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Spherical Aberrations of a Gaussian Beam Produced by Mirrors and Lenses

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1 Introduction

It is well known that interfaces between media, with spherical shape, produces aberrations on a beam. This kind of aberration are called "spherical".

Usually, in the case of a "small" gaussian beam transmitted trough a "long" focal length optic, this effect can be neglected.

In the case of VIRGO, where 2 cm waist beam is considered, this effect becomes to be not negligible.

Providing parabolic interfaces between media, no spherical aberration is produced, because of the perfect matching of the gaussian beam wavefront with interface surface.

The aim of this paper, is to estimate the percentage of power lost by a TEM_{00} gaussian beam emerging from an aligned optical element with spherical surfaces (i.e. lenses and mirrors).

Three methods for the TEM_{00} losses calculation are here presented, The first method is based on the phase shift introduced by the optical element [4]; the second one is an improvement of the first one [5]; the last one uses the Kirchhoff integral solution of the scalar wave equation [3] (in this case, no approximation is done).

This calculations have some importance for the definition of the detection bench telescope specifications for VIRGO-2000.

In all cases the beam will be assumed to be perfectly aligned with the optic element axis, but these calculations can be easily extended to the more general case of a not aligned optics.

2 Basics

Let consider a normalized pure TEM_{00} $\Psi_{in} = \psi_{00}(R, w)$ impinging on an optical system (R and w are the curvature radius and the dimension of the beam). If Ψ_{out} is the emerging beam, the percentage $|c_{mn}|^2$ of TEM_{mn} for the emerging beam is

$$|c_{mn}|^2 = |\langle \psi_{mn}(R', w') \, | \Psi_{out} \rangle|^2 ,$$

where in this case R' and w' are respectively the curvature radius and the dimension of the beam after the optical element.

The Losses $\mathcal{L}$ are defined as the total amount of energy which is transmitted from the TEM_{00} beam into the other modes

$$\mathcal{L} = 1 - |\langle \psi_{00}(R', w') \, | \Psi_{out} \rangle|^2 \tag{1}$$

The problem to compute the losses is at the end, the determination of Ψ_{out}, R' and w' and the computation of the projection of Ψ_{out} on the TEM_{00} mode.

3 First Method, Phase Shift

Let consider the case of a beam propagating in the z direction. In this case , the optical element introduces a phase shift of the beam that depends on the transverse coordinate r. The optical system acts as an operator on the incoming beam, (the so called transmission function) so that the general expression of the out coming beam is

$$\Psi_{out} = e^{i\phi(r)} \left| \psi_{00}(R, w) \right\rangle .$$

and the $\mathcal{L}$ expression becomes

$$\mathcal{L} = 1 - \left| \langle (\psi_{00}(R', w')| \, e^{i\phi(r)} \left| \psi_{00}(R, w) \right\rangle \right|^2$$

3.1 Mirror

Considering at first a spherical mirror (see figure 1), the phase shift due to the optical path δ_{tot} is

$$\delta_{tot}(r) = 2\delta = 2\left(R_1 - \sqrt{R_1^2 - r^2}\right),$$

where R_1 is the curvature radius of the mirror and r is the distance from the propagation axis .

Considering the usual sign conventions for the curvature radiuses of geometrical optics, and if $r/R << 1$, the Taylor expansion of the square root term of the previous expression gives

