe-Heitler formula [Heitl57], corrected and extended for various effects:

- the screening of the field of the nucleus
- the contribution to the brems from the atomic electrons
- the correction to the Born approximation
- the polarisation of the matter (dielectric suppression)
- the so-called LPM suppression mechanism
- ...

See Seltzer and Berger for a synthesis of the theories [Sel85].

screening effect

Depending of the energy of the projectile, the Coulomb field of the nucleus can be more on less screened by the electron cloud.

A screening parameter measures the ratio of an 'impact parameter' of the projectile to the radius of an atom, for instance given by a Thomas-Fermi approximation or a Hartree-Fock calculation.

Then, screening functions are introduced in the Bethe-Heitler formula.

Qualitatively:

- at low energy $\rightarrow$ no screening effect
- at ultra relativistic electron energy $\rightarrow$ full screening effect

electron-electron bremsstrahlung

The projectile feels not only the Coulomb field of the nucleus (charge Ze), but also the fields of the atomic electrons (Z electrons of charge e).

The bremsstrahlung amplitude is roughly the same in both cases, except the charge.

Thus the electron cloud gives an additional contribution to the bremsstrahlung, proportional to Z (instead of Z^2).

Born approximation

The derivation of the Bethe-Heitler formula is based on perturbation theory, using plane waves for the electron. If the validity of the Born approximation:

$$\beta \gg \alpha Z$$

is violated for the initial and/or final velocity (low energy) the Coulomb waves would be used instead of the plane waves.

To correct for this, a Coulomb correction function is introduced in the Bethe-Heitler formula.

high energies regime : $E \gg mc^2/(\alpha Z^{1/3})$

Above few GeV the energy spectrum formula becomes simple :

$$\left.\frac{d\sigma}{dk}\right]_{Tsai} \approx 4\alpha\, r_e^2\, \frac{1}{k} \times \left\{ \left(\frac{4}{3} - \frac{4}{3}y + y^2\right) \left(Z^2 \left[L_{rad} - f(Z)\right] + Z L'_{rad}\right) \right\} \quad (1)$$

where

k	energy of the radiated photon ; $y = k/E$
α	fine structure constant
r_e	classical electron radius: $e^2/(4\pi\epsilon_0 mc^2)$
$L_{rad}(Z)$	$\ln(184.15/Z^{1/3})$ (for $Z \geq 5$)
$L'_{rad}(Z)$	$\ln(1194/Z^{2/3})$ (for $Z \geq 5$)
$f(Z)$	Coulomb correction function

limits of the energy spectrum

$k_{min} = 0$: In fact the infrared divergence is removed by the dielectric suppression mechanism, which is not shown in the formula 1.
For $k/E \leq 10^{-4}$: $d\sigma/dk$ becomes proportional to k [Antho96]

$k_{max} = E - mc^2 \approx E$: In this limit, the screening is incomplete, and the expression 1 of the cross section is not completely accurate.

mean rate of energy loss due to bremsstrahlung

$$-\frac{dE}{dx} = n_{at} \int_{k_{min}=0}^{k_{max}\approx E} k\,\frac{d\sigma}{dk}\,dk \tag{2}$$

n_{at} is the number of atoms per volume.

The integration immediately gives:

$$-\frac{dE}{dx} = \frac{E}{X_0} \tag{3}$$

with:

$$\frac{1}{X_0} \stackrel{def}{=} 4\alpha\, r_e^2\, n_{at} \left\{Z^2 \left[L_{rad} - f(Z)\right] + Z L'_{rad}\right\}$$

Radiation Length

The radiation length has been calculated by Y. Tsai [Tsai74]

$$\frac{1}{X_0} = 4\alpha\, r_e^2\, n_{at}\left\{Z^2\left[L_{rad} - f(Z)\right] + Z L'_{rad}\right\}$$

where

α	fine structure constant
r_e	classical electron radius
n_{at}	number of atoms per volume: $\mathcal{N}_{av}\rho/A$
$L_{rad}(Z)$	$\ln(184.15/Z^{1/3})$ (for $Z \geq 5$)
$L'_{rad}(Z)$	$\ln(1194/Z^{2/3})$ (for $Z \geq 5$)
$f(Z)$	Coulomb correction function

$$f(Z) = a^2[(1+a^2)^{-1} + 0.20206 - 0.0369a^2 + 0.0083a^4 - 0.002a^6 \cdots]$$

with $a = \alpha Z$

main conclusion: The relation 3 shows that the average energy loss per unit path length due to the bremsstrahlung increases linearly with the initial energy of the projectile.

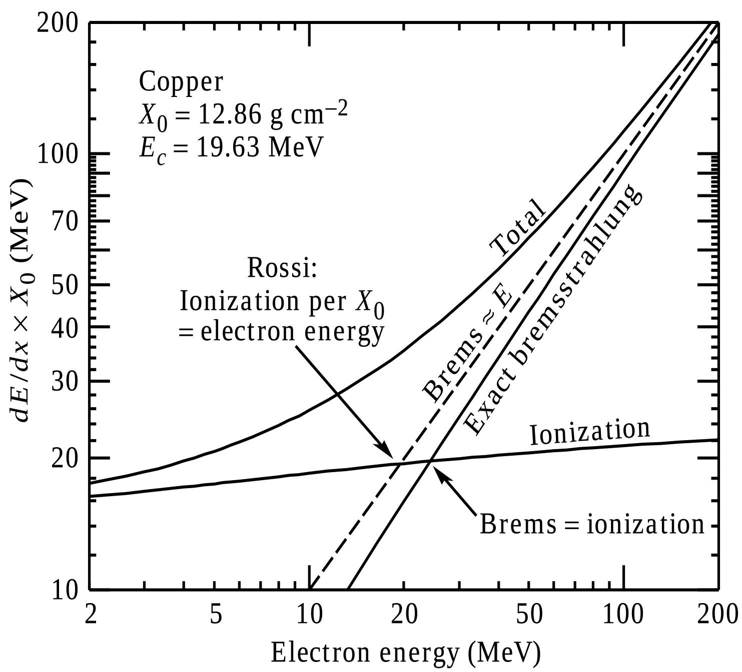
equivalent:

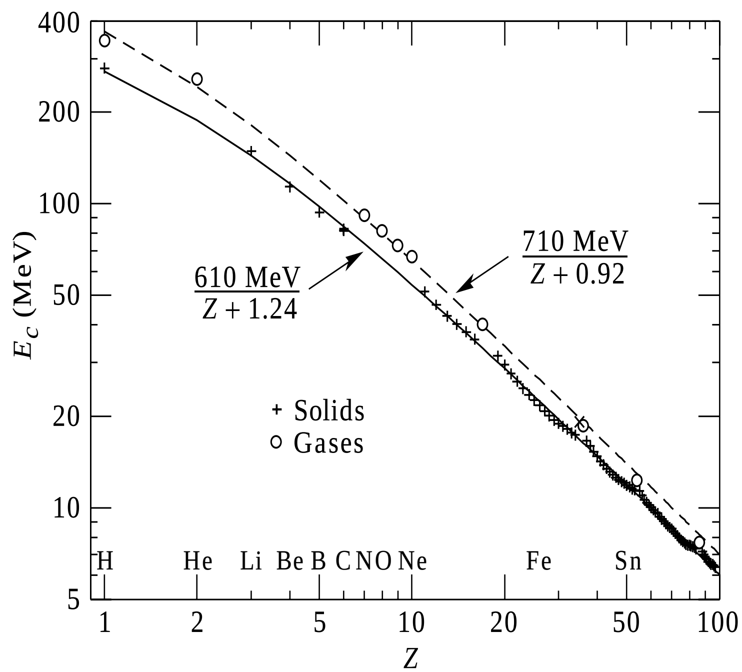
$$E(x) = E(0)\exp\left(-\frac{x}{X_0}\right)$$

This is the exponential attenuation of the energy of the projectile by radiation losses.

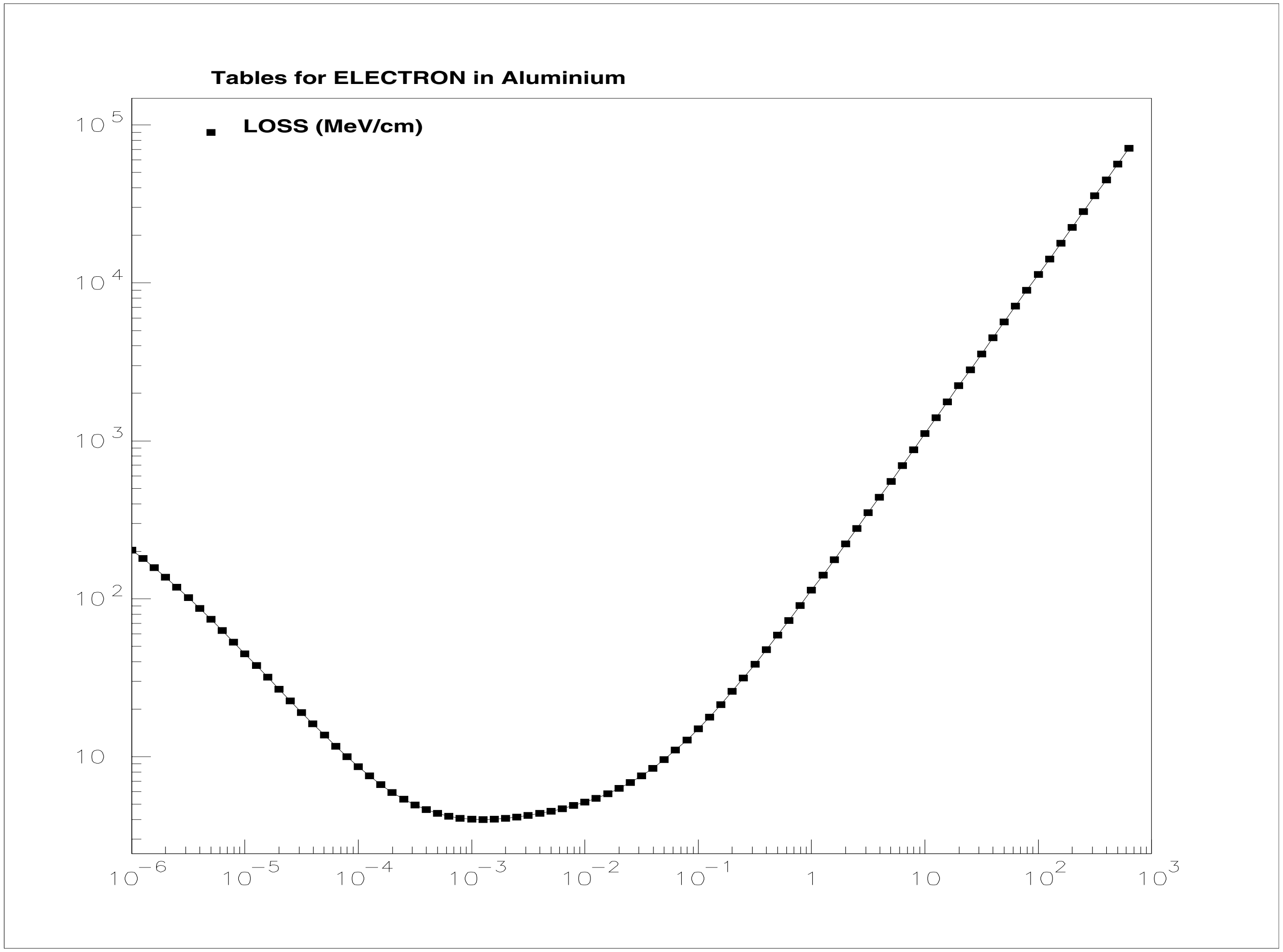
critical energy

The total mean rate of energy loss is the sum of ionization and bremsstrahlung.
The *critical energy* E_c is the energy at which the two rates are equal. Above E_c the total energy loss rate is dominated by bremsstrahlung.





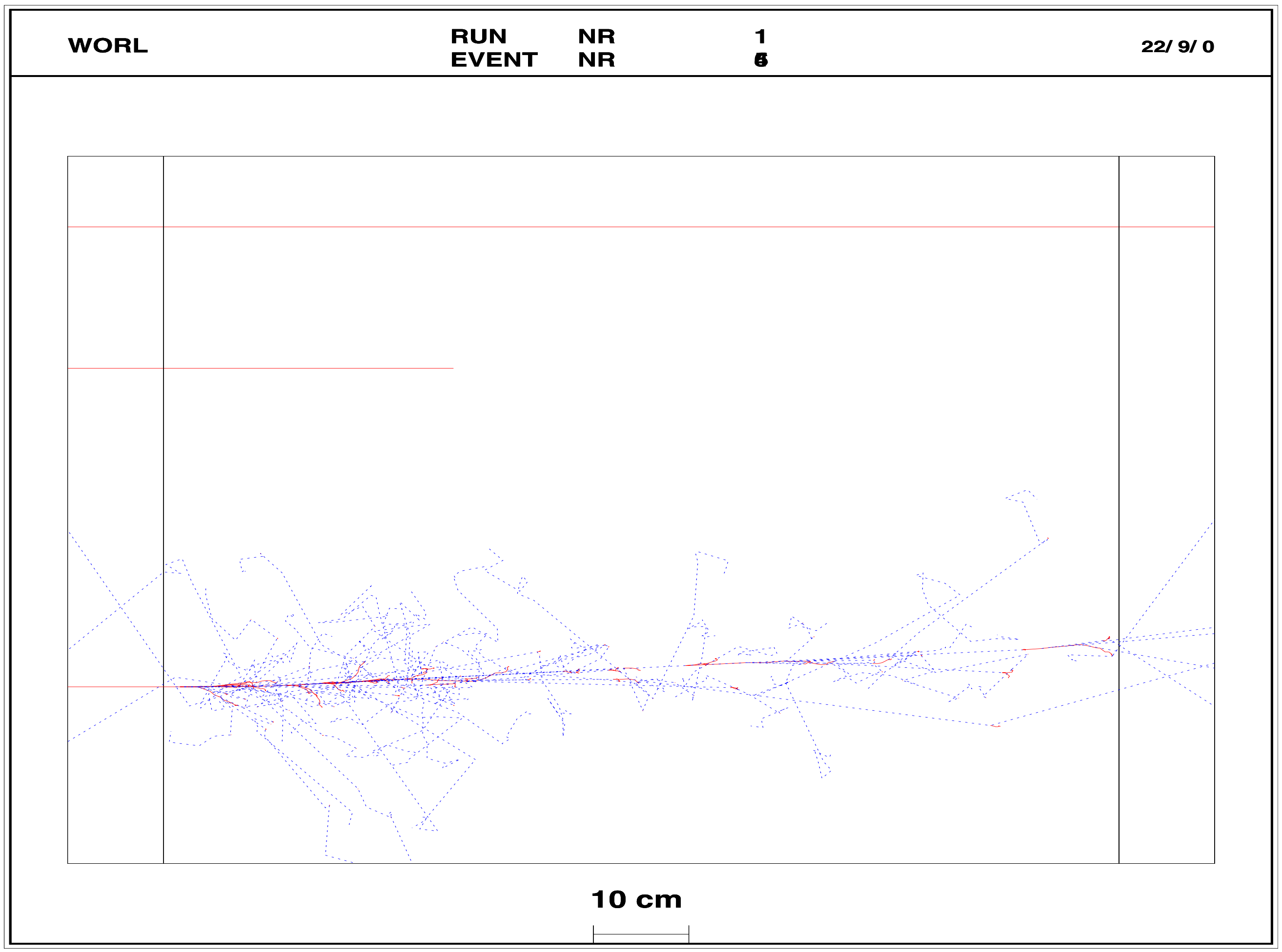
dE/dx for electrons



electron kinetic energy (GeV)

e^- 1 GeV in 1 meter of Aluminium.

Brems counted as continuous energy loss versus cascade development



fluctuations : Unlike the ionization loss which is quasicontinuous along the path length, almost all the energy can be emitted in one or two photons. Thus, the fluctuations on energy loss by bremstrahlung are large.

Energetic photons and truncated energy loss rate

One may wish to take into account separately the high-energy photons emitted above a given threshold k_{cut} (miss detection, explicit simulation ...).

Those photons must be excluded from the mean energy loss count.

$$-\frac{dE}{dx}\bigg]_{k<k_{cut}} = n_{at} \int_{k_{min}=0}^{k_{cut}} k \, \frac{d\sigma}{dk} \, dk \tag{4}$$

n_{at} is the number of atoms per volume.

Then, the truncated total cross-section for emitting 'hard' photons is:

$$\sigma(E, k_{cut} \le k \le k_{max}) = \int_{k_{cut}}^{k_{max} \approx E} \frac{d\sigma}{dk} \, dk \tag{5}$$

In the high energies regime, one can use the complete screened expression 1 of $d\sigma/dk$ to integrate 5. This gives:

$$\text{for } E \gg k_{cut}: \quad \sigma_{br}(E, k \geq k_{cut}) \approx \frac{4}{3}\,\frac{1}{n_{at}\,X_0} \ln\left[\frac{E}{k_{cut}}\right] \tag{6}$$

The bremsstrahlung total cross section increases as $\ln E$.

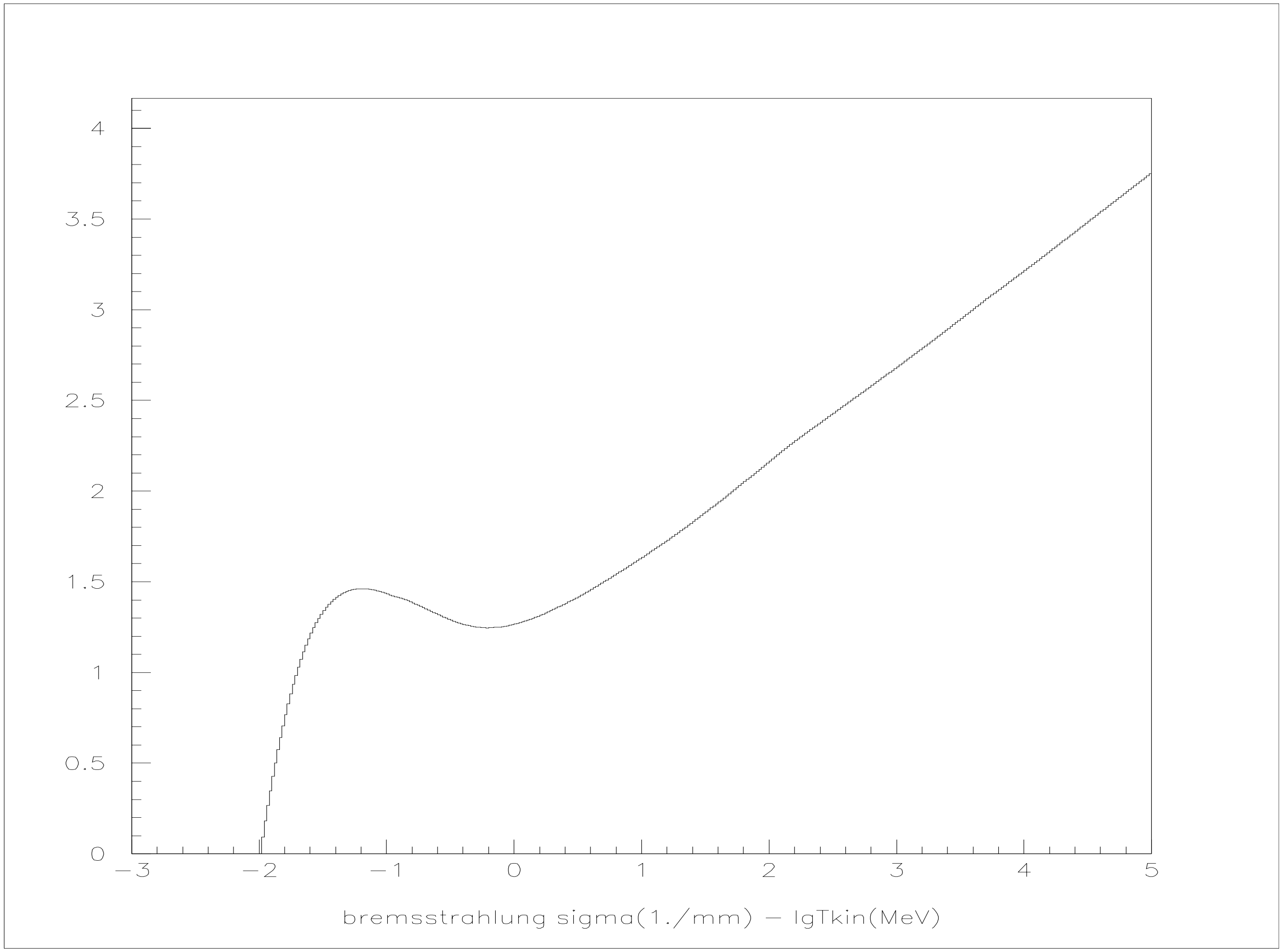
(in fact, taking into account the LPM effect leads to $\sigma_{br} \propto 1/\sqrt{E}$ when $E > E_{lpm}$)

the average angle of emission of the photon is

$$\theta = \frac{mc^2}{E}$$

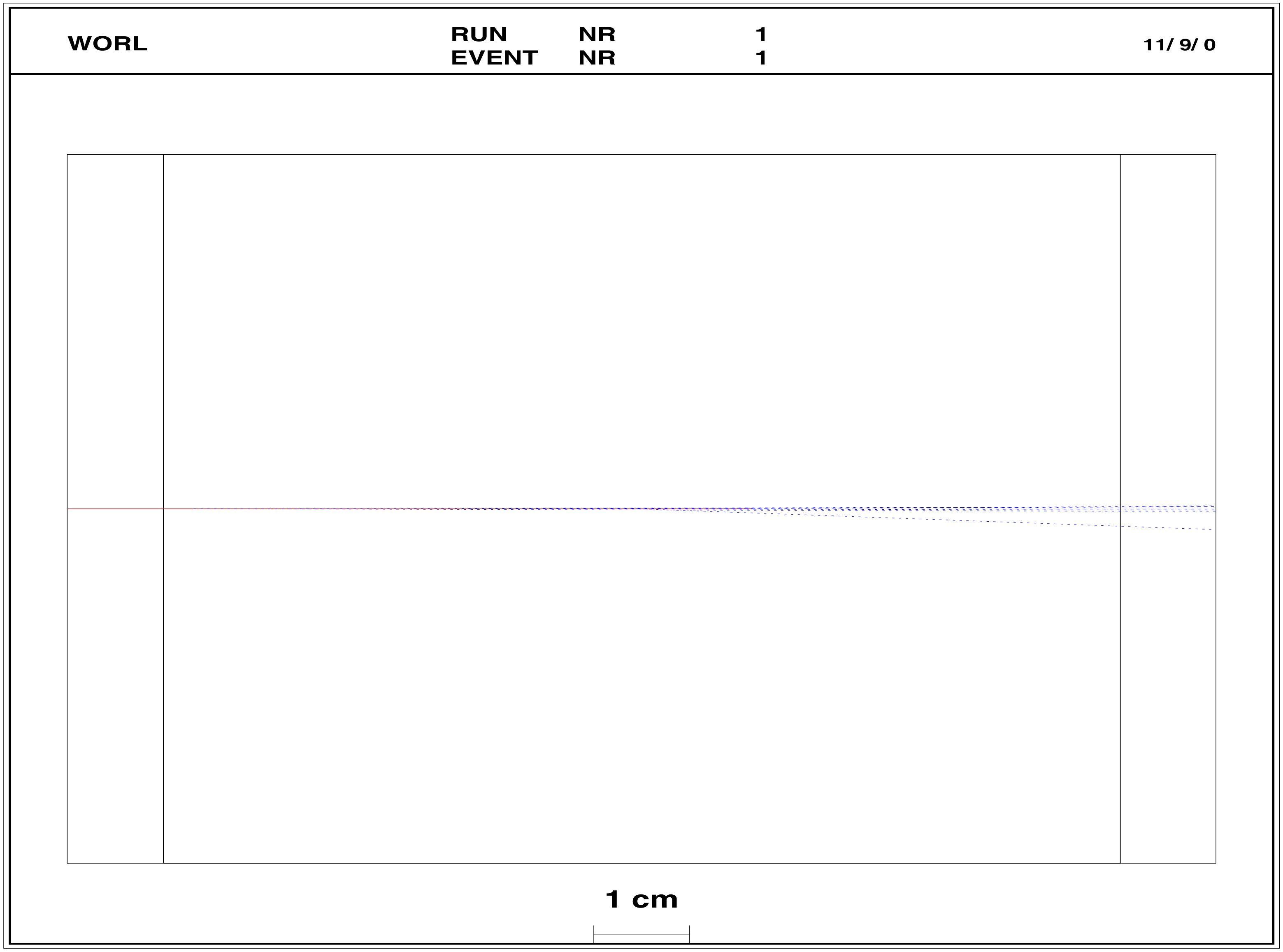
which is independent of the energy of the emitted photon.

number of interactions per mm in Lead (cut 10 keV)

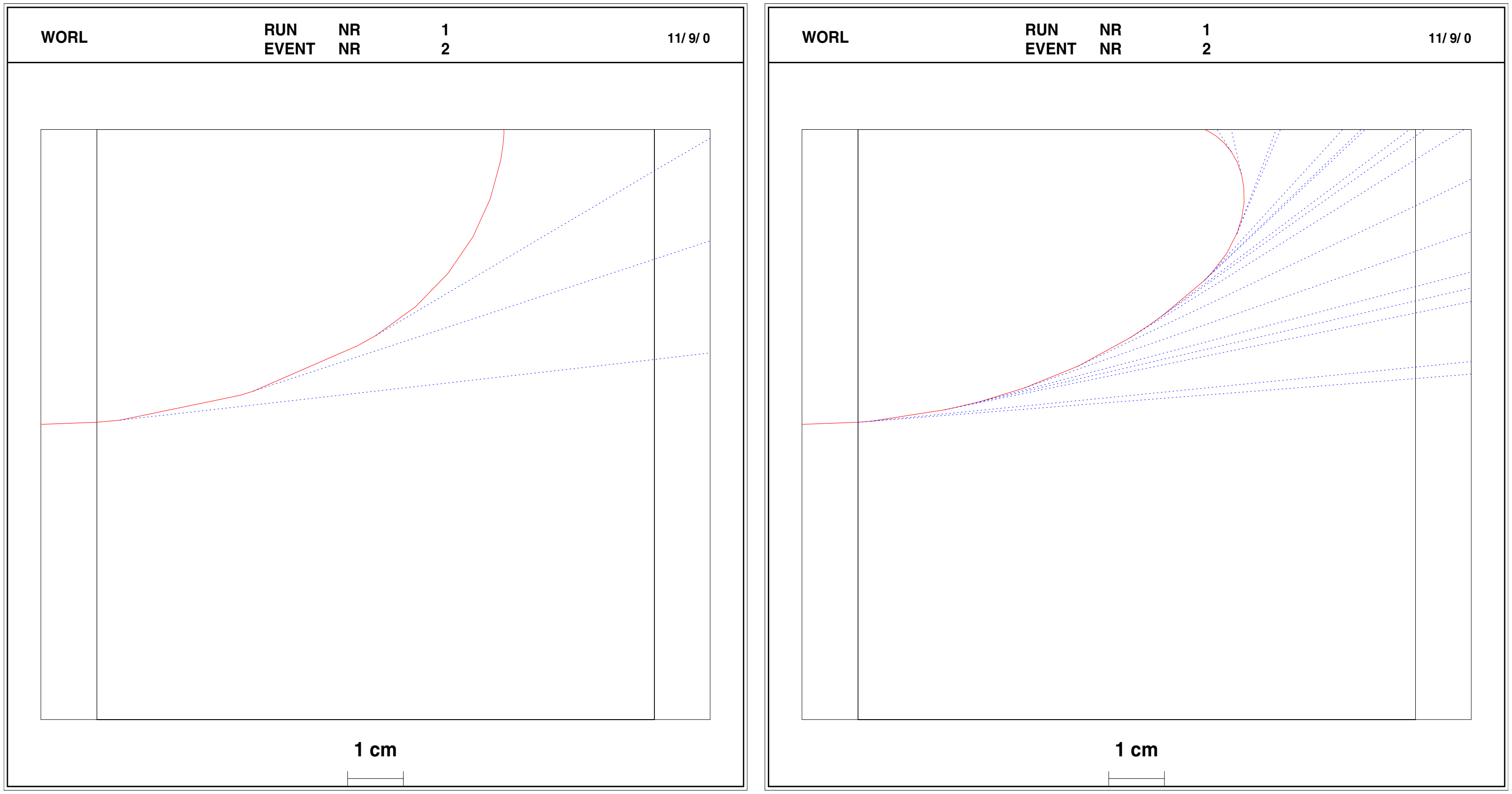


electron kinetic energy (ln(MeV))

e^- 200 MeV in 10 cm Aluminium



e^- 200 MeV in 10 cm Aluminium (cut: 1 MeV, 10 keV). Field 5 tesla



Bremsstrahlung by high energy muon

The bremsstrahlung cross section is proportional to $1/m^2$. Thus the critical energy for muons scales as $(m_\mu/m_e)^2$, i.e. near the TeV.

Bremsstrahlung dominates other muon interaction processes in the region of catastrophic collisions ($y \geq 0.1$), at "moderate" muon energies - above knock–on electron production kinematic limit.

At high energies ($E \geq 1$ TeV) this process contributes about 40% to average muon energy loss.

See [Keln97] for up-to-date review and formulae.

formation length ([Antho96])

In the bremsstrahlung process the longitudinal momentum transfer from the nucleus to the electron can be very small. For $E \gg mc^2$ and $E \gg k$:

$$q_{long} \sim \frac{k(mc^2)^2}{2E(E-k)} \sim \frac{k}{2\gamma^2}$$

Thus, the uncertainty principle requires that the emission take place over a comparatively long distance :

$$f_v \sim \frac{2\hbar c\gamma^2}{k} \tag{7}$$

f_v is called the formation length for bremsstrahlung in vacuum. It is the distance of coherence, or the distance required for the electron and photon to separate enough to be considered as separate particles. If anything happens to the electron or photon while traversing this distance, the emission can be disrupted.

dielectric suppression mechanism

The suppression is due to the photon interaction with the electron gas (Compton scattering) within the formation length.
The magnitude of the process can be evaluated using classical electromagnetism [Ter72].

A dielectric medium is characterized by his dielectric function:

$$\epsilon(k) = 1 - (\hbar\omega_p/k)^2 \qquad \text{with } \hbar\omega_p = \sqrt{4\pi n_{el} r_e^3}\, mc^2/\alpha$$

(n_{el} is the electron density of the medium $= Z n_{at}$.)

It can be shown that the formation length in this medium is reduced:

$$f_m \sim \frac{2\hbar c\gamma^2\, k}{k^2 + (\gamma\hbar\omega_p)^2} \tag{8}$$

The effect is factorized in the bremsstrahlung differential cross section:

$$\frac{d\sigma}{dk} = S_d(k) \left[\frac{d\sigma}{dk}\right]_{Tsai} \tag{9}$$

S_d is called the suppression function :

$$S_d(k) \stackrel{def}{=} \frac{f_m(k)}{f_v(k)} = \frac{k^2}{k^2 + (\gamma\hbar\omega_p)^2} \tag{10}$$

$S_d(k)$ is vanishing for $k^2 \ll (\gamma\hbar\omega_p)^2$ i.e. :

$$\frac{k}{E} \ll \frac{\hbar\omega_p}{mc^2} \quad (\sim 10^{-4},\ 10^{-5} \text{ in all materials}) \tag{11}$$

Then,

$$S_d(k) \approx \frac{k^2}{(\gamma\hbar\omega_p)^2} \tag{12}$$

which cancel the infrared divergence of the Bethe-Heitler formula.

Landau-Pomeranchuk-Migdal suppression mechanism

The electron can multiple scatter with the atoms of the medium while it is still in the formation zone. If the angle of multiple scattering, θ_{ms}, is greater than the typical emission angle of the emitted photon, $\theta_{br} = mc^2/E$, the emission is suppressed.

In the gaussian approximation : $\theta_{ms}^2 = \frac{2\pi}{\alpha}\frac{1}{\gamma^2}\frac{f_v(k)}{X_0}$ where f_v is the formation length in vacuum, defined in equation 7.

Writing $\theta_{ms}^2 > \theta_{br}^2$ show that suppression becomes signifiant for photon energies below a certain value, given by

$$\frac{k}{E} < \frac{E}{E_{lpm}} \tag{13}$$

E_{lpm} is a characteristic energy of the effect :

$$E_{lpm} = \frac{\alpha}{4\pi}\frac{(mc^2)^2}{\hbar c}\, X_0 \qquad \sim (7.7\; TeV/cm) \times X_0\; (cm) \tag{14}$$

The multiple scattering increases q_{long} by changing the direction and the momentum of the electron. One can show :

$$q_{long} = \left(\frac{k}{2\gamma^2}\right)\left[1 + \frac{(mc^2)^2}{2\hbar c\, E_{lpm}}\, f_m(k)\right]$$

where f_m is the formation length in the material.

On the other hand the uncertainty principle says : $q_{long} = \hbar c / f_m$

These two equations can be solved in f_m:

$$f_m(k) \approx \frac{2\hbar c\gamma^2}{k}\sqrt{\frac{k\, E_{lpm}}{E^2}} \qquad \text{if } k\, E_{lpm} \ll E^2 \tag{15}$$

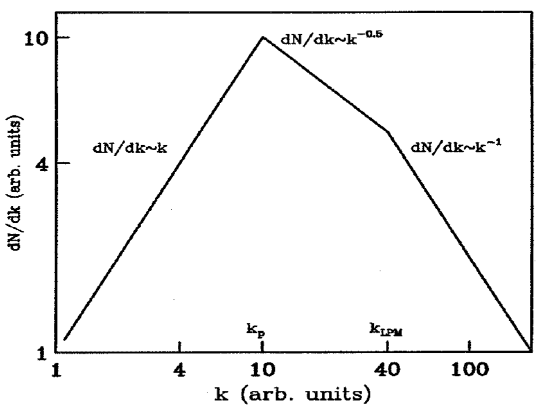
Hence, the suppression function S_{lpm} :

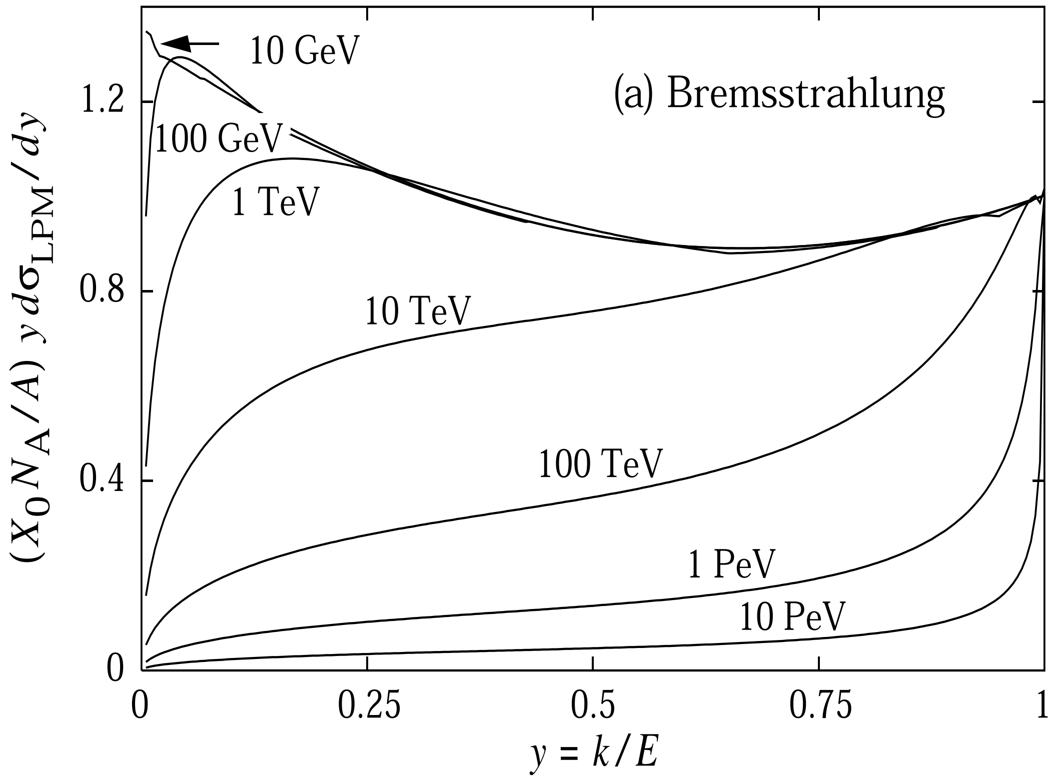
$$S_{lpm}(k) \stackrel{def}{=} \frac{f_m(k)}{f_v(k)} = \sqrt{\frac{k\, E_{lpm}}{E^2}} \equiv \sqrt{\frac{(k/E)}{(E/E_{lpm})}} \tag{16}$$

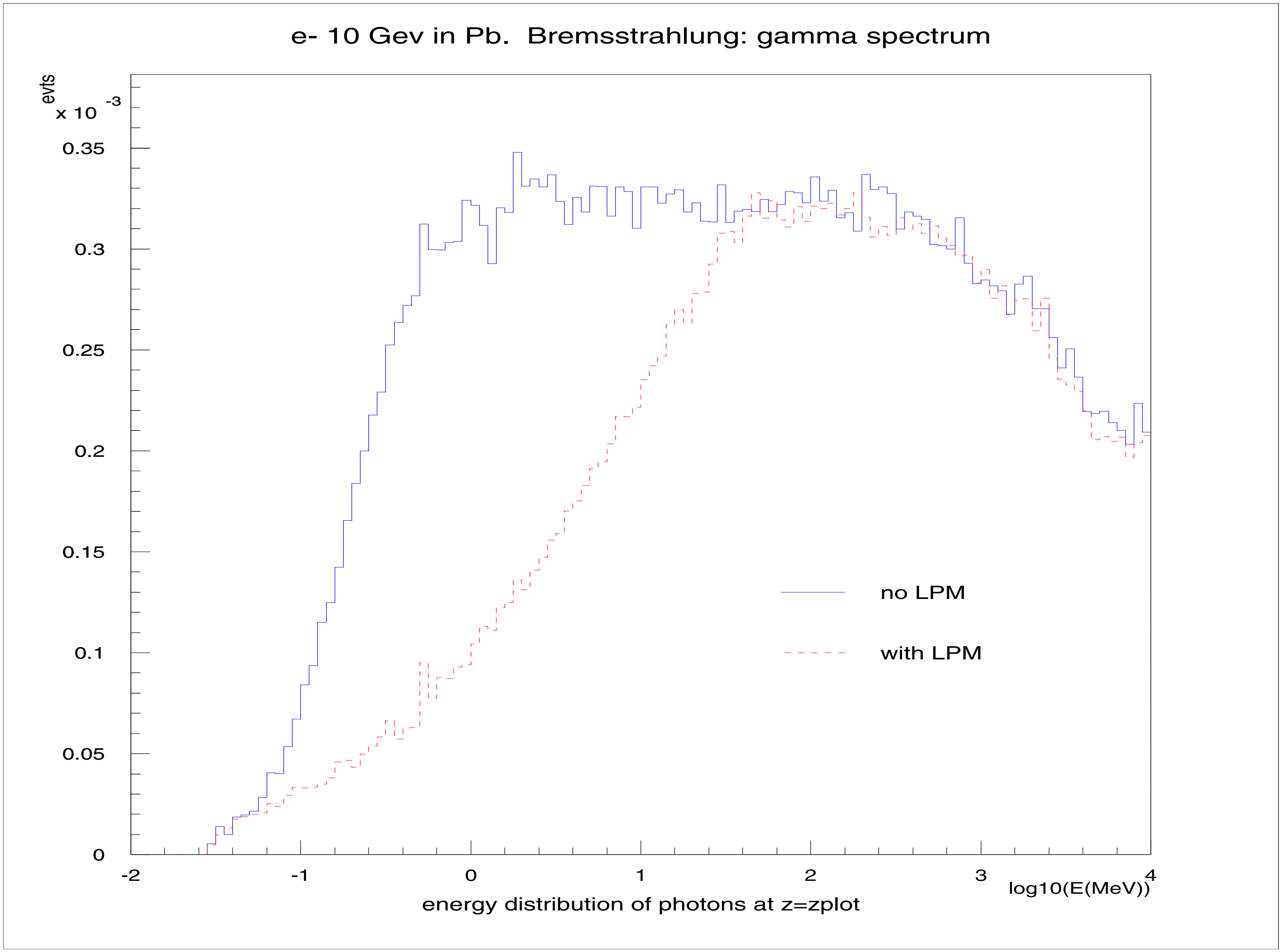
total suppression

The LPM and dielectric mechanism both reduce the effective formation length. The suppressions do not simply multiply. The total suppression follows [Gal64]

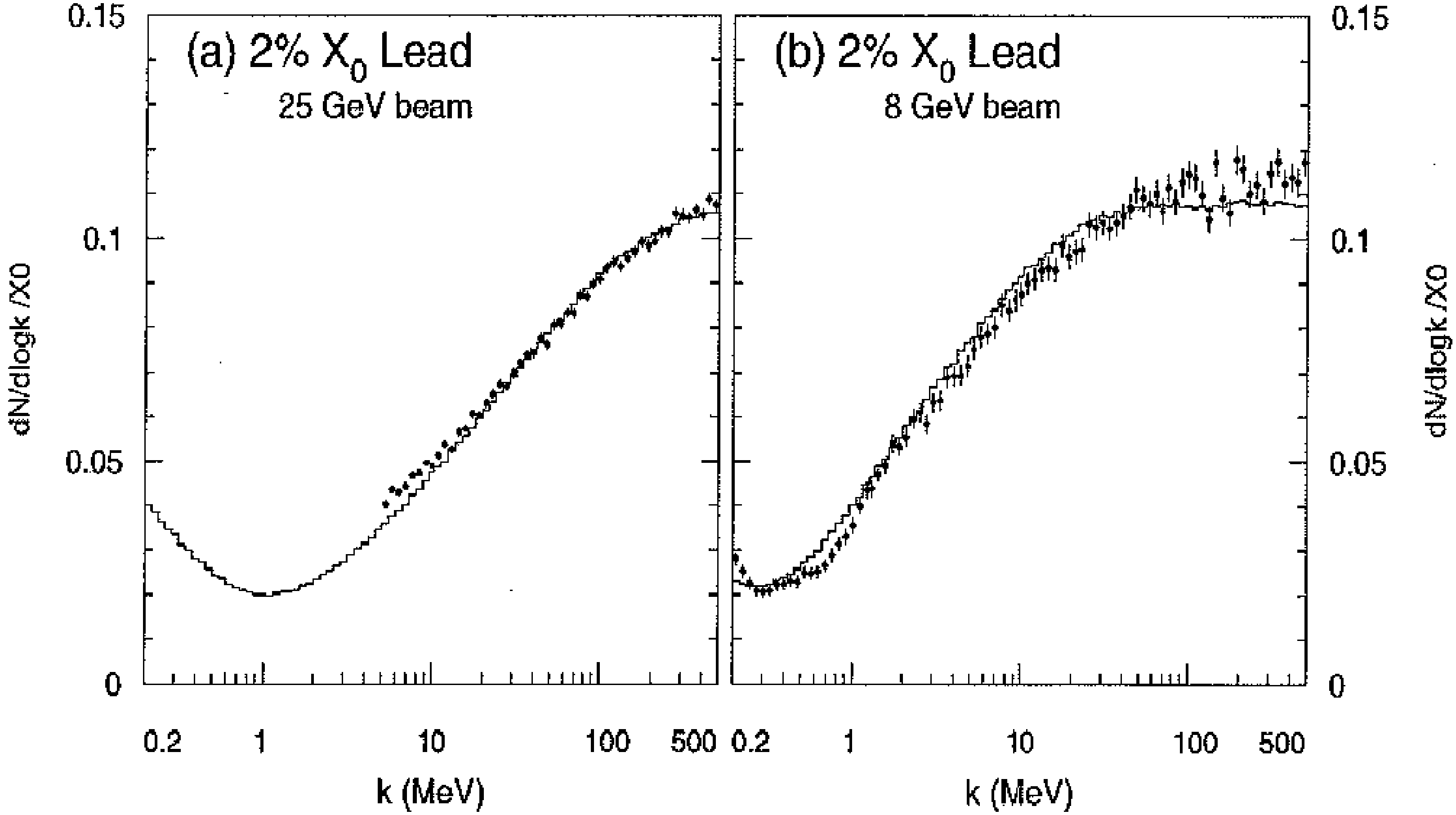
$$\frac{1}{S} = 1 + \frac{1}{S_d} + \frac{S}{S_{lpm}^2}$$


dN/dk (arb. units)
10
4
1
dN/dk~k
dN/dk~k^-0.5
dN/dk~k^-1
k_p
k_LPM
1
4
10
40
100
k (arb. units)


10 GeV
(a) Bremsstrahlung
100 GeV
1 TeV
10 TeV
100 TeV
1 PeV
10 PeV
1.2
0.8
0.4
0
0.25
0.5
0.75
1
$(X_0 N_A/A)\, y\, d\sigma_{LPM}/dy$
$y = k/E$


e- 10 Gev in Pb. Bremsstrahlung: gamma spectrum
evts
x 10 -3
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-2
-1
0
1
2
3
4
log10(E(MeV))
energy distribution of photons at z=zplot
no LPM
with LPM

E146 (SLAC) data [Antho96]



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(e^+, e^-) annihilation

(e^+, e^-) annihilation into two photons

$$e^+ + e^- \to \gamma + \gamma$$

(need two γ for momentum conservation, if the e^- is assumed to be free).