$$\delta_{tot}(r) \simeq 2\left(-\frac{r^2}{4f} + \frac{r^4}{8R_1^3}\right).$$

where f is the mirror focal length

$$\frac{1}{f} = -\frac{2}{R}.$$

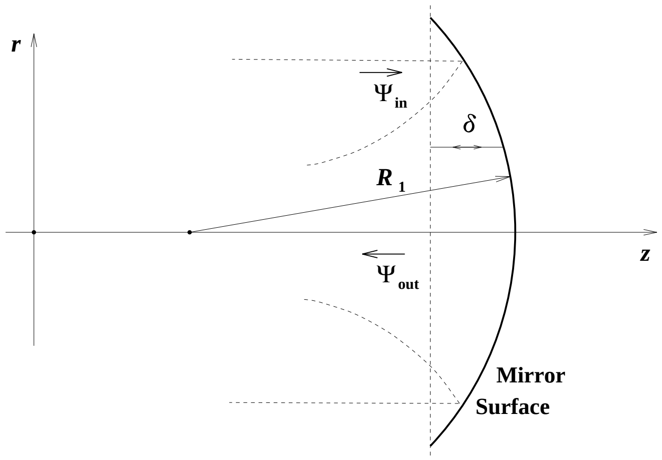

Figure 1: Optical path due to a mirror surface. ($R_1 < 0$)

Neglecting a constant phase shift term, the expression of the reflected beam becomes

$$\begin{aligned} \Psi_{out} &= e^{-ik\delta_{tot}} \left| \psi_{00}(R, w) \right\rangle & (2) \\ &= A_0 e^{-\frac{r^2}{w^2} - ik\frac{r^2}{2}\left(\frac{1}{R} - \frac{1}{f}\right) - ik\frac{r^4}{4R_1^3}} & (3) \\ &= e^{-ik\frac{r^4}{4R_1^3}} \left| \psi_{00}(R', w) \right\rangle , & (4) \end{aligned}$$

where the following relation for R' it has been used

$$\frac{1}{R'} = \frac{1}{R} - \frac{1}{f}.$$

In the case of the Fresnel approximation, we can neglect the term containing r^4 in the Ψ_{out} expression and therefore the emerging beam becomes a pure TEM_{00} with radius R' and waist $w' = w$

$$\Psi_{out} \simeq A_0 e^{-\frac{r^2}{w^2} - ikr^2\left(\frac{1}{R} - \frac{1}{f}\right)} = |\psi_{00}(R', w)\rangle\,.$$

3.2 Thin Lens

Using the same procedure done for the mirror, one obtains the optical path for a thin lens whose expression is

$$\delta_{tot}(r) \simeq -\frac{r^2}{2f} - \frac{r^4}{8}(n-1)\left(\frac{1}{R_1^3} - \frac{1}{R_2^3}\right),$$

where f is the lens focal length

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right),$$

n is the refraction index of the lens, R_1 and R_2 the radiuses of curvature of the two lens surfaces

The emerging beam becomes

$$\begin{aligned}
\Psi_{out} &= e^{-ik\delta_{tot}}\,|\psi_{00}(R, w)\rangle & (5)\\
&= A_0 e^{-\frac{r^2}{w^2} - ik\frac{r^2}{2}\left(\frac{1}{R} - \frac{1}{f}\right) + ik(n-1)\frac{r^4}{8}\left(\frac{1}{R_1^3} - \frac{1}{R_2^3}\right)} & (6)\\
&= e^{+ik(n-1)\frac{r^4}{8}\left(\frac{1}{R_1^3} - \frac{1}{R_2^3}\right)}\,|\psi_{00}(R', w)\rangle\,, & (7)
\end{aligned}$$

It turns out that in this case, we can minimize the losses, minimizing the following term

$$g(R_1, R_2) = \frac{1}{R_1^3} - \frac{1}{R_2^3}.$$

For a given focal length one can express R_2 as a function of R_1 and f and then $g(R_1, R_2)$ as a function of R_1 only.

One obtains a minimun value from $g\left[R_1, R_2(R_1, f)\right]$, if the following condition is fulfilled

$$R_2 = -R_1,$$

which corresponds to a biconvex lens with the same curvature radius.

Figure 2 shows the behavior of $g(R_1)$ as a function of R_1, for a focal length of $f = 1$ and a refraction index $n = 1.5$.