Theoretically, (e^+, e^-) annihilation is related to Compton scattering by crossing symmetry:

- incoming $e^+ \leftrightarrow$ outgoing e^-
- outgoing $\gamma \leftrightarrow$ incoming γ

total cross section per atom

$$\sigma(Z,E) = \frac{Z\pi r_e^2}{\gamma+1}\left[\frac{\gamma^2+4\gamma+1}{\gamma^2-1}\ln\left(\gamma+\sqrt{\gamma^2-1}\right) - \frac{\gamma+3}{\sqrt{\gamma^2-1}}\right]$$

$$\begin{aligned} E &= \text{total energy of the incident positron} \\ \gamma &= E/mc^2 \\ r_e &= \text{classical electron radius} \end{aligned}$$

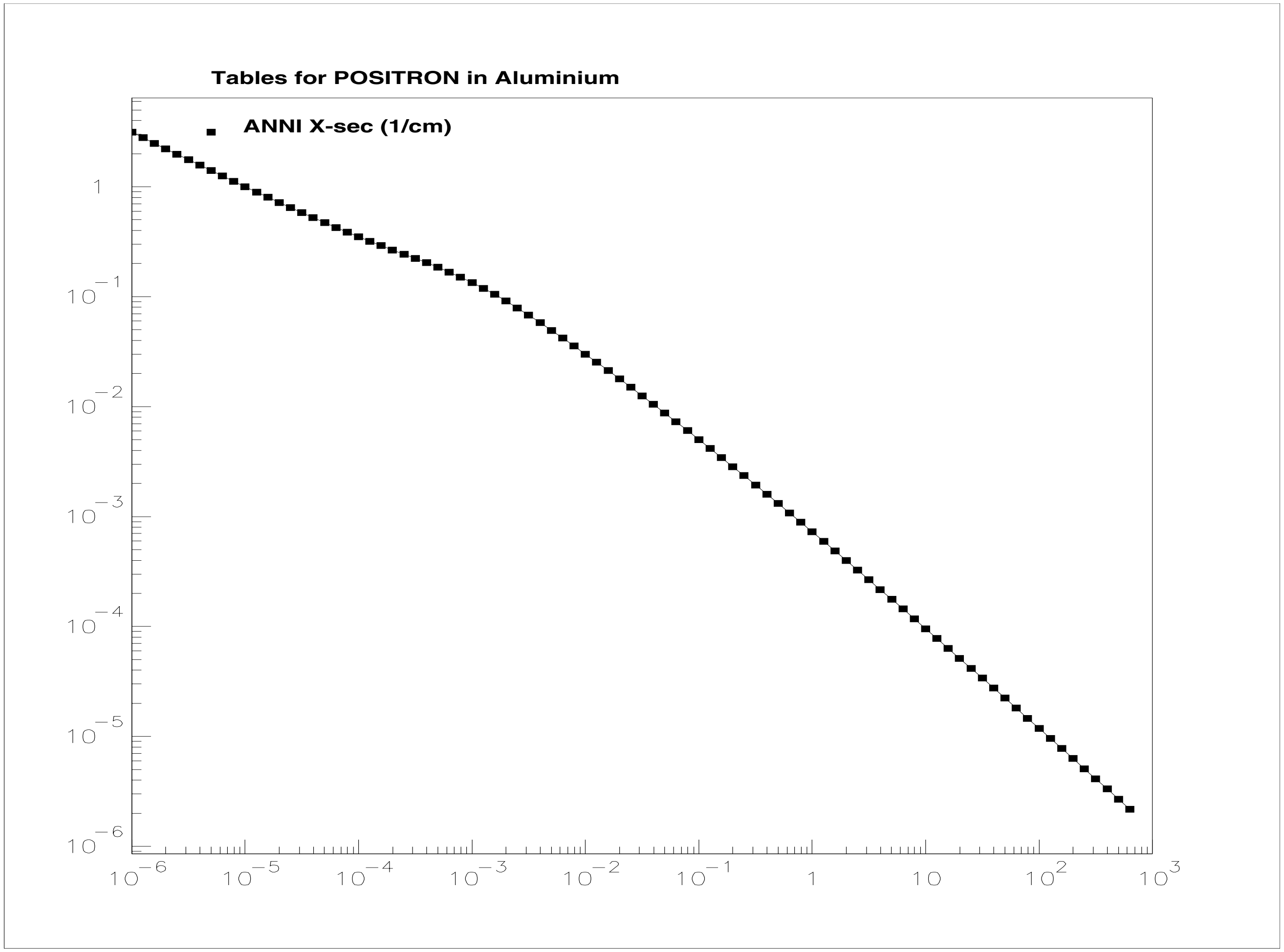
The cross section decreases with increasing E.

The nonrelativistic limit is:

$$\sigma_{nr}(Z,E) \sim \frac{Z\pi r_e^2}{\beta}$$

The angles of emitted photons are determined by momentum conservation.

number of interactions per cm in Aluminium



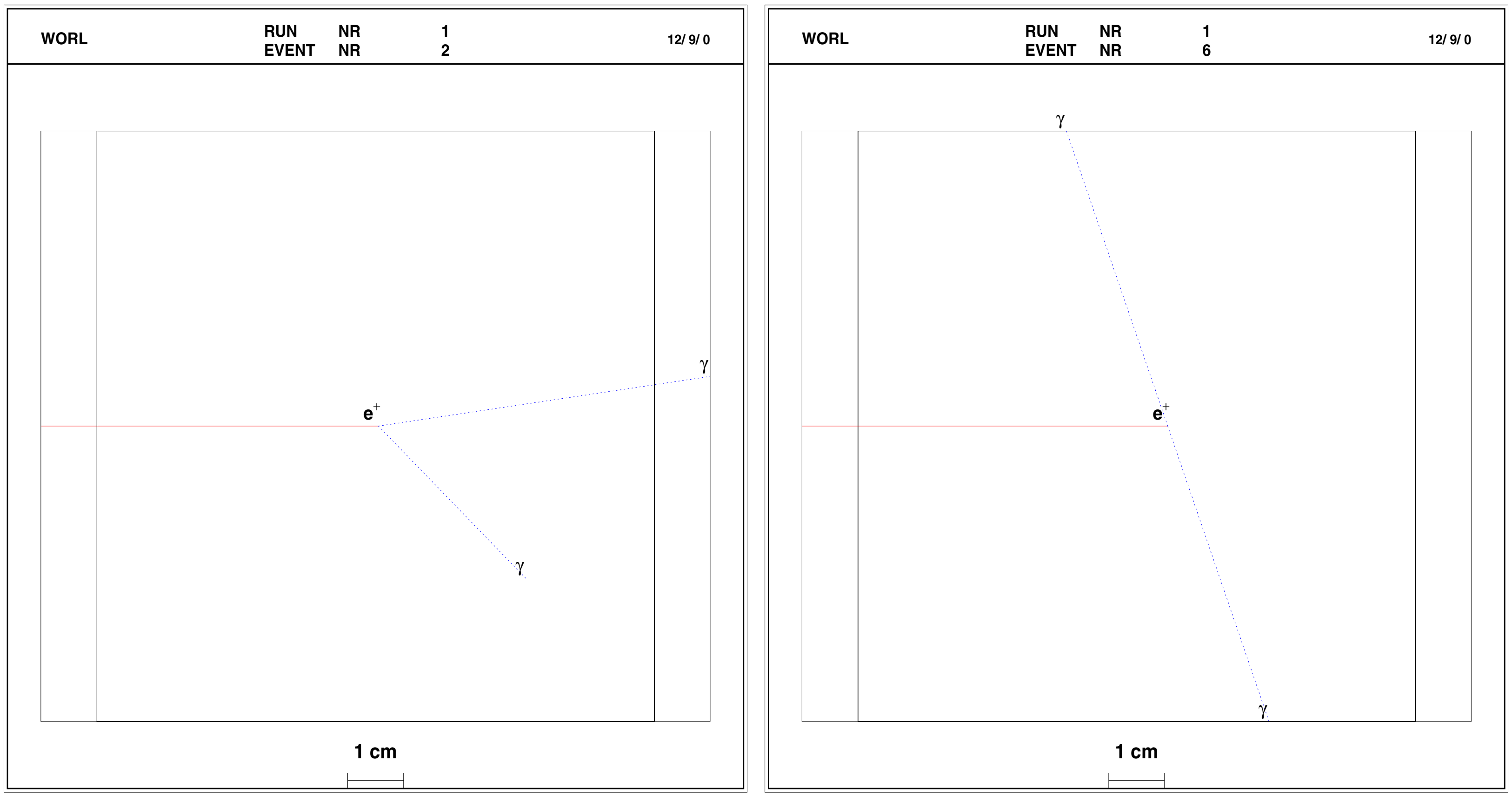
positron kinetic energy (GeV)

The annihilation in fly is not the dominant process. Most of the time the positron comes at rest and does a *positronium* with the electron.

The positronium decays in two-photon (in 0.125 *nanosec*) or three-photon state (in 142 *nanosec.*)

The (e^+, e^-) can also annihilate in a single photon: the other photon is absorbed by the recoil mucleus. However this mechanism is suppressed by a factor α^4.

e^+ 30 MeV in 10 cm Aluminium. Annihilation in fly (left), at rest (right).



(e^+, e^-) pair creation by photon

(e^+, e^-) pair creation by photon

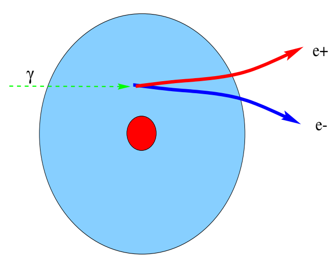
This is the transformation of a photon into an (e^+, e^-) pair in the Coulomb field of atoms (for momentum conservation).

To create the pair, the photon must have at least an energy of $2mc^2(1 + m/M_{rec})$.

Theoretically, (e^+, e^-) pair production is related to bremsstrahlung by crossing symmetry:

- incoming $e^- \leftrightarrow$ outgoing e^+
- outgoing $\gamma \leftrightarrow$ incoming γ

For $E_\gamma \geq$ few tens MeV, (e^+, e^-) pair creation is the dominant process for the photon, in all materials.



differential cross section

The differential cross section is given by the Bethe-Heitler formula [Heitl57], corrected and extended for various effects:

- the screening of the field of the nucleus
- the pair creation in the field of atomic electrons
- the correction to the Born approximation
- the LPM suppression mechanism
- ...

See Seltzer and Berger for a synthesis of the theories [Sel85].

screening effect

Depending of the energy of the projectile, the Coulomb field of the nucleus can be more on less screened by the electron cloud.

A screening parameter measures the ratio of an 'impact parameter' of the projectile to the radius of an atom, for instance given by a Thomas-Fermi approximation or a Hartree-Fock calculation.

Then, screening functions are introduced in the Bethe-Heitler formula.

Qualitatively:

- at low energy $\rightarrow$ no screening effect
- at ultra relativistic electron energy $\rightarrow$ full screening effect

triplet creation in the electron field

The projectile feels not only the Coulomb field of the nucleus (charge Ze), but also the fields of the atomic electrons (Z electrons of charge e).

As for bremsstrahlung, the amplitude is roughly the same in both cases, except the charge.

Thus the electron cloud gives an additional contribution to the pair creation, proportional to Z (instead of Z^2).

However, in the Coulomb field of an electron, the threshold is : $E_\gamma \geq 4mc^2$.

The recoil electron may be ejected from the atom, thus the final state can be a triplet (e^+, e^-, e^-). The kinetic energy of this e^- is small.

Born approximation

The derivation of the Bethe-Heitler formula is based on perturbation theory, using plane waves for the electron. If the validity of the Born approximation:

$$\beta \gg \alpha Z$$

is violated for the e^+ and/or e^- velocity (low energy photon) the Coulomb waves would be used instead of the plane waves.

To correct for this, a Coulomb correction function is introduced in the Bethe-Heitler formula.

high energies regime : $k \gg mc^2/(\alpha Z^{1/3})$

Above few GeV the energy spectrum formula becomes simple :

$$\left.\frac{d\sigma}{dE}\right]_{Tsai} \approx 4\alpha\, r_e^2 \,\frac{1}{k} \times \left\{\left(1 - \frac{4}{3}x(1-x)\right)\left(Z^2\left[L_{rad} - f(Z)\right] + Z L'_{rad}\right)\right\} \quad (1)$$

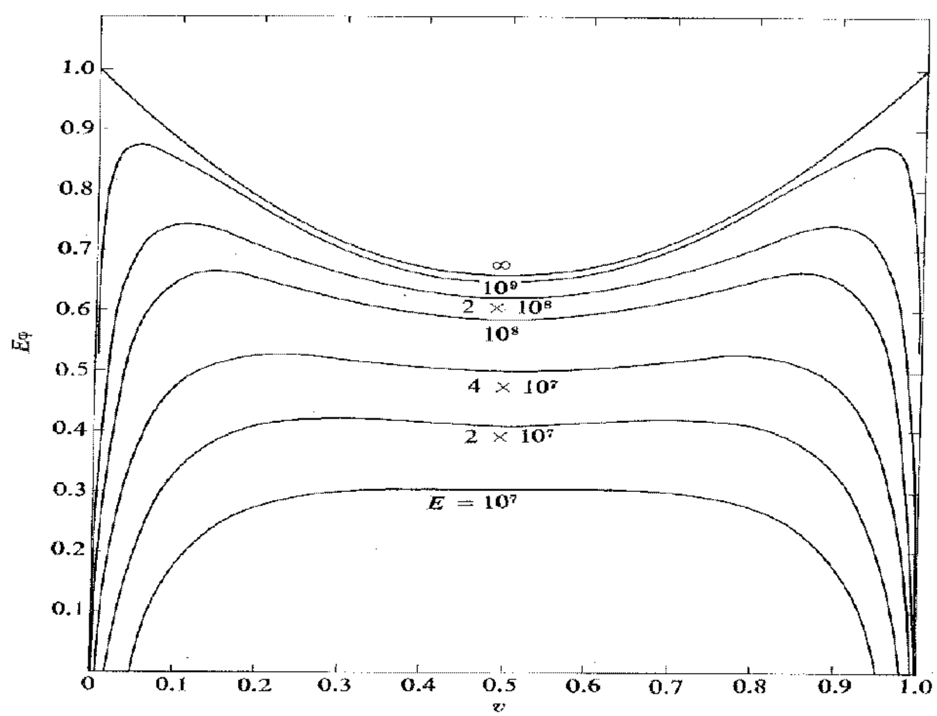
where

k	energy of the incident photon
E	total energy of the created e^+ (or e^-) ; $x = E/k$
$L_{rad}(Z)$	$\ln(184.15/Z^{1/3})$ (for $Z \geq 5$)
$L'_{rad}(Z)$	$\ln(1194/Z^{2/3})$ (for $Z \geq 5$)
$f(Z)$	Coulomb correction function

energy spectrum

limits: $E_{min} = mc^2$: no infrared divergence. $E_{max} = k - mc^2$.

The partition of the photon energy between e^+ and e^- is flat at low energy ($k \leq 50\ MeV$) and increasingly asymmetric with energy. For $k >\ TeV$ the LPM effect reinforces the asymmetry.



In the high energies regime, one can use the complete screened expression 1 of $d\sigma/dE$ to compute the total cross section.

$$\sigma(k) = \int_{E_{min}\approx 0}^{E_{max}\approx k} \frac{d\sigma}{dE}\, dE \tag{2}$$

which gives:

$$\sigma_{pair}(k) \approx \frac{7}{9}\,\frac{1}{n_{at}\,X_0} \tag{3}$$

n_{at} is the number of atoms per volume.

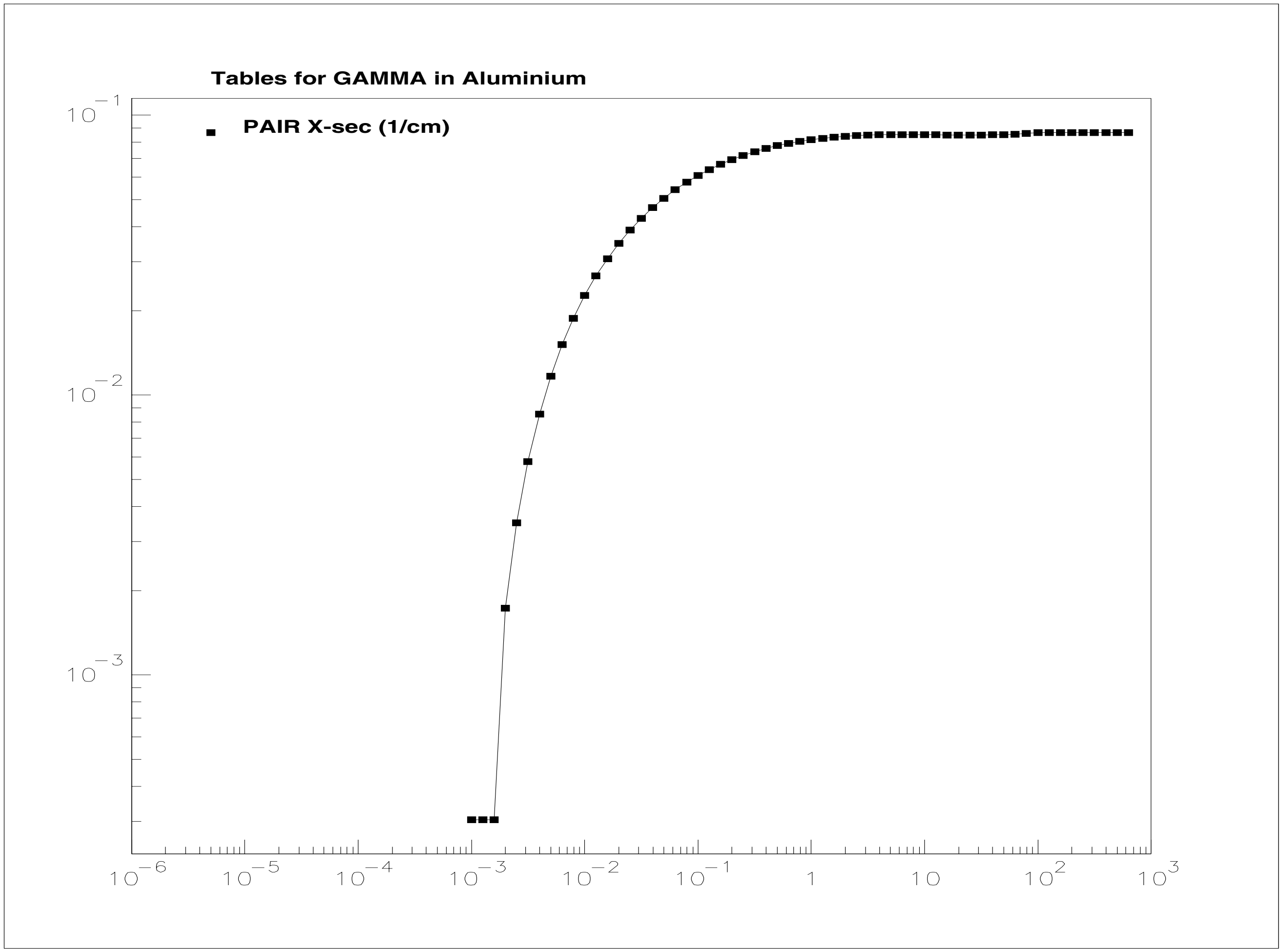
The pair creation total cross section is approximately constant above few GeV, for at least 4 decades (then, LPM effect).

the average angle of emission of the electron, respective to the incident photon is

$$\theta = \frac{mc^2}{k}$$

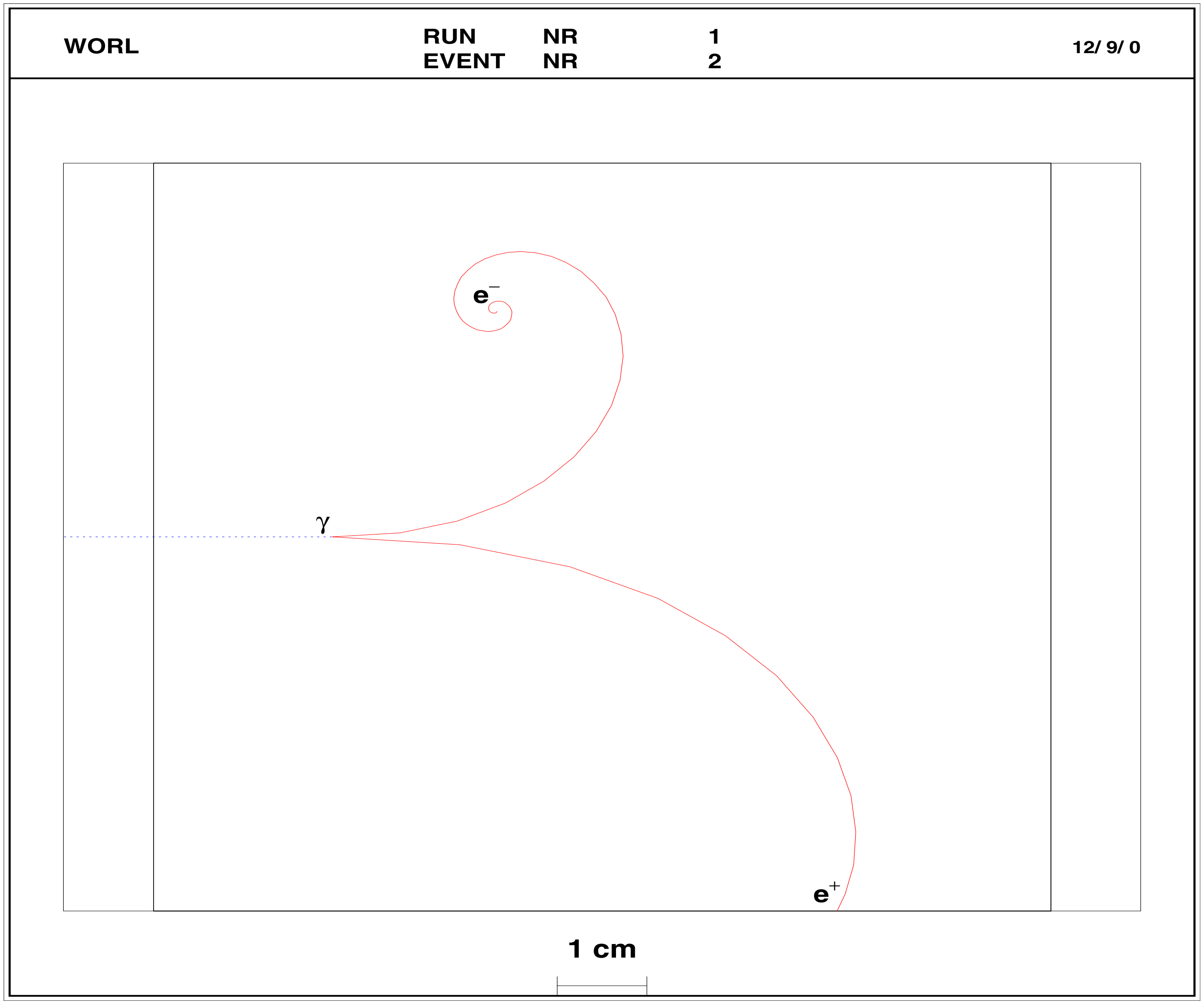
which is independent of the energy of the emitted electron.

number of interactions per cm in Aluminium



photon energy (GeV)

γ 200 MeV in 10 cm Aluminium. Field 5 tesla



Landau-Pomeranchuk-Migdal suppression mechanism

Due to the LPM mechanism, the (e^+, e^-) pair creation is reduced for [PDG00] :

$$E(k-E) > k\ E_{lpm} \qquad \Longleftrightarrow\ x(1-x) > \frac{E_{lpm}}{k} \tag{4}$$

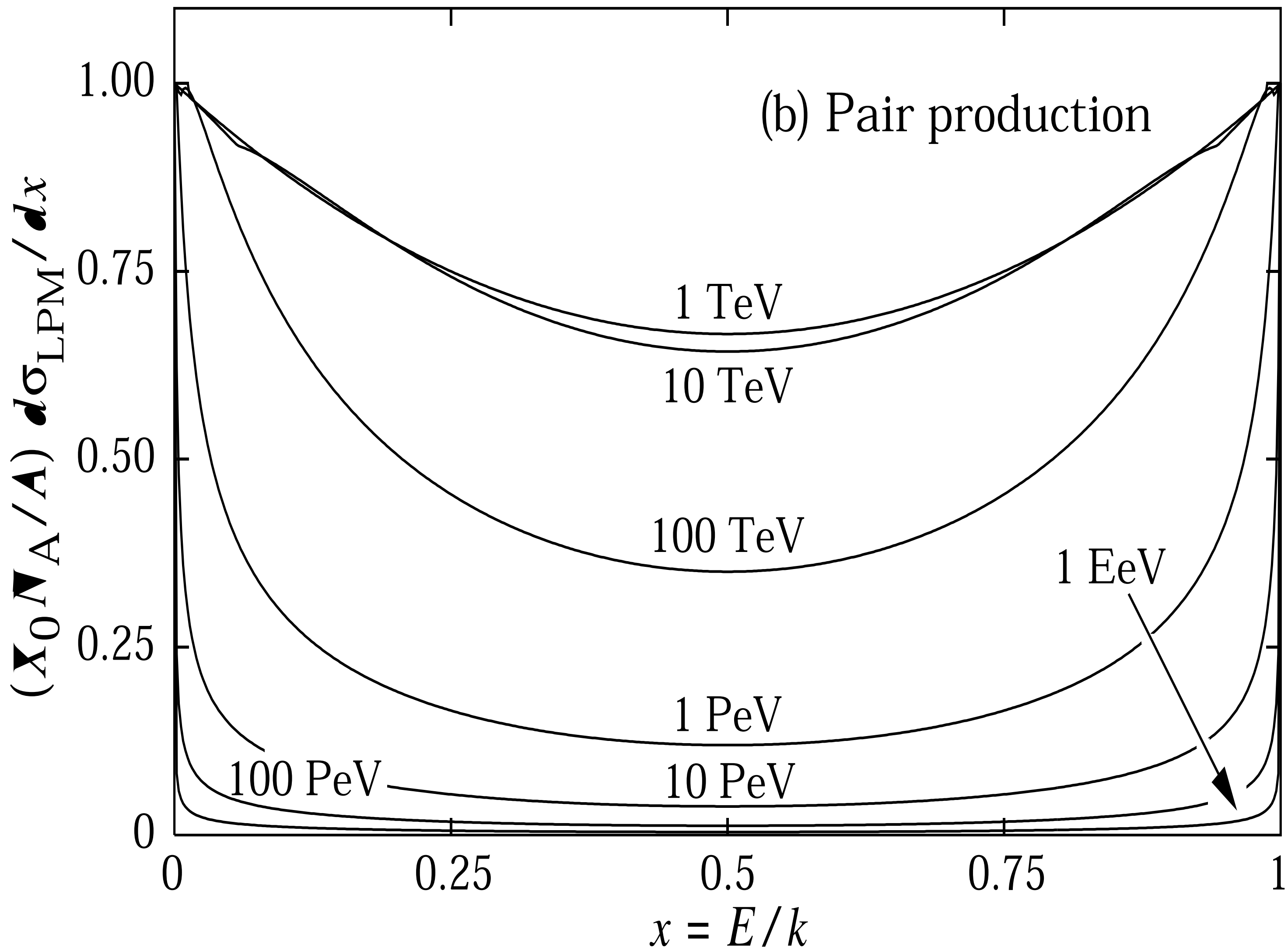
where: $x = E/k \qquad \Rightarrow k > 4E_{lpm}$

E_{lpm} is a characteristic energy of the effect :

$$E_{lpm} = \frac{\alpha}{4\pi}\frac{(mc^2)^2}{\hbar c}\ X_0 \qquad \sim (7.7\ TeV/cm) \times X_0\ (cm) \tag{5}$$

The suppression function S_{lpm} is:

$$S_{lpm}(E) = \sqrt{\frac{k\ E_{lpm}}{E(k-E)}} \equiv \sqrt{\frac{E_{lpm}/k}{x(1-x)}} \tag{6}$$


(b) Pair production
1.00
0.75
0.50
0.25
0
0
0.25
0.5
0.75
1
$(X_0 N_A / A)\ d\sigma_{LPM}/dx$
$x = E/k$
1 TeV
10 TeV
100 TeV
1 EeV
1 PeV
100 PeV
10 PeV

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Compton scattering

Compton scattering

The Compton effect describes the scattering off quasi-free atomic electrons :

$$\gamma + e \rightarrow \gamma' + e'$$

Each atomic electron acts as an independent cible; Compton effect is called incoherent scattering. Thus:

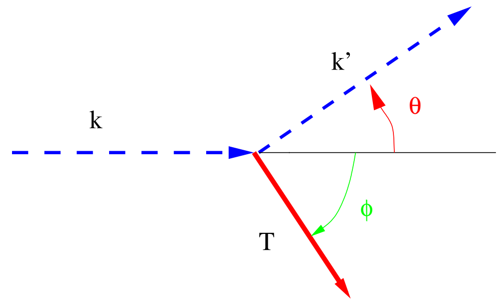
$$\text{cross section per atom} = Z \times \text{cross section per electron}$$

The inverse Compton scattering also exists: an energetic electron collides with a low energy photon which is blue-shifted to higher energy. This process is of importance in astrophysics.

Compton scattering is related to (e^+, e^-) annihilation by crossing symmetry.

kinematic

Assuming the initial electron free and at rest, the kinematic is given by energy-momentum conservation of two-boby scattering.



$$k' = \frac{k}{1+\kappa(1-\cos\theta)} \qquad where\ \kappa = \frac{k}{mc^2}$$

$$T = k - k'$$

$$\cot\phi = (1+\kappa)\tan(\theta/2)$$

limits $\theta = 0:\quad k'_{max} = k \quad T_{min} = 0 \quad \phi_{max} = \frac{\pi}{2}$

$\theta = \pi:\quad k'_{min} = k\frac{1}{2\kappa+1} \quad T_{max} = k\frac{2\kappa}{2\kappa+1} \quad \phi_{min} = 0$

energy spectrum

Under the same assumption,the unpolarized differential cross section per atom is given by the Klein-Nishina formula [Klein29] :

$$\frac{d\sigma}{dk'} = \frac{\pi\, r_e^2}{mc^2}\, \frac{Z}{\kappa^2} \left[\epsilon + \frac{1}{\epsilon} - \frac{2}{\kappa}\left(\frac{1-\epsilon}{\epsilon}\right) + \frac{1}{\kappa^2}\left(\frac{1-\epsilon}{\epsilon}\right)^2\right] \qquad (1)$$

where

k'	energy of the scattered photon ; $\epsilon = k'/k$
r_e	classical electron radius
κ	k/mc^2

total cross section

$$\sigma(k) = \int_{k'_{min}=k/(2\kappa+1)}^{k'_{max}=k} \frac{d\sigma}{dk'}\, dk'$$

$$\sigma(k) = 2\pi\, r_e^2\, Z\, \left[\left(\frac{\kappa^2 - 2\kappa - 2}{2\kappa^3}\right)\ln(2\kappa+1) + \frac{\kappa^3 + 9\kappa^2 + 8\kappa + 2}{4\kappa^4 + 4\kappa^3 + \kappa^2}\right]$$

limits

$k \to \infty :$ $\quad \sigma$ goes to 0 : $\sigma(k) \sim \pi\, r_e^2\, Z\, \dfrac{\ln 2\kappa}{\kappa}$

$k \to 0 :$ $\quad \sigma \to \dfrac{8\pi}{3}\, r_e^2\, Z$ (classical Thomson cross section)

low energy limit

In fact, when $k \leq 100\ keV$ the binding energy of the atomic electron must be taken into account by a corrective factor to the Klein-Nishina cross section:

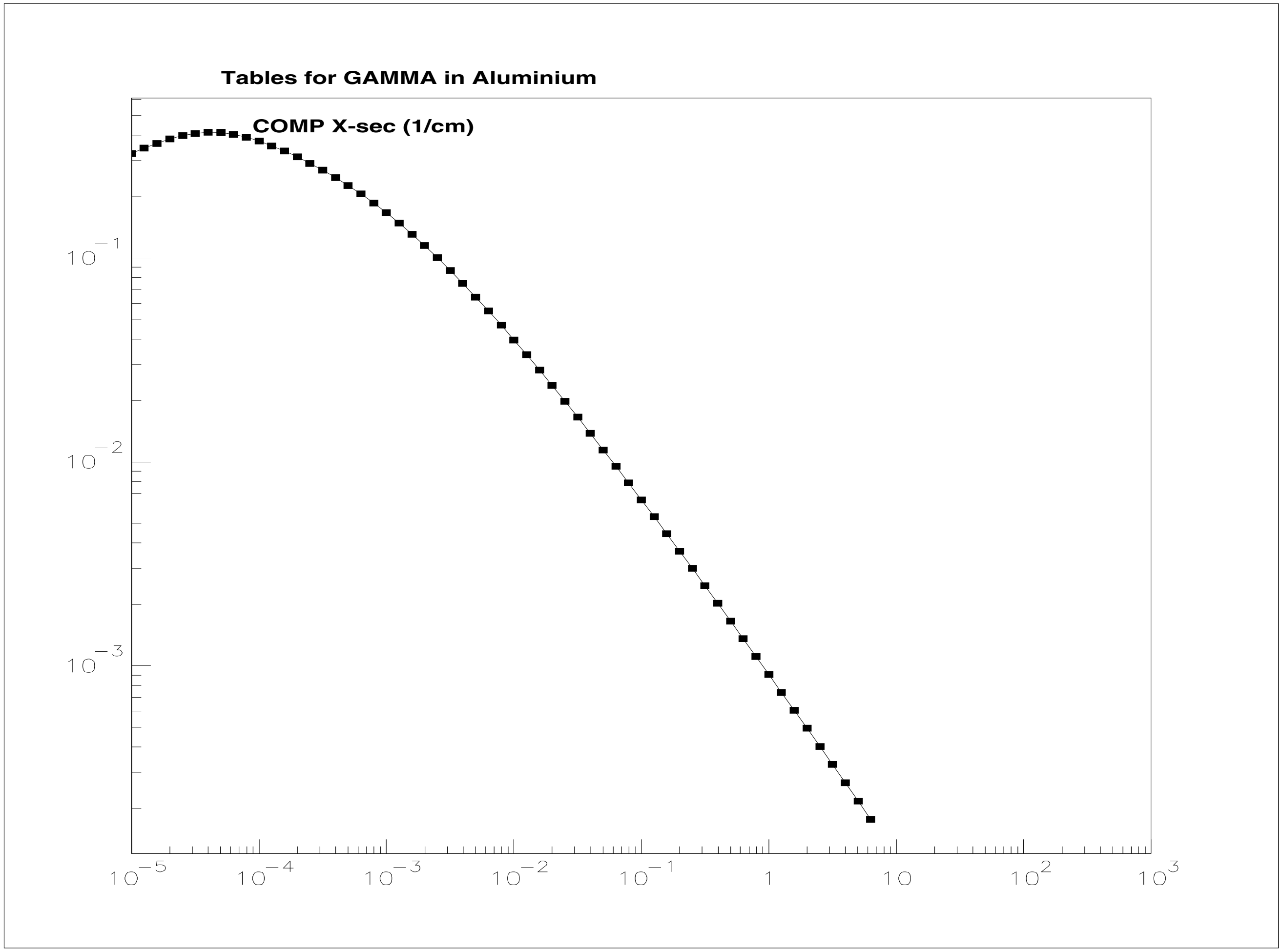
$$\frac{d\sigma}{dk'} = \left[\frac{d\sigma}{dk'}\right]_{KN} \times S(k,k')$$

See for instance [Cullen97] or [Salvat96] for derivation(s) and discussion of the *scattering function* S(k,k').

As a consequence, at very low energy, the total cross section goes to 0 like k^2. It also suppresses the forward scattering.

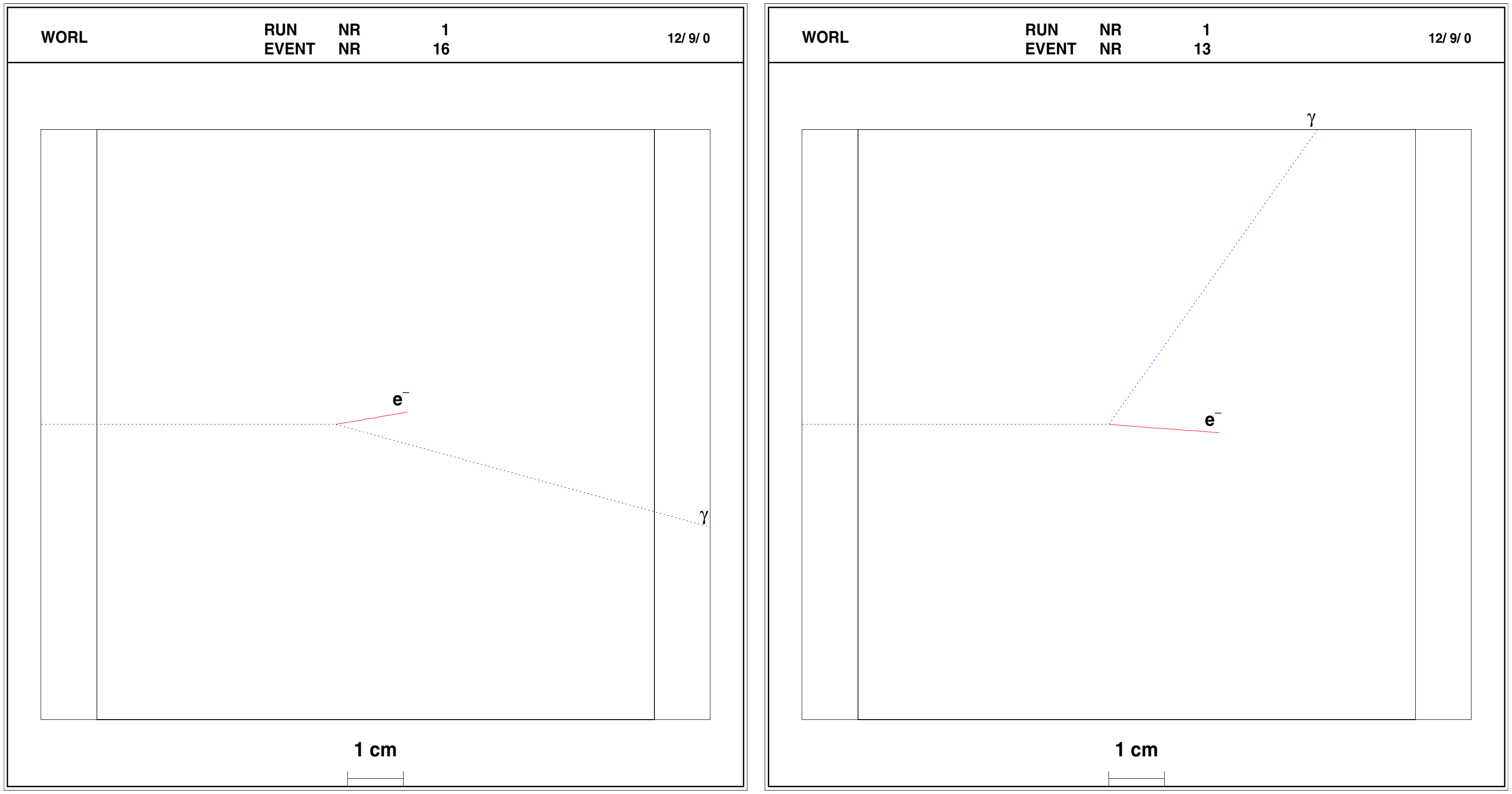
At X-rays energies the scattering function has little effect on the Klein-Nishina energy spectrum formula 1. In addition the Compton scattering is not the dominant process in this energy region.

number of interactions per cm in Aluminium

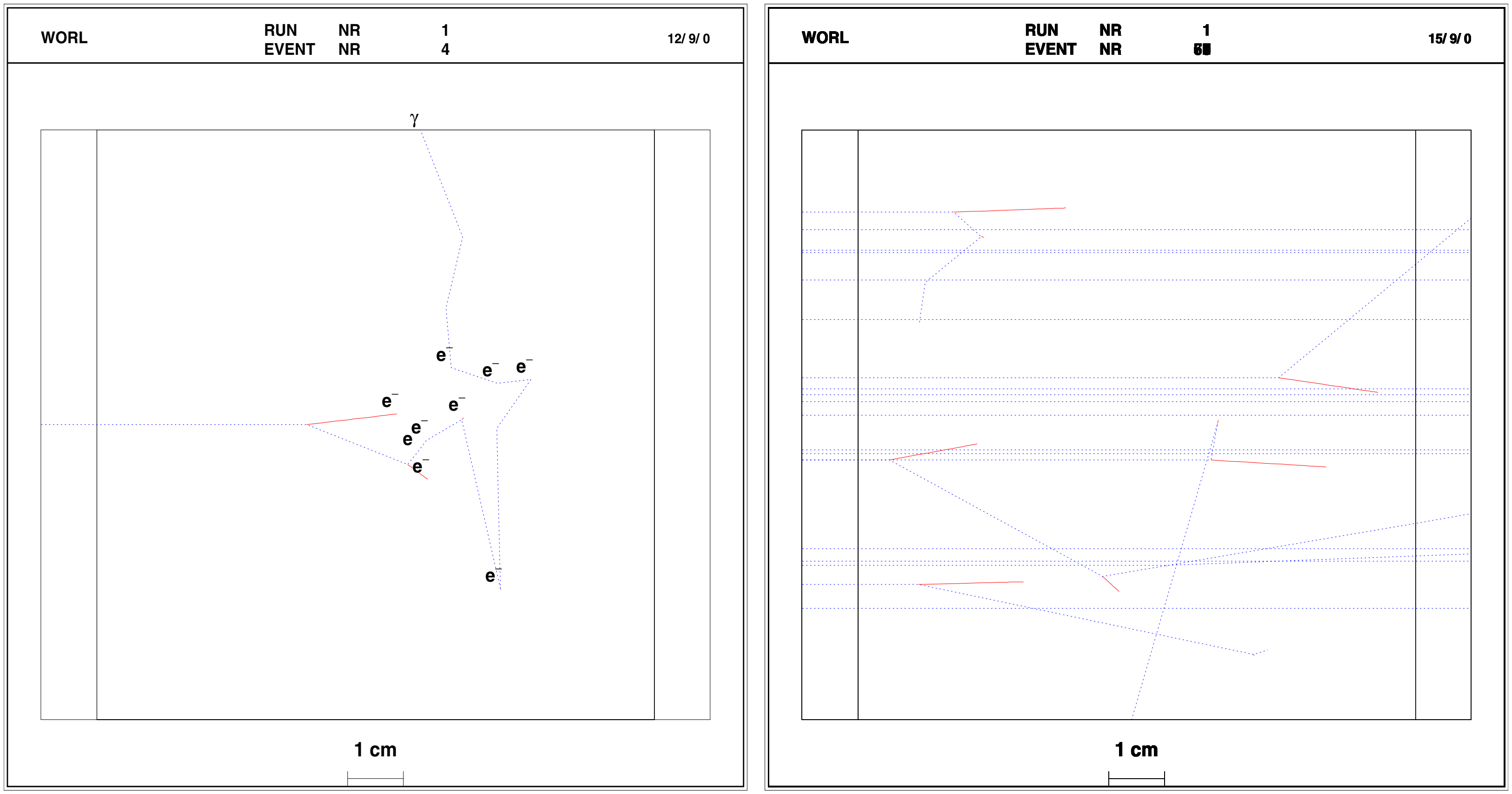


photon energy (GeV)