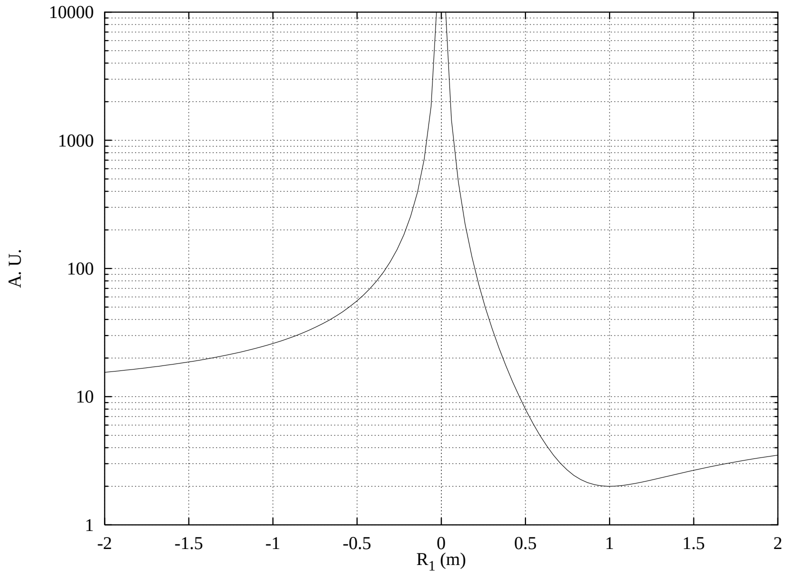

Figure 2: "Losses term" $g(R)$, as function of the lens curvature radius, for a fixed focal length.

3.3 Losses Calculation

Starting from the eq. (1) and with some algebraic manipulations, one can obtain an expression of the TEM_{00}

$$\mathcal{L}(\epsilon) = 1 - 2\left|\int_0^\infty d\zeta\,\zeta\,e^{-\zeta^2 - i\epsilon\zeta^4}\right|^2,$$

where ϵ is a real variable that depends on the optical systems considered.

For the mirror case we have

$$\epsilon = k\frac{w_0^4}{16R_1^3},$$

and for the thin lens with curvature radiuses R_1, R_2,

$$\epsilon = -kw_0^4\frac{(n-1)}{32}\left(\frac{1}{R_1^3} - \frac{1}{R_2^3}\right).$$

To obtain some numerical results, the following values (VIRGO-2000) has been used

$$\begin{aligned} n &= 1.5, \\ w_0 &= 1.984\cdot 10^{-2}\ m. \\ R &= \infty \end{aligned}$$

For the numerical integration of $\mathcal{L}(\epsilon)$, it has been used Maple V package.

The curves in figure 3 computed considering a spherical mirror, a biconvex symmetric lens and a plane-convex lens, shows the expected losses if one changes the focal length.