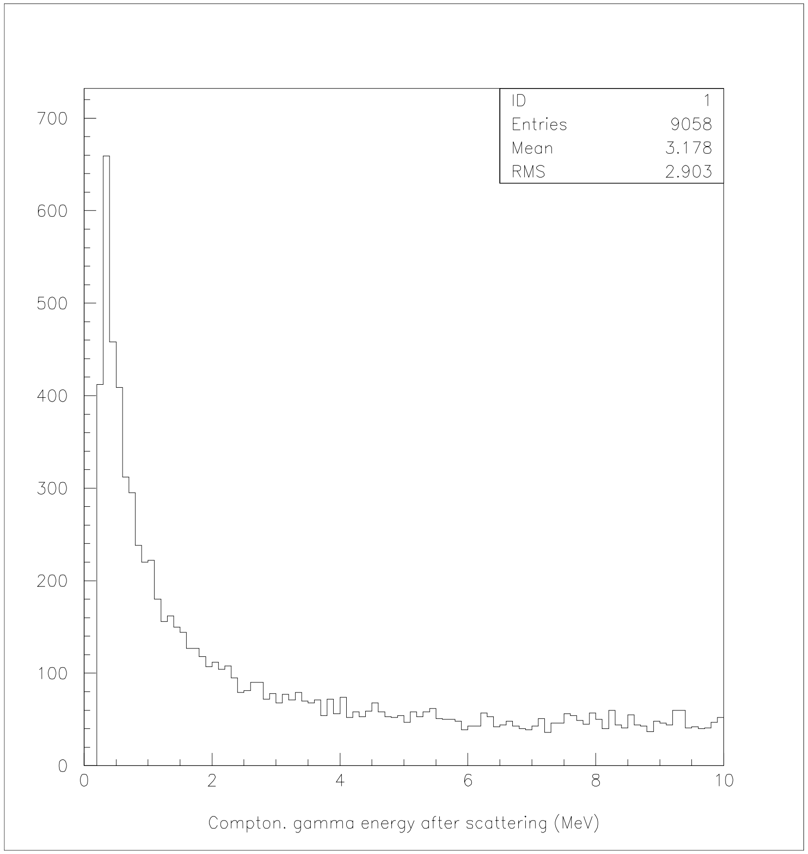
γ 10 MeV in 10 cm Aluminium: Compton scattering

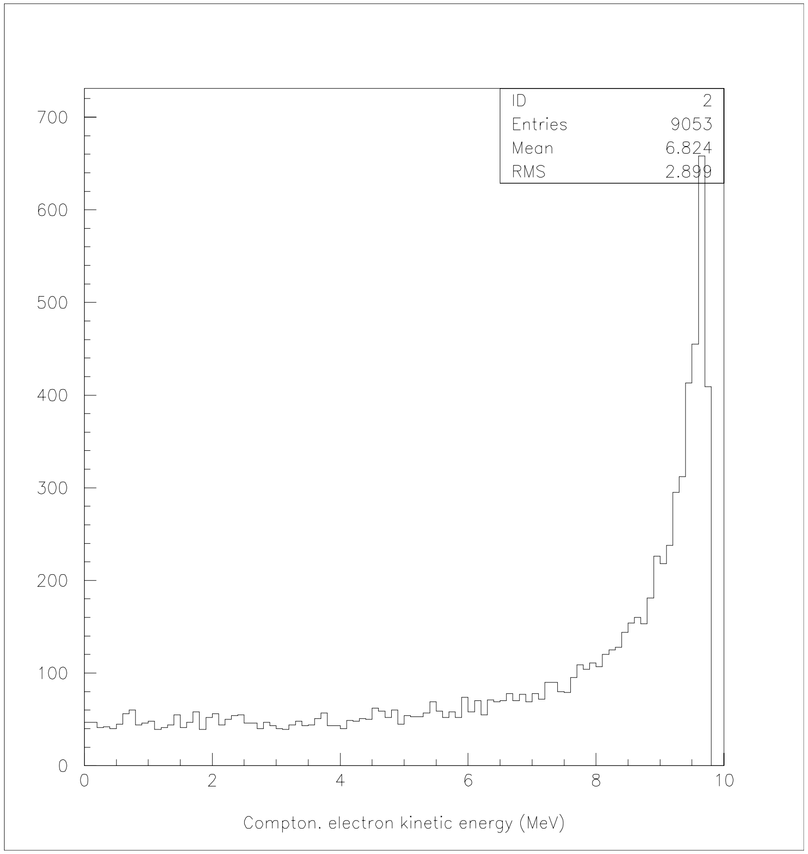


γ 10 MeV in 10 cm Aluminium: Compton scattering



Compton scattering : γ 10 MeV in Aluminium. *Compton edge*: energy spectrum of scattered photon (left) and emitted e^- (right)





absorption, diffusion, attenuation

Only a fraction of the energy of the incident photon is transferred to the recoil electron, which is generally stopped into the material. Thus this energy is absorbed by the material.
The mean kinetic energy of the electron is :

$$\langle T(k) \rangle = \int_{k'_{min}}^{k} T \, \frac{d\sigma}{dk'} \, dk'$$

Then the absorption coefficient of Compton scattering is defined as

$$\mu_{abs} = n_{at} \, \frac{\langle T(k) \rangle}{k} \qquad (n_{at} \text{ is the nb of atoms per volume})$$

$\dfrac{\mu_{abs}}{\rho}$ is the mass absorption coefficient.

Similar, from the mean energy of the diffused photon one defines the scattered coefficient of Compton scattering

$$\langle k' \rangle = \int_{k'_{min}}^{k} k' \, \frac{d\sigma}{dk'} \, dk' \qquad \longrightarrow \mu_{sca} = n_{at} \, \frac{\langle k' \rangle}{k}$$

The attenuation coefficient of Compton scattering is

$$\mu_{att} \stackrel{def}{=} n_{at}\sigma = \mu_{abs} + \mu_{sca}$$

and similar relations for the mass coefficients.

Rayleigh scattering

Rayleigh scattering is the scattering of photons by an atom as a whole : all the electrons of the atom participate in a coherent manner.

It is an elastic collision: no energy transfer from photon to atom (no ionisation nor excitation).

At x-rays and γ-rays energy region, Rayleigh scattering is small compared to the photo electric effect, and can be generally neglected.

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J.H.Hubbell et al. Rad. Phys. Chem. vol50, 1 (1997)

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Photoelectric absorption

Photoelectric absorption

A bound electron can absorb completely the energy of a photon :

$$\gamma + atom \rightarrow atom^{+} + e^{-}$$

The electron is ejected with kinetic energy $T = k - B_s$.
(k: energy of the incident photon, B_s: binding energy of the corresponding subshell). The nucleus absorbs the recoil momentum.

The cross section per shell can be parametrized [Biggs87] :

$$\sigma_s = r_e^2\ \alpha^4\ Z^5\ f\left(\frac{1}{k^{a(k)}}\right)$$

with f : nonsimple function of $1/k$, and $1 \leq a(k) \leq 4$.

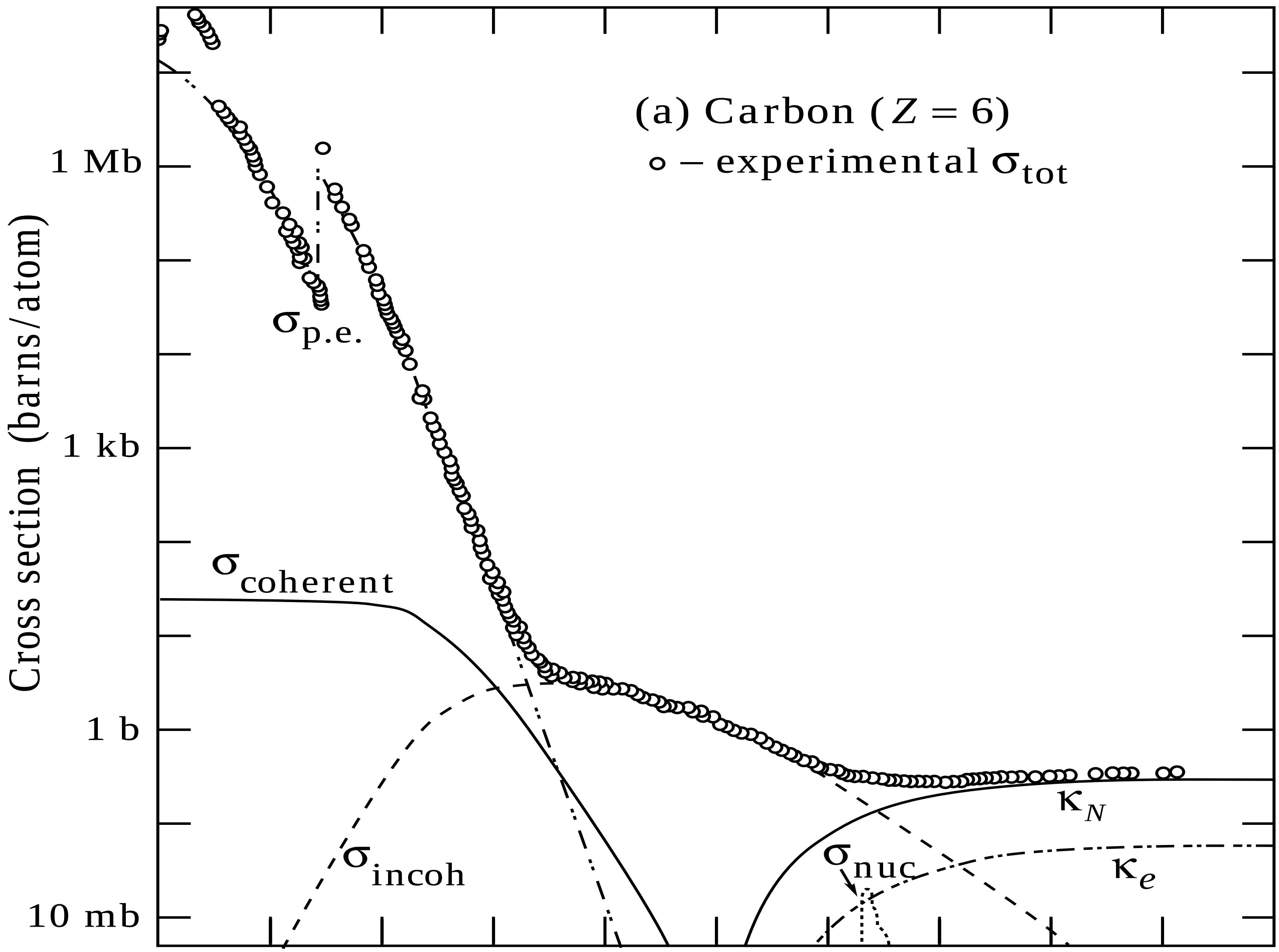
The total cross section has discontinuities at $k = B_s$ (absorption edge).

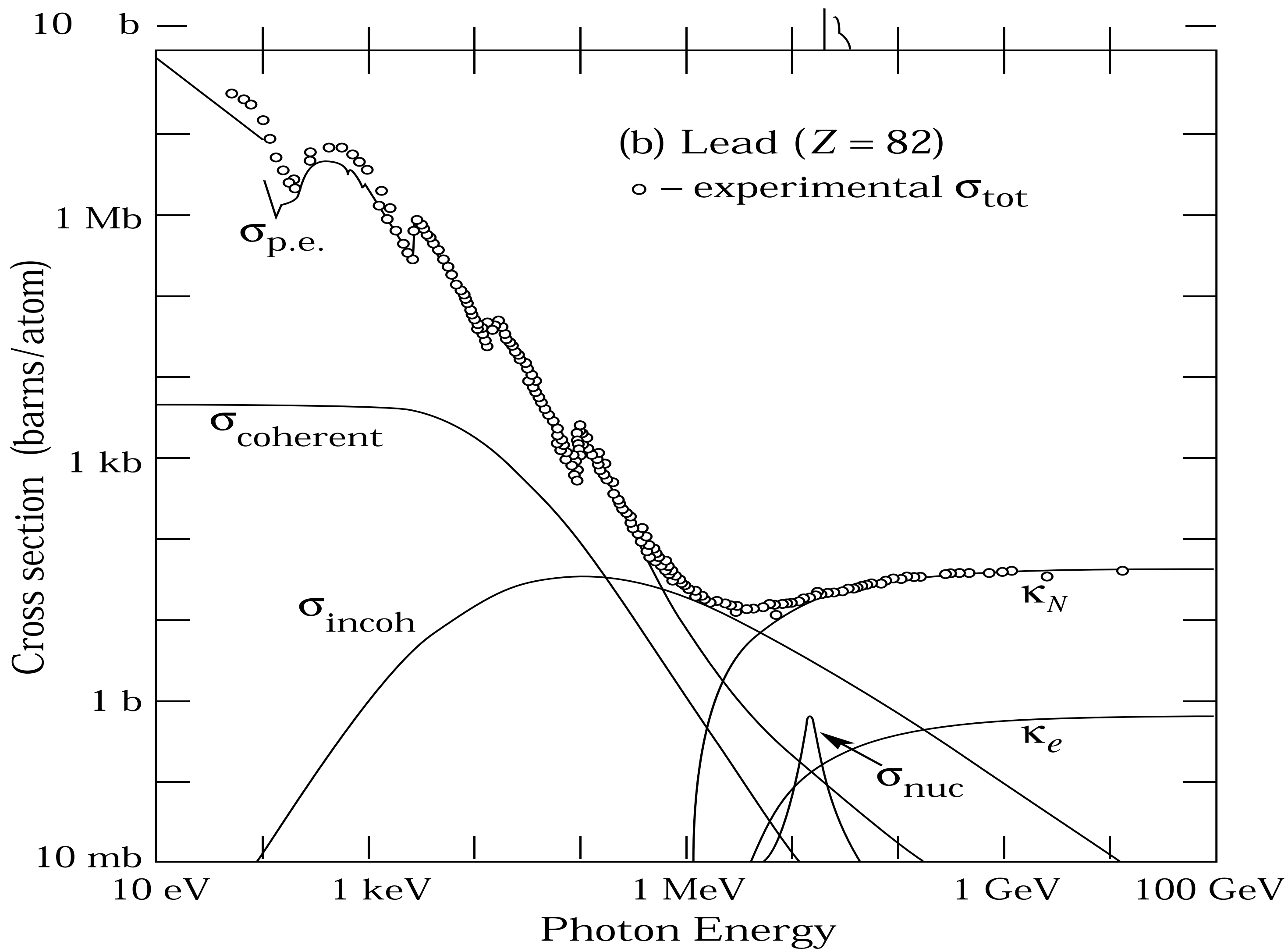
There are several parametrizations and tables of the cross sections. See [Cullen97, Biggs87].

If $k > B_K$ the absorption occurs mainly on the K-shell (80% of the cases).

The electron is emitted forward in the direction of the incident photon at high k, and perpendicular to the photon at low k [Sauter31, Gavri61].

Following the photoabsorption in the K-shell, characteristic X-rays or Auger electrons are emitted [Perkin91].


(a) Carbon (Z = 6)
○ – experimental σtot
1 Mb
1 kb
1 b
10 mb
Cross section (barns/atom)
σp.e.
σcoherent
σincoh
σnuc
κN
κe



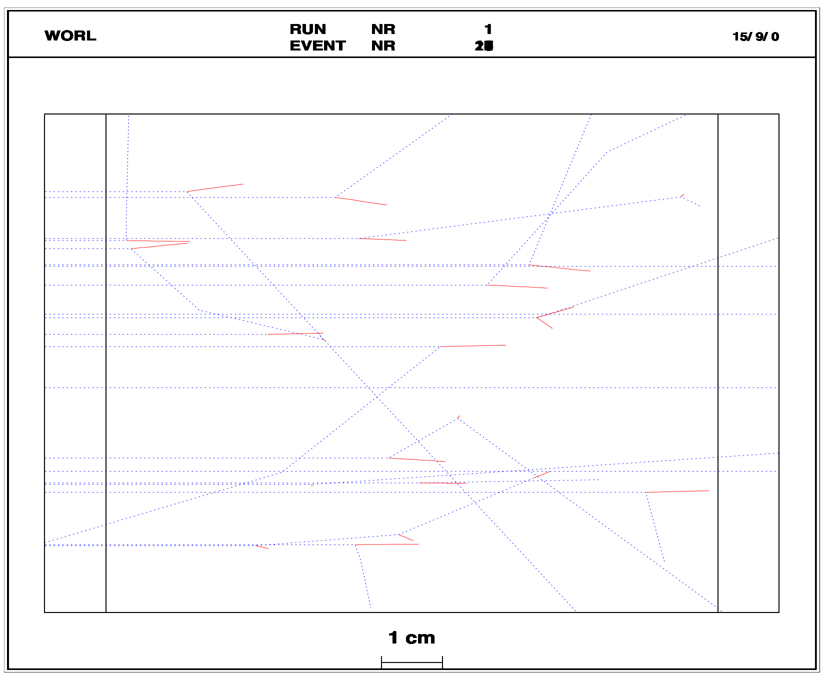
10 b
1 Mb
1 kb
1 b
10 mb
Cross section (barns/atom)
(b) Lead (Z = 82)
○ – experimental σtot
σp.e.
σcoherent
σincoh
σnuc
κN
κe
10 eV
1 keV
1 MeV
1 GeV
100 GeV
Photon Energy

attenuation

$$\sigma_{tot} = \sigma_{pair} + \sigma_{comp} + \sigma_{phot} + \sigma_{rayl} \quad \longrightarrow \mu = n_{at}\,\sigma_{tot}$$

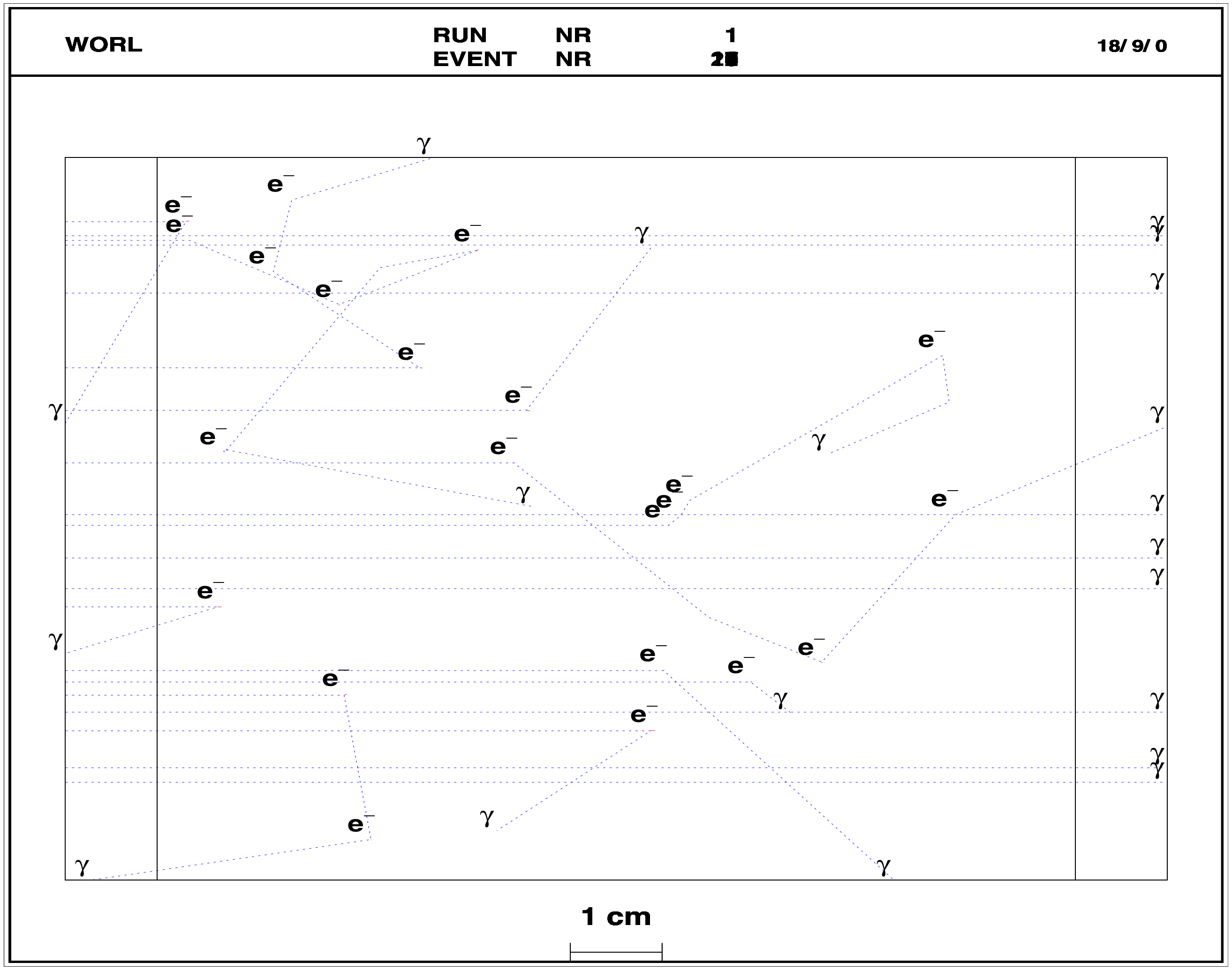
A beam of monoenergetic photons is attenuated in intensity (not in energy) according : $I(x) = I(0)\exp(-\mu\, x) = I(0)\exp(-x/\lambda)$

Below : 20 photons, 5 MeV, entering 10 cm of Al. 4 exit unaltered.