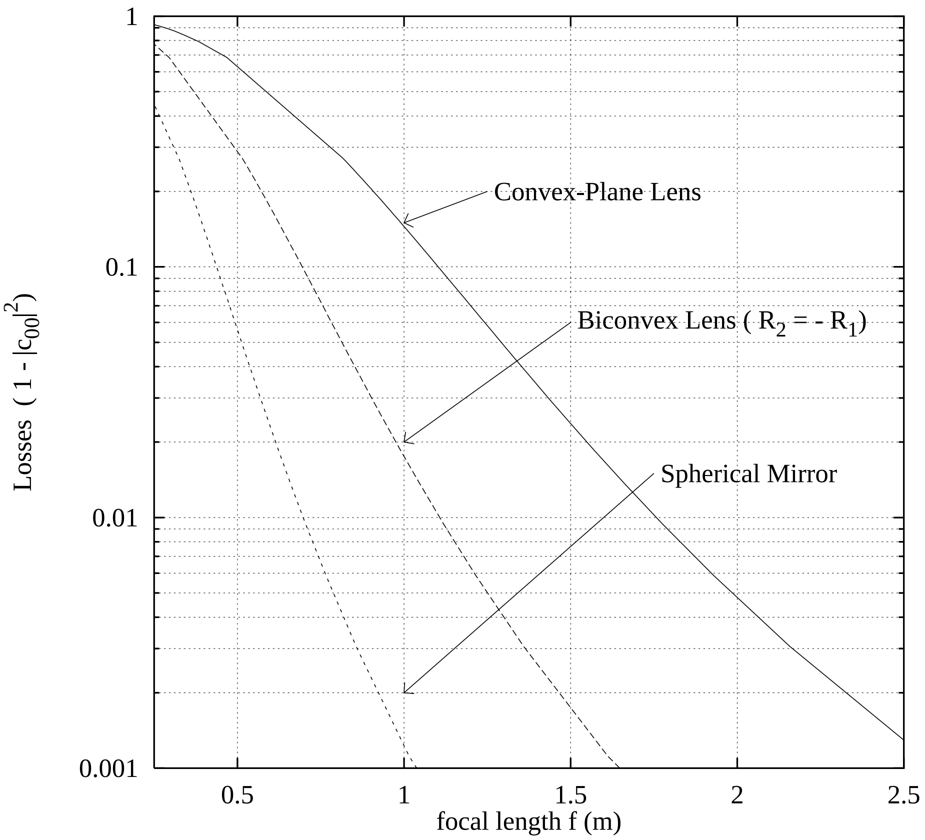

Figure 3: TEM_{00} losses versus focal length.

The following table contains the minimum focal lengths that can be used if we want losses around 0.1% and 1%, for the 3 considered cases.

Losses (%)	Spherical Mirror f (m)	Bi-Convex Lens $(R_2 = -R_1)$ f (m)	Convex-Plane Lens f (m)
1	0.7	1.1	1.8
0.1	1.0	1.7	2.2

These result does not agree with the prediction of the geometrical optics in the third order approximation necessary to depict the spherical aberration [2].

Spherical aberration are measured using two parameters :

- the distance between the focus f and the "focus" f_i of the ray coming outside the paraxial region (Longitudinal Spherical Aberration).

- the distance between the the focus f and intersection of the ray coming outside the paraxial region with the focal plane (Transverse Spherical Aberration).

The results of geometrical optics are summarized in figure 4, for the case of a plane wave getting through three possible kind of lenses.

It is reasonable to expect that the losses are smaller in the case of minimum spherical aberration which corresponds to the convex-plane lens.

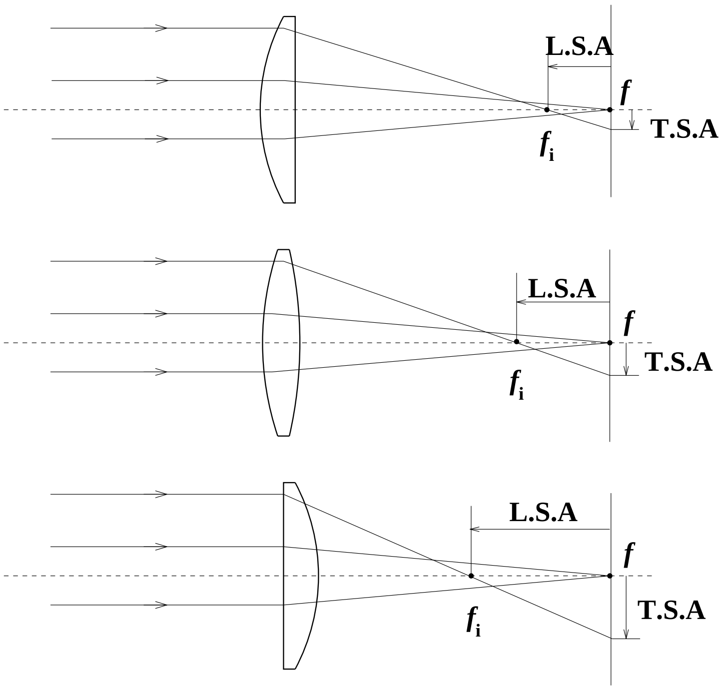

Figure 4: transverse (T.S.A) and longitudinal (L.S.A) spherical aberration for a plane wave getting through three kind of converging lenses.