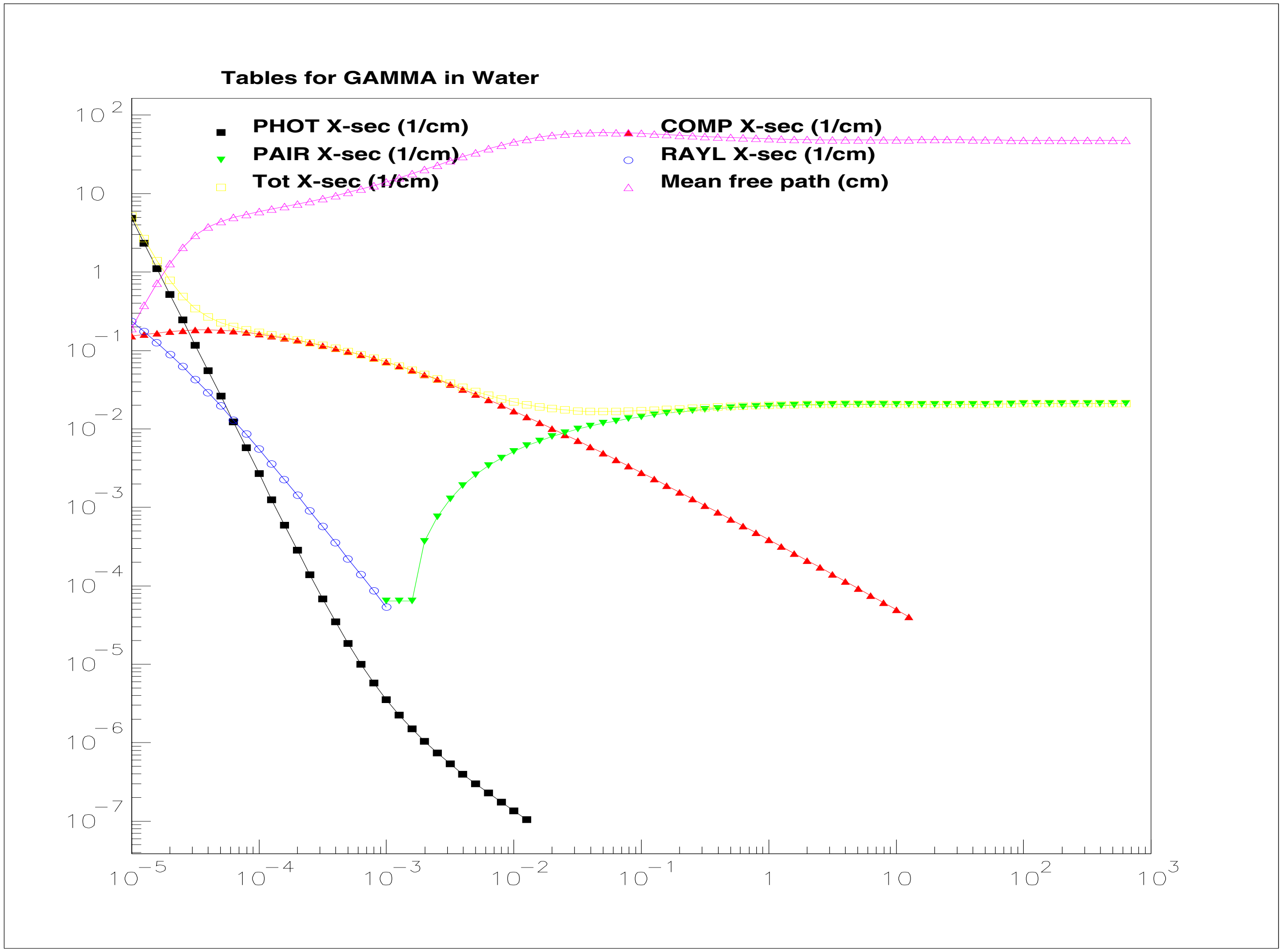


20 photons, 400 keV, entering 10 cm of water.

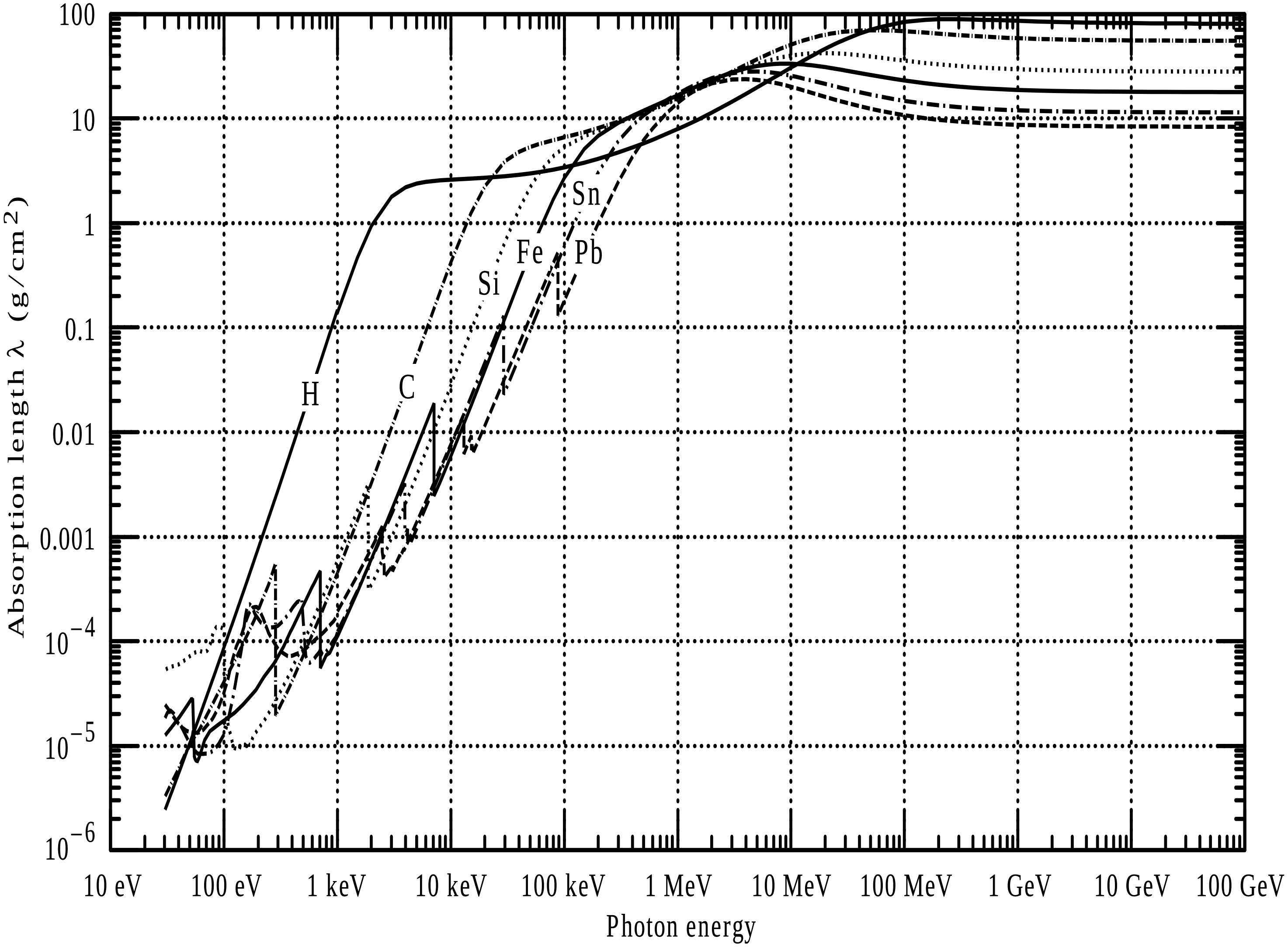
(compare with e^- and protons)



Macroscopic cross sections for photon in water. ($\longrightarrow$ mean free path)



photon energy (GeV)


100
10
1
0.1
0.01
0.001
10^{-4}
10^{-5}
10^{-6}
Absorption length λ (g/cm^2)
H
C
Si
Fe
Sn
Pb
10 eV
100 eV
1 keV
10 keV
100 keV
1 MeV
10 MeV
100 MeV
1 GeV
10 GeV
100 GeV
Photon energy

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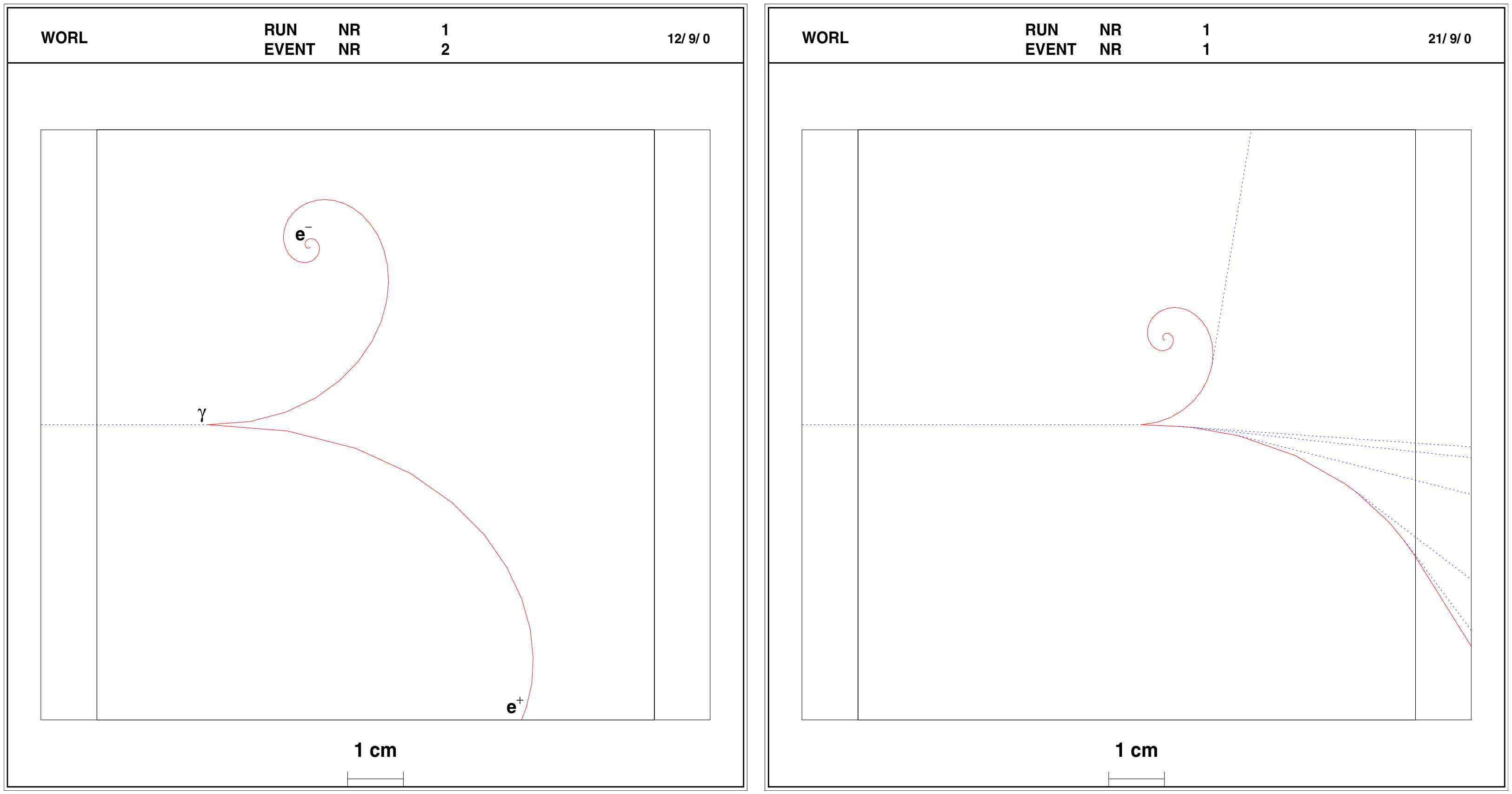
Electromagnetic cascades

Electromagnetic cascades

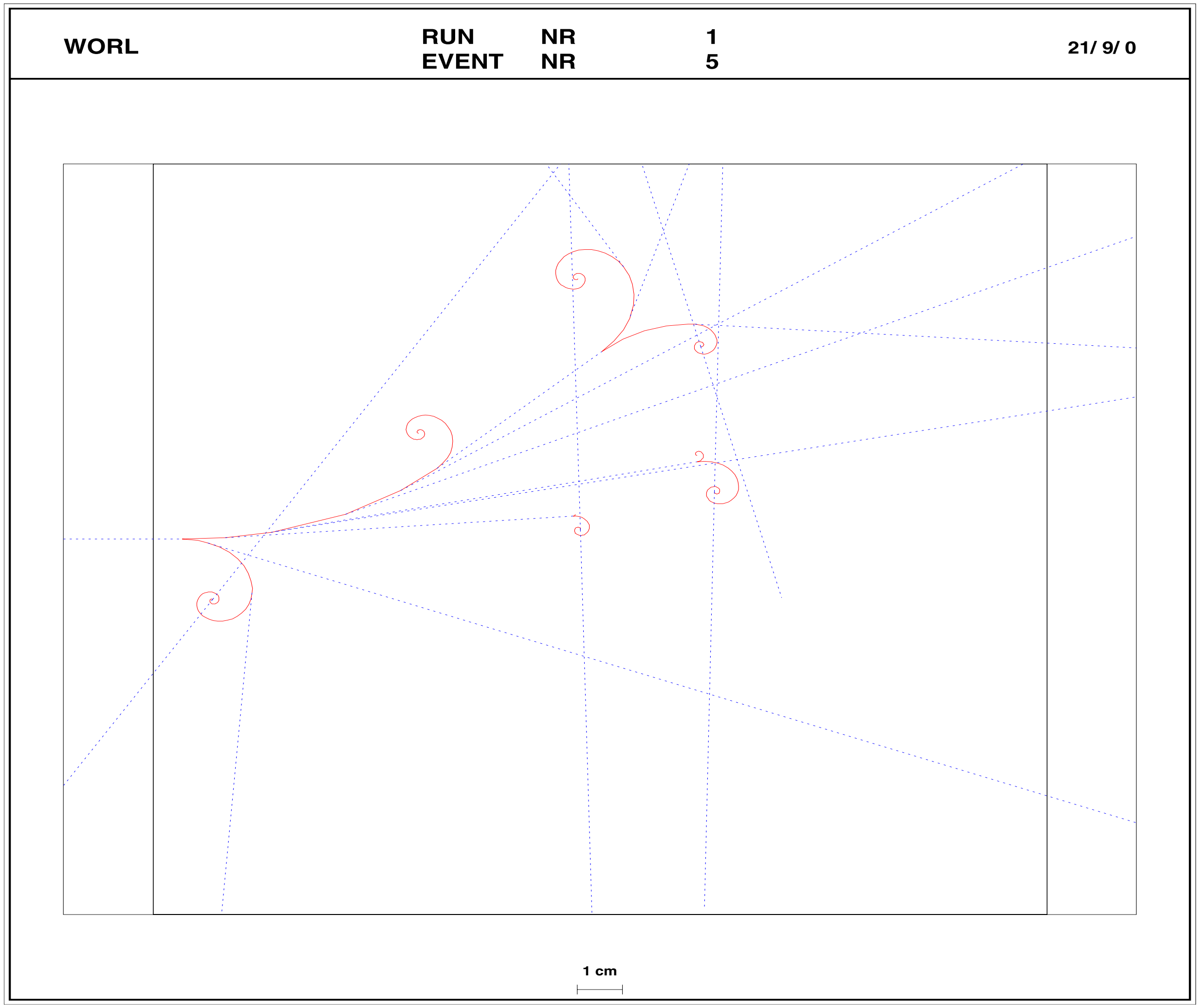
The development of cascades induced by electrons or photons is governed by bremsstrahlung of electrons/positrons and (e^+, e^-) pair creation by photons, until the energy of secondary electrons/positrons fall below the critical energy E_c.

Then the electrons and positrons lose their energy preferentially by ionization, halting the cascade.

γ 200 MeV in 1 X_0 Aluminium. left: pair only; right: pair + brem



γ 200 MeV in 2 X_0 Aluminium. Pair + brem



longitudinal profile in homogeneous material

Beyond the first radiation length, the mean longitudinal profile of the energy deposition is well described by a gamma function [Long75] :

$$\frac{dE}{dt} = E_0 \cdot \mathrm{const} \cdot t^a \cdot \exp(-bt) \quad \text{with } t = x/X_0 \tag{1}$$

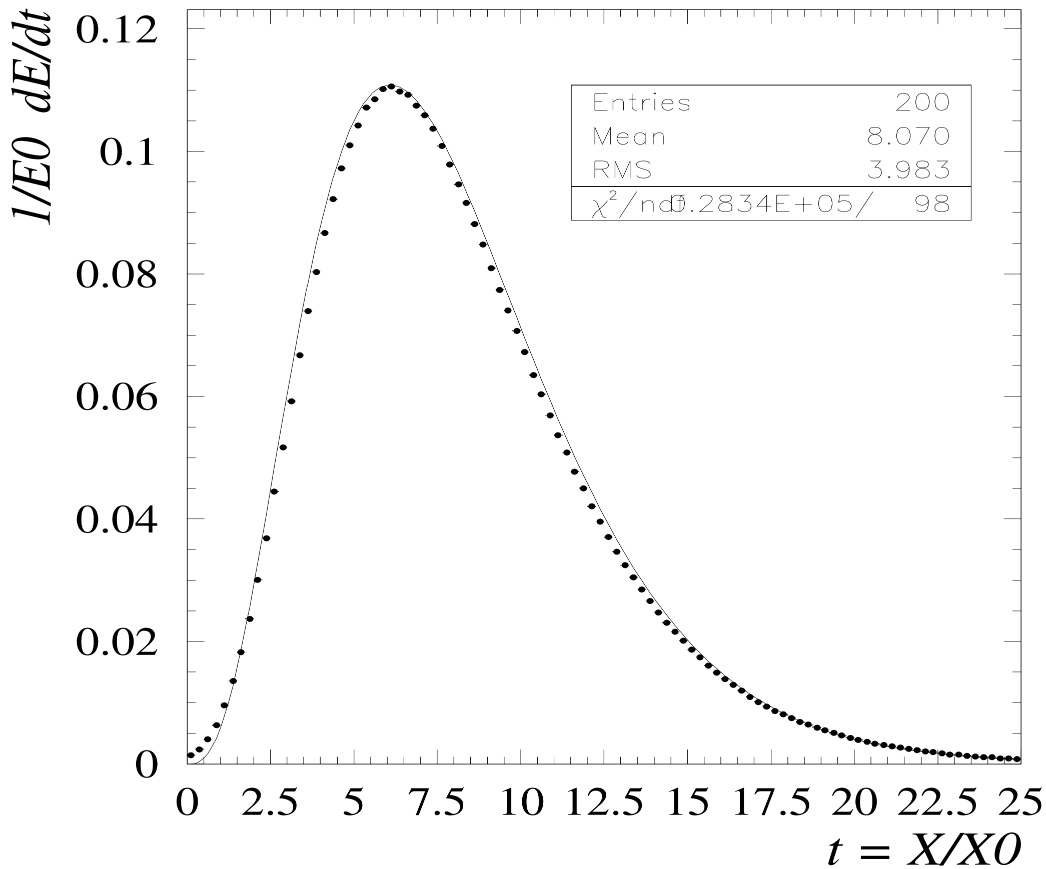
a and b are fit parameters dependent on the material.

On the other hand, a simple model of shower development ([Leo94]) can predict the maximum of the distribution :

$$t_{max} = \frac{a}{b} = k \ln\left(\frac{E_0}{E_c}\right)$$

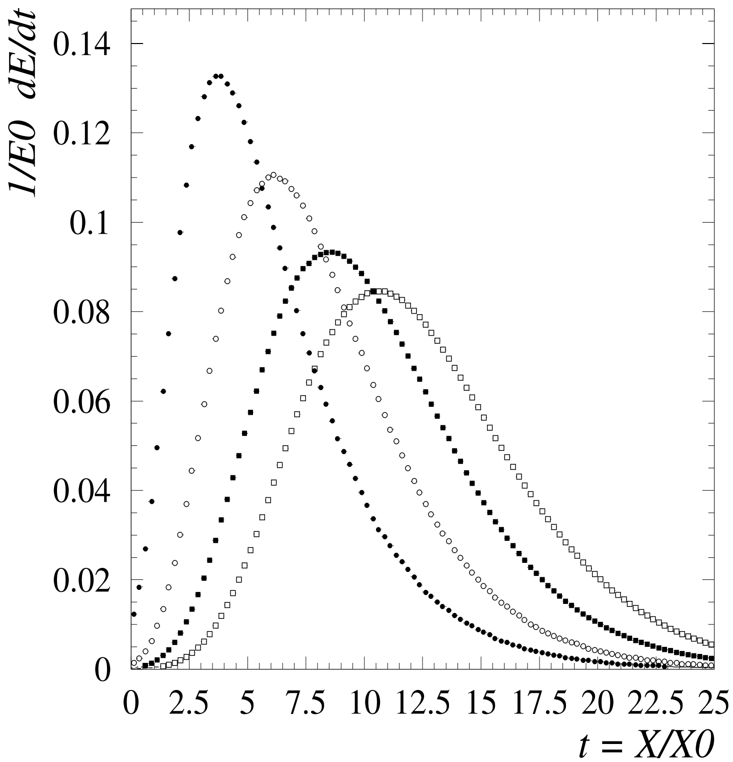
e^- 10 GeV in PbWO4

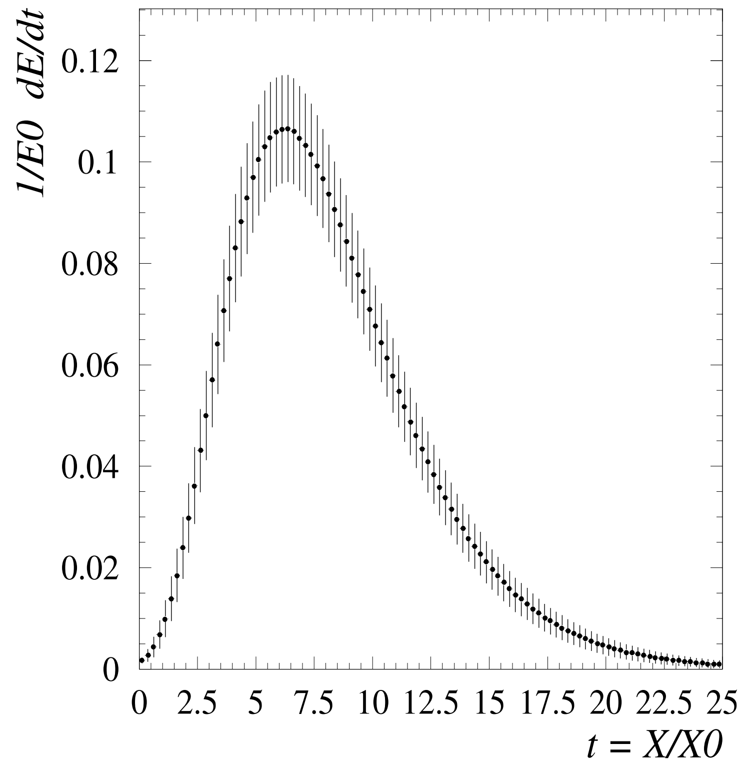
Simulation and gamma-fit of the longitudinal profile [Melo99]



left: e^- 1 GeV, 10 GeV, 100 GeV, 1 Tev in PbWO4

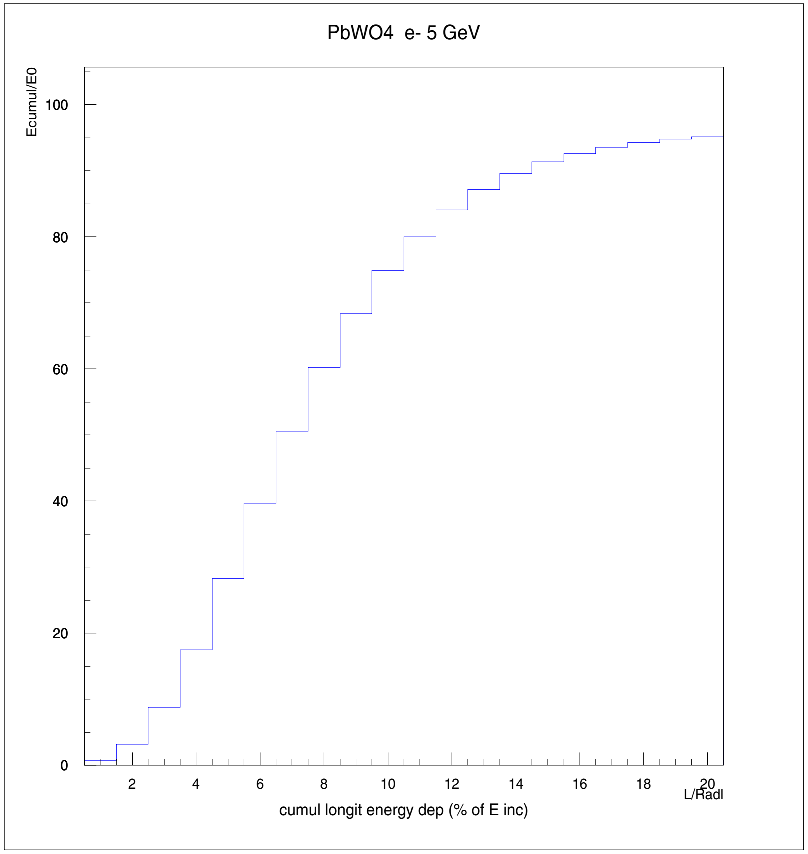
right: e^- 10 GeV; profile and its intrinsic fluctuations

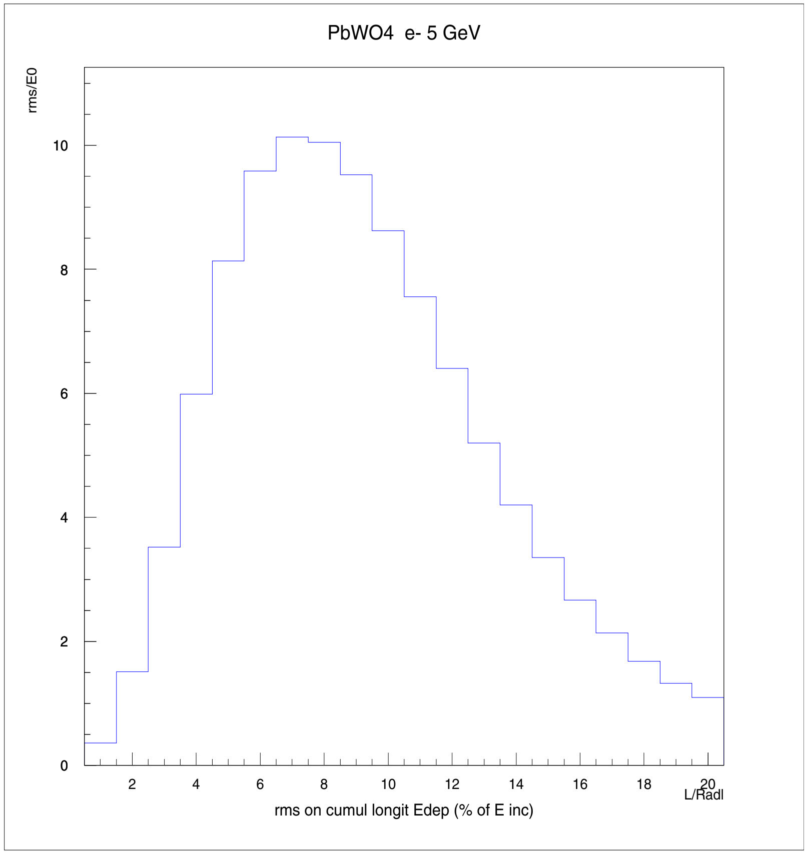




e^- 5 GeV in PbWO4

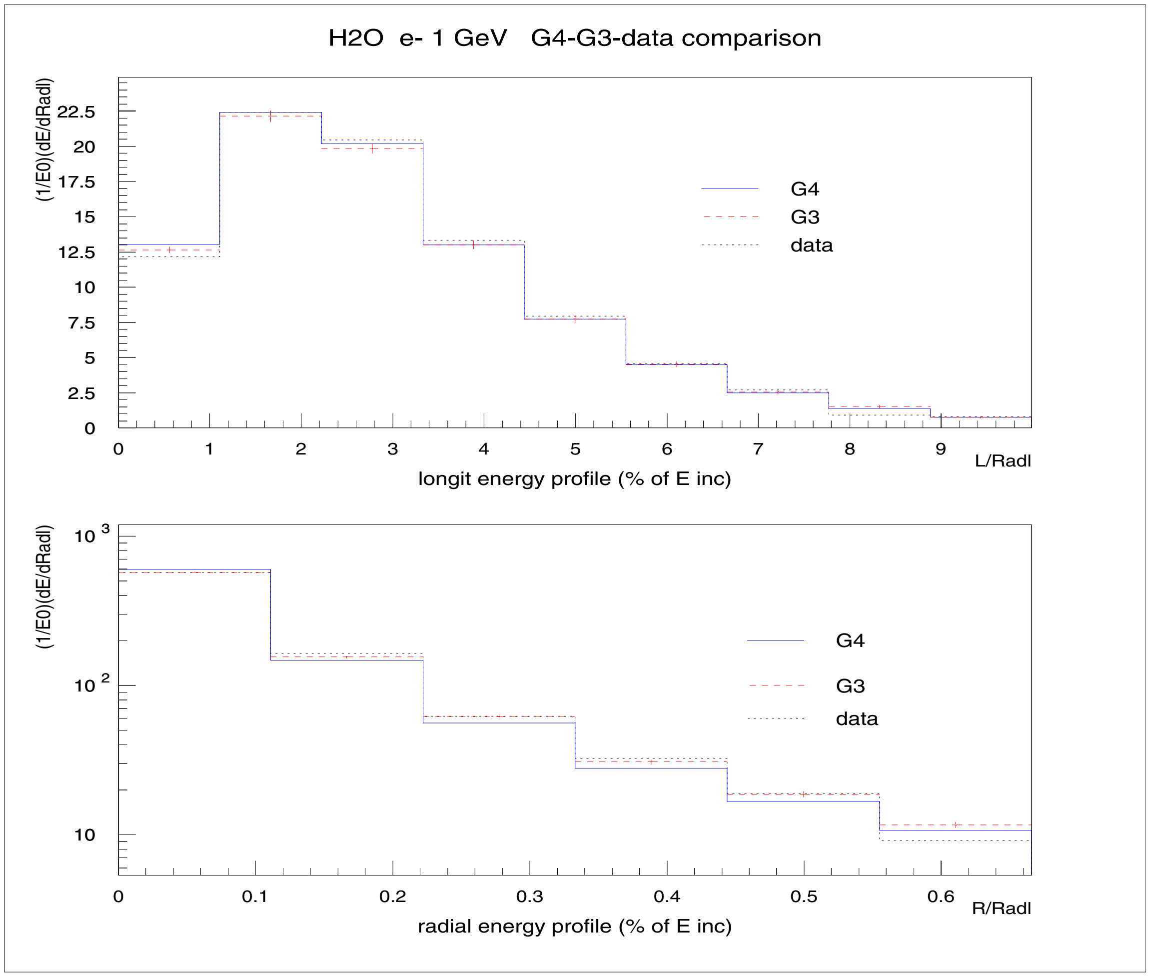
cumulative longitudinal profile and its intrinsic fluctuations ($\longrightarrow$ leakage)





e^- 1 GeV in Water

data [Cran69] and simulation of the longitudinal profile



radial profile in homogeneous material

The lateral spread of an electromagnetic shower is mainly caused by multiple scattering.

The Molière radius is defined :

$$R_m = \frac{21\ \text{MeV}}{E_c}\ X_0$$

On average, 90% of the shower energy is contained in a (semi infinite) cylinder of radius R_m, in any material.

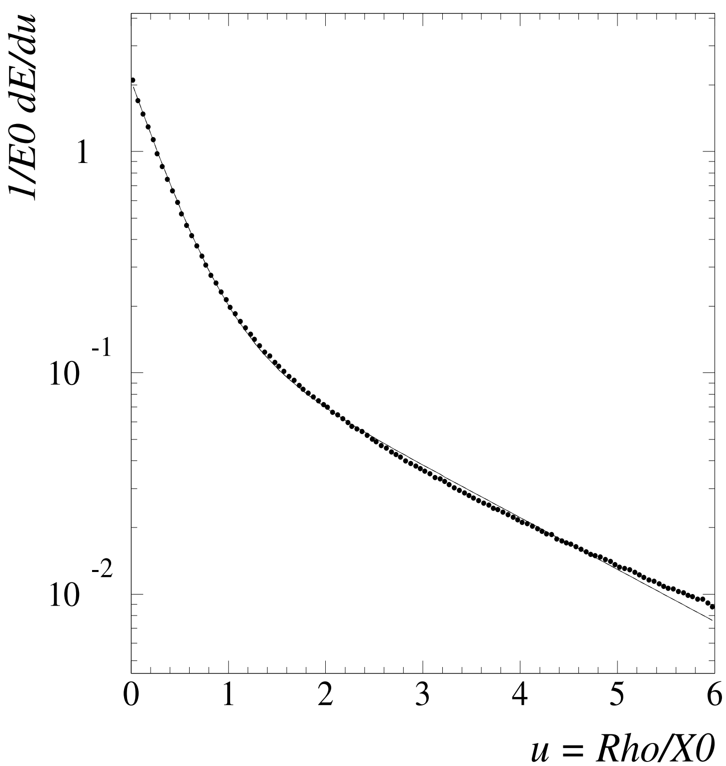
This radial distribution is well represented by the sum of two gaussian.

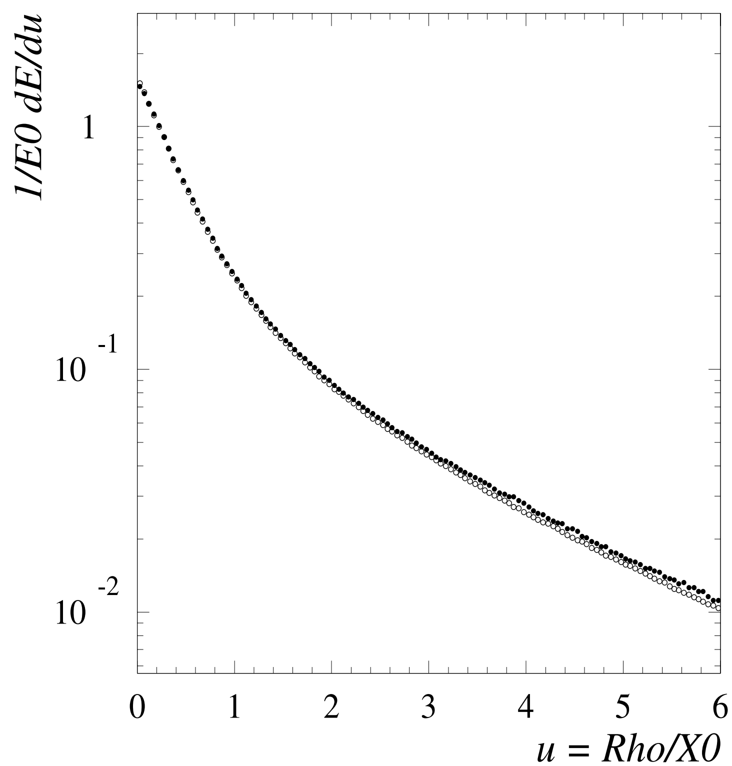
The distribution is nearly independent of the incident energy E_0

radial profile

left: e^- 10 GeV in PbWO4, simulation and two-gaussian fit.

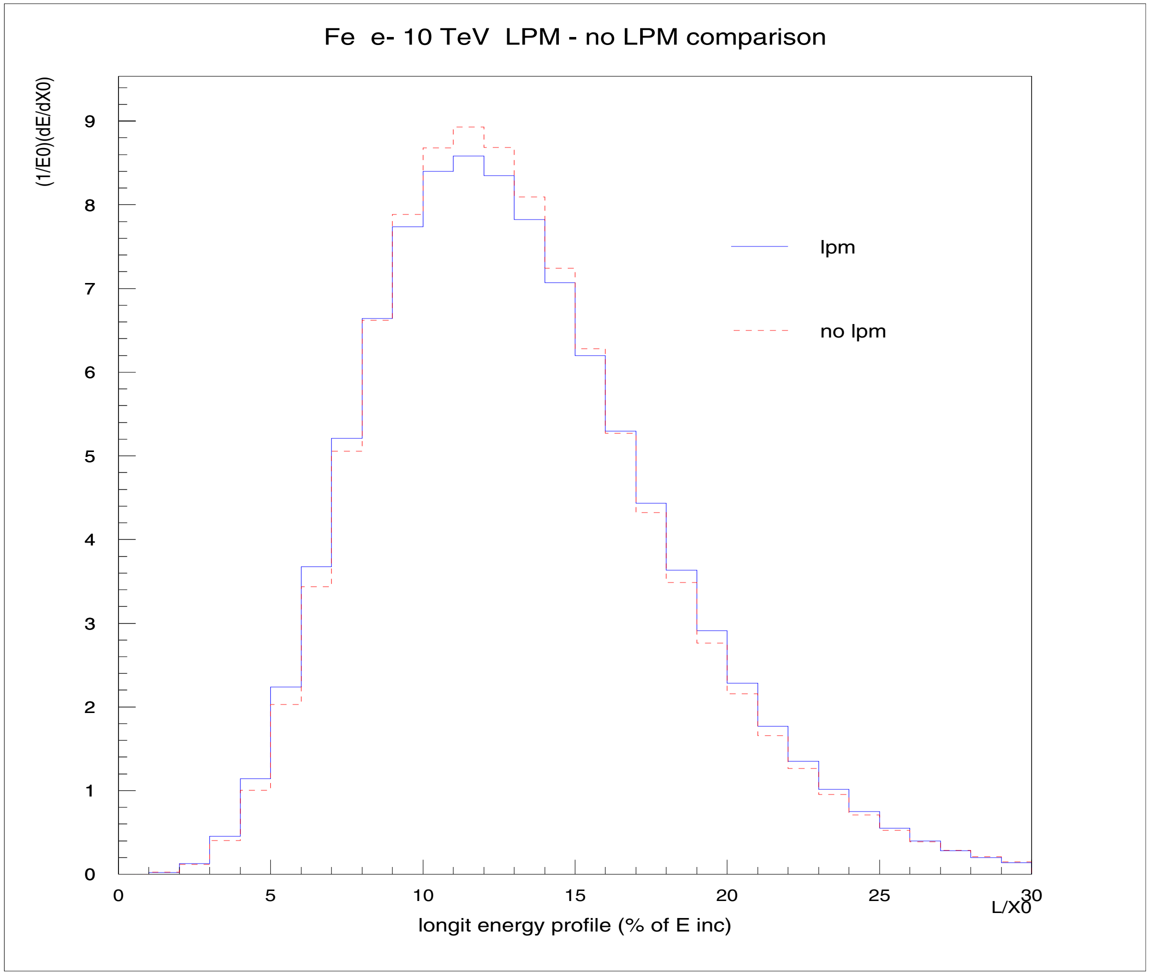
right: 1 GeV and 1 TeV profiles





At high enough energies, the LPM effect can cause signifiant elongation of electromagnetic cascades ...

apparently, not yet at 10 TeV ...



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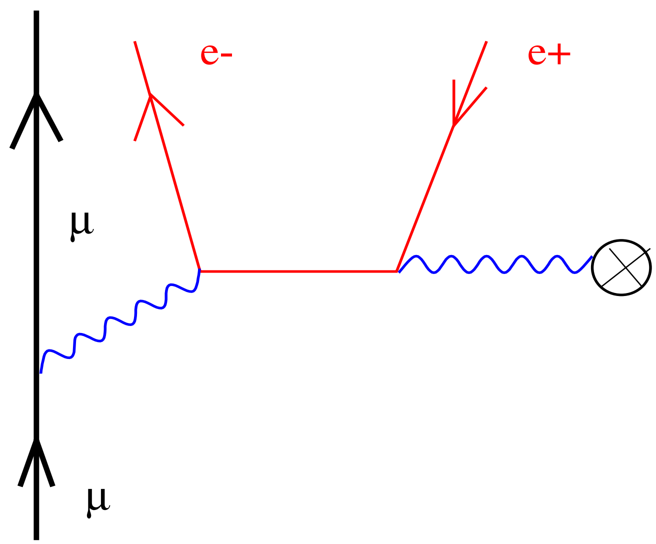
Direct (e^+, e^-) pair creation by muon

Direct (e^+, e^-) pair creation by muon

Creation of a (e^+, e^-) pair by virtual photon in the Coulomb field of the nucleus (for momentum conservation).

$$\mu + \text{nucleus} \longrightarrow \mu + e^+ + e^- + \text{nucleus}$$



It is one of the most important processes of muon interaction.