The previous method doesn't distinguish between a plane-concave lens to a convex-plane one, and the minimum losses is achieved for the bincovex-lens with the same curvature radius.

4 Second Method, OPD

An attempt to improve the approximation done with the previous method, is to consider the exact optical path according to the Snell-laws [4]. In figure 5 the mirror case is shown. Respect to the previous method, one consider the optical phase shift that due to the path $\delta_1 + \delta_2$

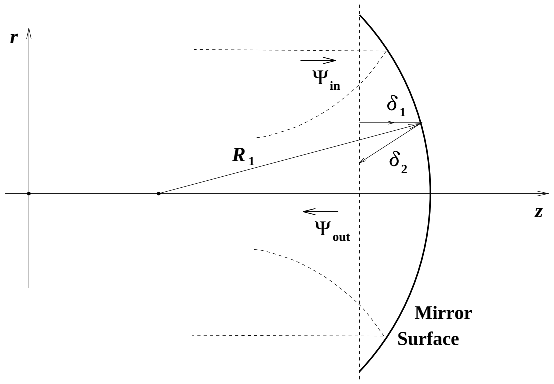

Figure 5: .

Using the previous notations, the explicit the total path expression will be

$$\delta_{tot} = \sqrt{1 - \frac{r^2}{R_1^2}} \left(1 + \frac{1}{1 - 2\frac{r^2}{R_1^2}} \right)$$

The results obtained with this new approximation, gives results very different compared to the previous method for the spherical mirror. Figure 6 shows the differences between the two methods.

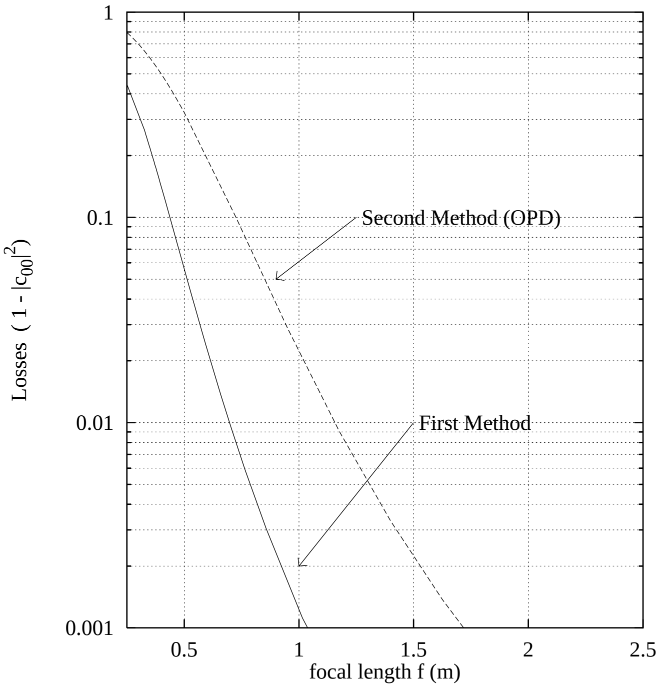

Figure 6: .

In this case a disagreement with geometrical optics has been found also [4].

A more rigorous approach is necessary for clarify the right way to compute correctly the out coming beam and the losses.

5 Kirchhoff Integral Method

Kirchhoff formula is the integral solution of the Helmholtz equation, which is the wave equation form in the scalar wave approximation, for a monochromatic wave [3].

Kirchhoff integral formula, allows to calculate the field in each point of the space knowing the field distribution on a closed surface enclosing the point.

Considering the mirror surface or the lens surface as the surface on which we known the field, we can compute respectively the reflected and the emerging beam.