At TeV muon energies, pair creation cross section exceeds those of other muon interaction processes in a wide region of energy transfers :

$$100 \text{ MeV} \leq \epsilon \leq 0.1 \; E_\mu$$

Average energy loss for pair production increases linearly with muon energy, and in TeV region this process contributes over 50 % to the total energy loss rate.

energy transfers

Main contribution to the total cross section is given by transferred energies:

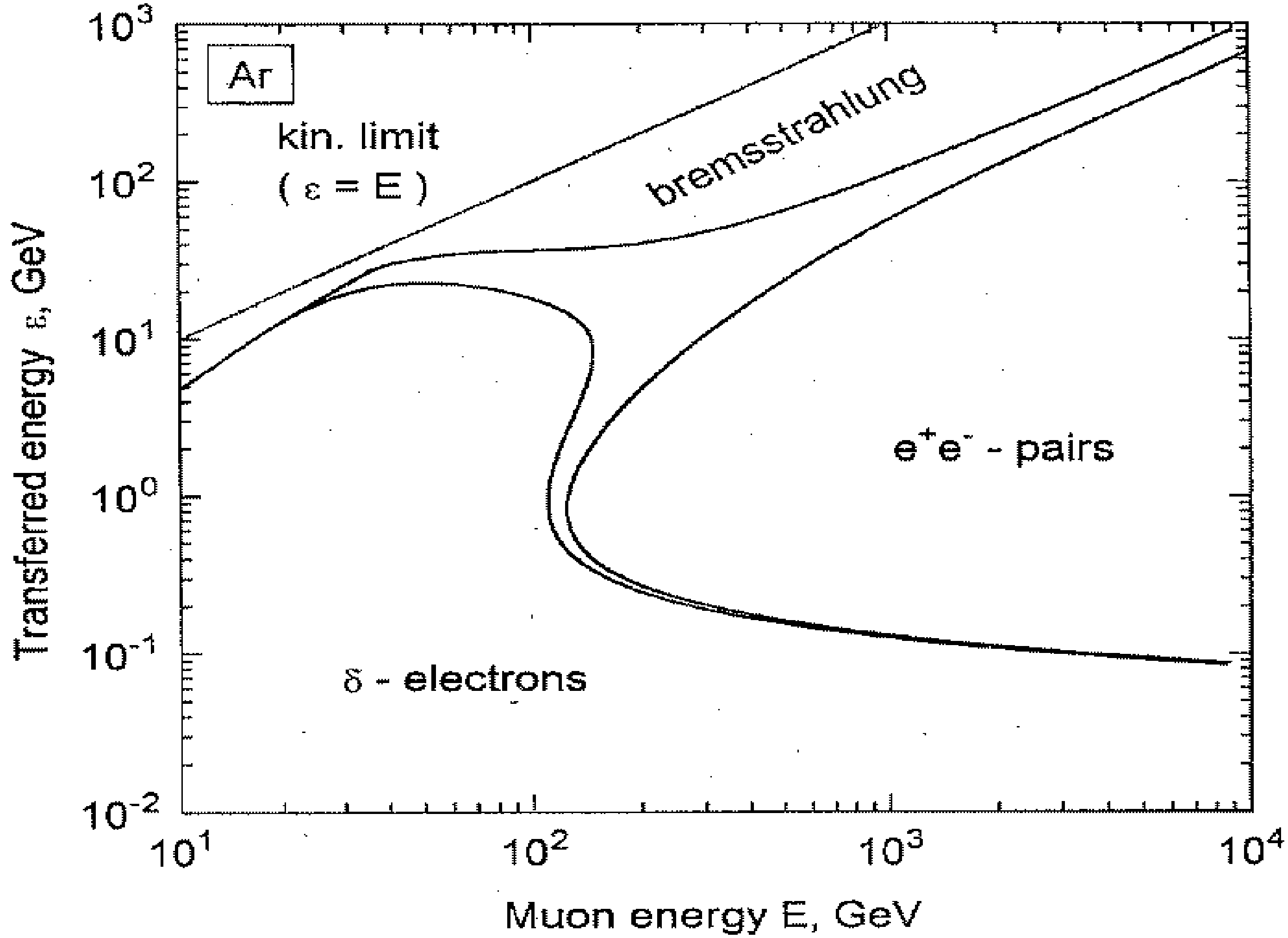
$$5\text{MeV} \leq \epsilon \leq 0.01\ E_\mu$$

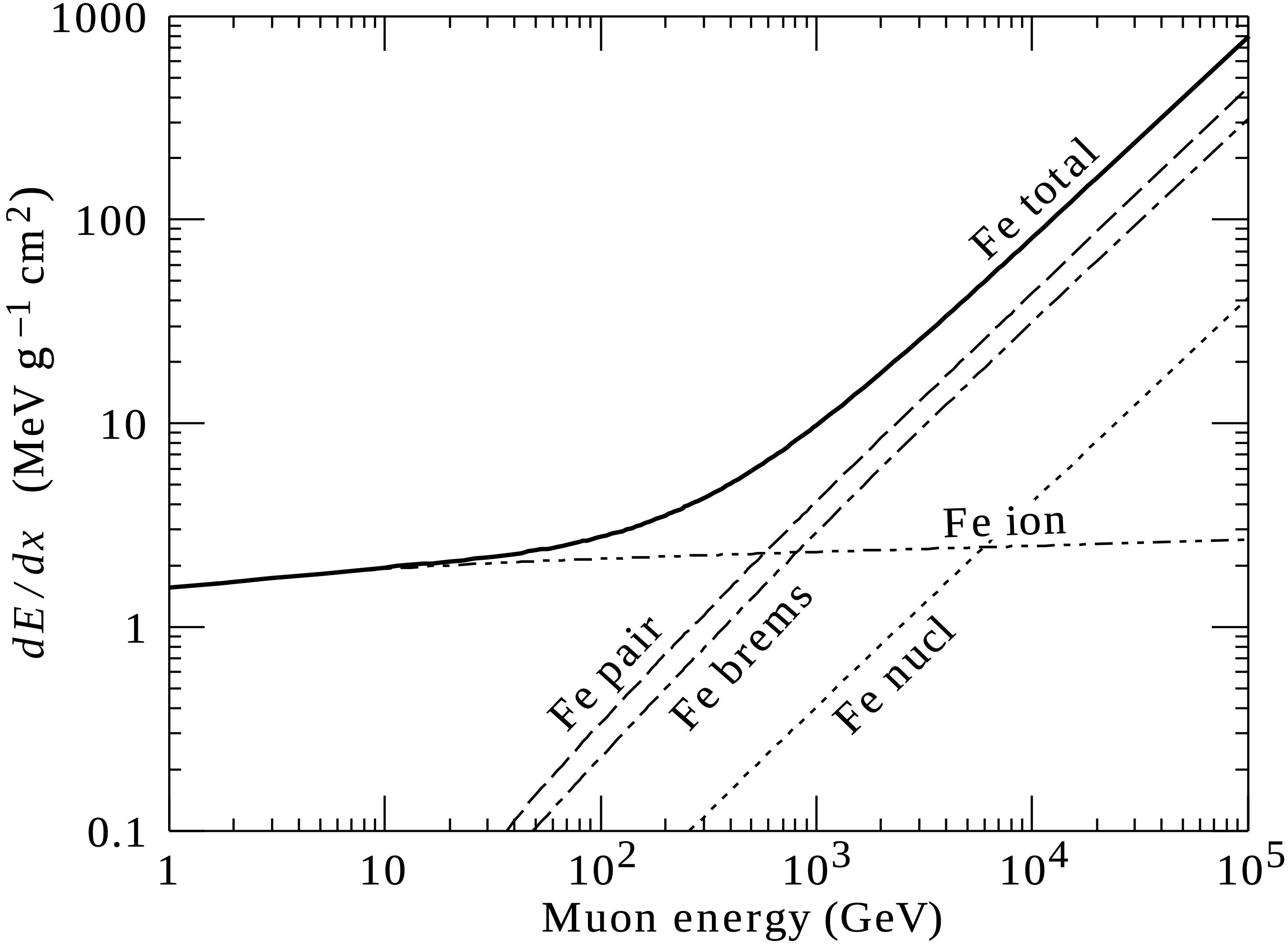
The contribution to average muon energy loss is determined mostly by region:

$$10^{-3}\ E_\mu \leq \epsilon \leq 0.1\ E_\mu$$

Thus, to adequatly describe the number of pairs produced, average energy loss and stochastic energy loss distribution one need to reproduce with a sufficient accuracy the differential cross section behaviour in a wide range of energy transfers:

$$5\text{MeV} \leq \epsilon \leq 0.1\ E_\mu$$


Ar
kin. limit
(ε = E)
bremsstrahlung
e⁺e⁻ - pairs
δ - electrons
Transferred energy ε, GeV
Muon energy E, GeV
10¹
10²
10³
10⁴
10⁻²
10⁻¹
10⁰


1000
100
10
1
0.1
1
10
10^2
10^3
10^4
10^5
dE/dx (MeV g^{-1} cm^2)
Muon energy (GeV)
Fe total
Fe ion
Fe pair
Fe brems
Fe nucl

differential cross section

The differential cross section is given by Kokoulin et al. [Koko71].
It includes :

- screening of the field of the nucleus
- correction for finite nuclear size
- contribution from the atomic electrons [Keln97]
- ...

See [Koko71] for a complete discussion.

differential cross section

The differential cross section per atom can be written as :

$$\frac{d\sigma}{d\epsilon} = \frac{4\,\alpha^2\, r_e^2}{3\pi}\,\frac{1-v}{\epsilon}\,[Z(Z+\zeta)]\,F(Z,E,\epsilon) \tag{1}$$

with

$$F(Z,E,\epsilon) = \int_0^{\rho_{max}} \left[\Phi_e(v,\rho) + \left(\frac{m_e}{m_\mu}\right)^2\,\Phi_\mu(v,\rho)\right]\,d\rho \tag{2}$$

where

$$\begin{aligned}
\epsilon &= \epsilon^+ + \epsilon^- && = \text{total energy of the created pair;}\\
v &= \epsilon/E \\
\rho &= (\epsilon^+ - \epsilon^-)/\epsilon && = \text{asymmetry coefficient;}
\end{aligned}$$

The functions Φ_e, Φ_μ, ζ can be found in [Koko00].

limits

$$\begin{aligned}
\epsilon_{min} &= 4 m_e c^2 \\
\epsilon_{max} &= E - \frac{3\sqrt{e}}{4}\, m_\mu c^2\ Z^{1/3} \\
\rho_{min} &= 0 \\
\rho_{max} &= \left[1 - \frac{6(m_\mu c^2)^2}{E(E-\epsilon)}\right] \sqrt{1 - \frac{\epsilon_{min}}{\epsilon}}
\end{aligned}$$

Energetic pairs and truncated energy loss rate

One may wish to take into account separately the high-energy pairs emitted above a given threshold ϵ_{cut} (miss detection, explicit simulation ...).

Those pairs must be excluded from the mean energy loss count.

$$-\frac{dE}{dx}\Big]_{\epsilon<\epsilon_{cut}} = n_{at}\int_{\epsilon_{min}}^{\epsilon_{cut}} \epsilon\,\frac{d\sigma}{d\epsilon}\,d\epsilon \tag{3}$$

n_{at} is the number of atoms per volume.

Then, the truncated total cross-section for emitting 'hard' pairs is:

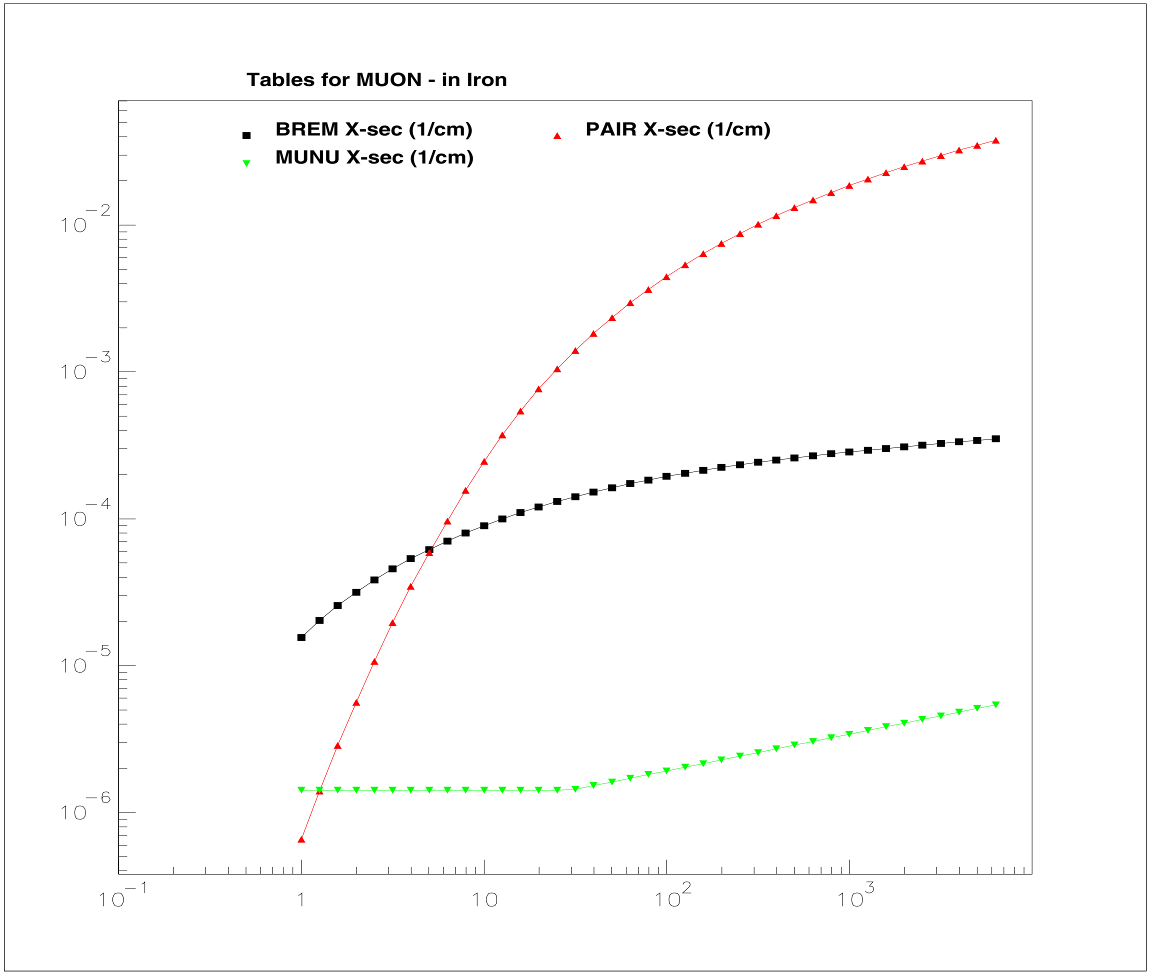
$$\sigma(E, \epsilon_{cut} \le \epsilon \le \epsilon_{max}) = \int_{\epsilon_{cut}}^{\epsilon_{max}} \frac{d\sigma}{d\epsilon}\,d\epsilon \tag{4}$$

The muon deflection angle is of the order of:

$$\theta = \frac{mc^2}{E}$$

Above ~1000 TeV the LPM suppression mechanism may have an effect.

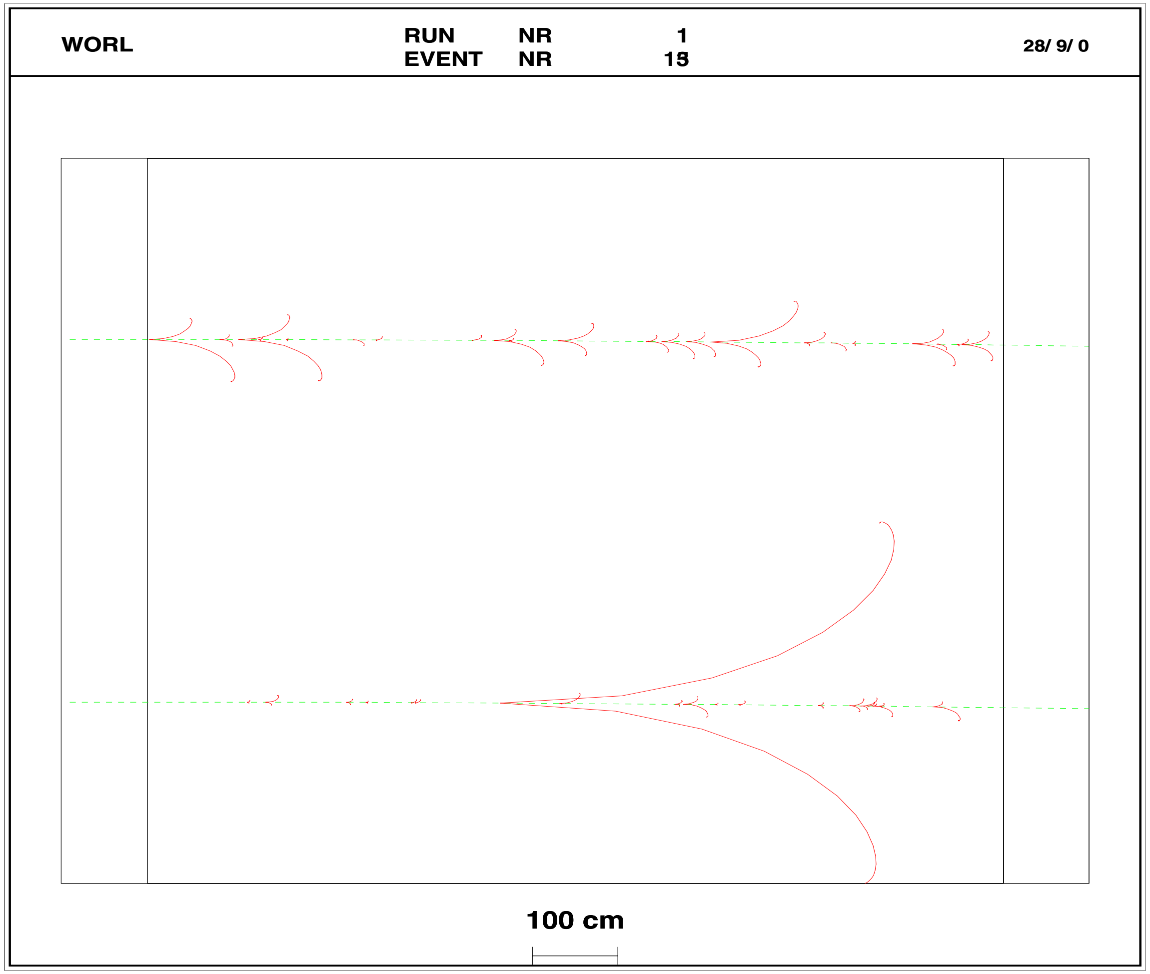
number of interactions per cm in Iron. (cut 100 MeV)



muon kinetic energy (GeV)

1 TeV muon in 10 meter of Fe (field 5 tesla).

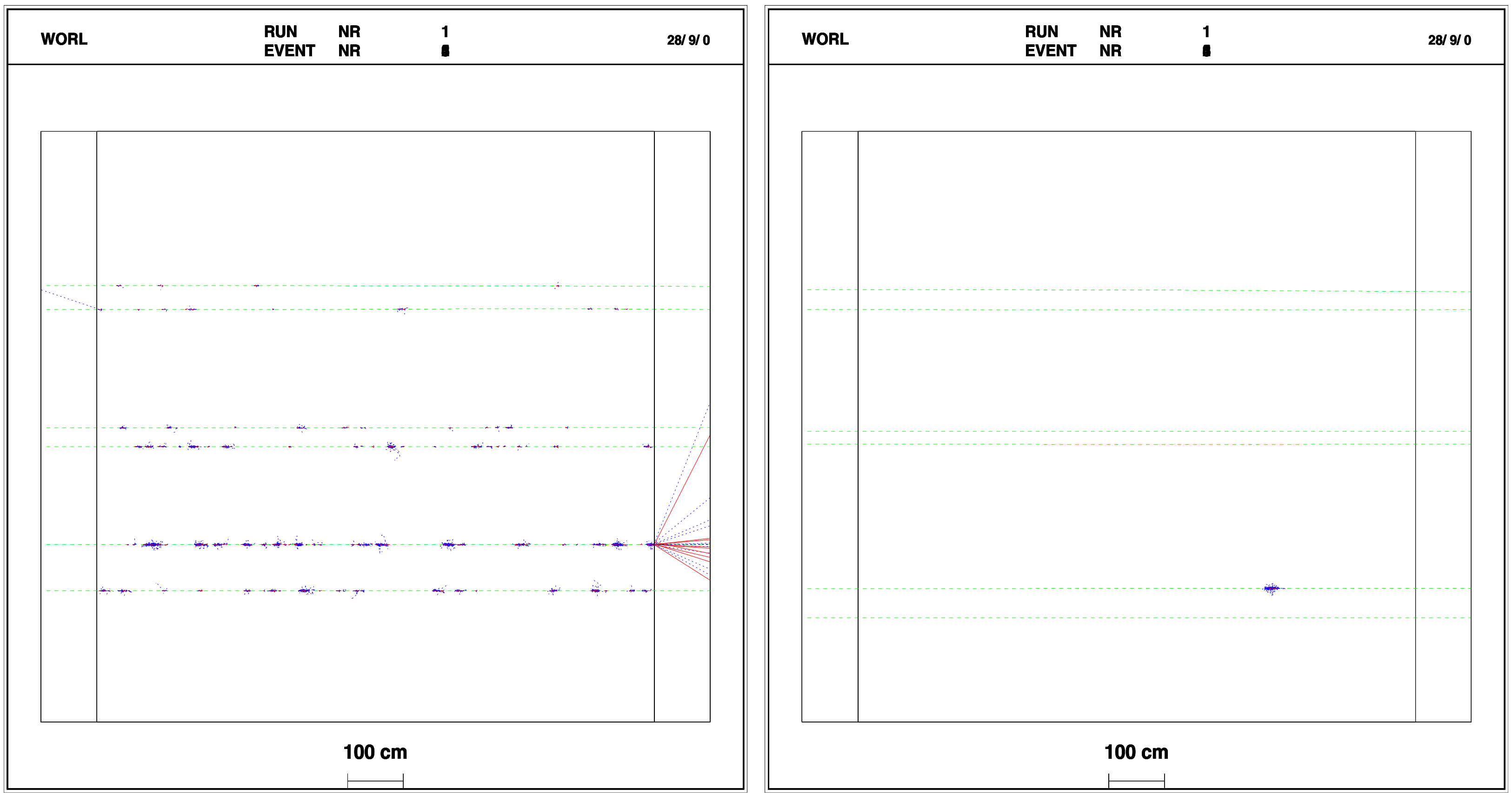
direct pair creation only



10 meter of Fe : muons 100 GeV, 1 TeV, 5 TeV.

right : direct pair creation + brems

left : brems only



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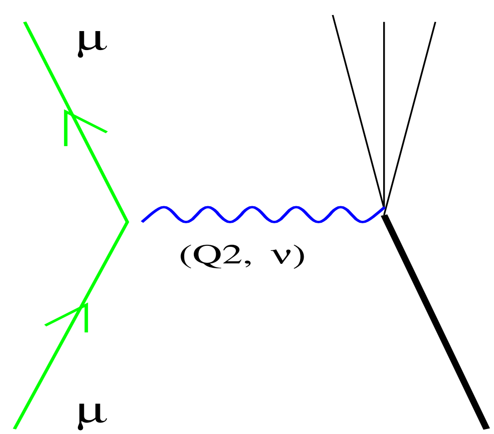
Muon photonuclear interaction

Muon photonuclear interaction

The inelastic interaction of muons with nuclei is important for high muon energies $E \geq 10$ TeV, at relatively high energy transfers $\nu/E \geq 10^{-2}$, in particular, in light materials, from view–point of the detector response for high energy muons, muon propagation and muon–induced hadronic background.



dE/dx : Average energy loss for this process is almost lineary increasing with energy, and at TeV muon energies constitutes about 10% in standard rock.

differential cross section : The main contribution to the cross section $d\sigma/d\nu$ and energy loss is given by low Q^2–region :

$$Q^2 \ll 1\ \mathrm{GeV}^2$$

Most widely used are the expressions given by Borog and Petrukhin [BOR75] and Bezrukov and Bugaev [BEZ81]. Results of these authors agree within 10% for differential cross section and within about 5% for the average energy loss (if the same photonuclear cross section $\sigma_{\gamma N}$ is used in calculations).

Theoretical estimates show that inelastic muon scattering gives, along with multiple Coulomb scattering, appreciable contribution to muon deflection (and dominates at large angles).

see [Koko00] for a review of Borog and Petrukhin cross section.

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