5.1 Mirror

For the spherical mirror (see figure 7), the Kirchhoff formula that gives the reflected field Ψ_{out} at the point P, is the following surface integral

$$\Psi_{out}(P) = \frac{1}{4\pi}\int_{Mirror\ \ Surface}\left[\Psi_{in}(Q)\frac{\partial}{\partial\hat{n}}\left(\frac{e^{-ik\rho}}{\rho}\right) - \frac{e^{-ik\rho}}{\rho}\frac{\partial}{\partial\hat{n}}\Psi_{in}(Q)\right] dS,$$

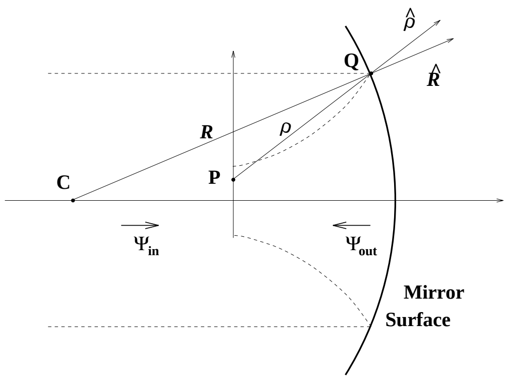

Figure 7: Optical path for the Kirchhoff integral for the mirror case.

where $\partial/\partial\hat{n}$ is the derivative in the direction of the unit vector perpendicular to the mirror surface , Q is a point on the mirror , ρ is the optical path between P and Q and

$$\frac{e^{-ik\rho}}{\rho},$$

is a spherical wave, a particular solution of the Helmholtz equation.

5.2 Thick Lens

For the thick lens, the Kirchhoff integral formula assumes the form (see figure 8)

$$\Psi_{out}(P) = \frac{1}{4\pi}\int_{Lens\ \ Left\ \ Surface}\left[\Psi_{in}(Q)\frac{\partial}{\partial\hat{n}}\left(\frac{e^{-ik(\rho_1+n\rho_2)}}{\rho_1+n\rho_2}\right) - \frac{e^{-ik(\rho_1+n\rho_2)}}{\rho_1+n\rho_2}\frac{\partial}{\partial\hat{n}}\Psi_{in}(Q)\right] dS,$$

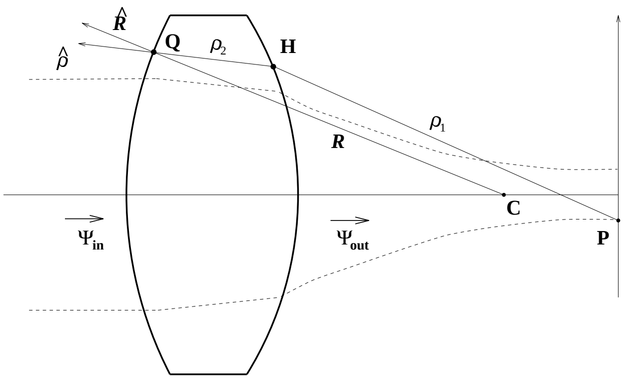

Figure 8: Optical path for the Kirchhoff integral for the lens case.

where n is the lens refraction index. In this case the particular solution is

$$\frac{e^{-ik(\rho_1+n\rho_2)}}{\rho_1+n\rho_2},$$

where the total optical path $\rho_1 + n\rho_2$ from the point P to Q is computed considering the refraction Snell law.

5.3 Numerical Results

In figure 9 are reported the numerical results obtained for the the three considered cases.

The computation was done using the same input parameters used for the phase-shift method.

Results obtained with the Kirchhoff method are in agree with the geometrical optics predictions :

- The convex-plane lens (the less aberrating one for the geometrical optics) produces less TEM_{00} losses.

- the biconvex lens with the same curvature radiuses produces more losses than the concave-plane lens and less loss than the plane-convex one.

- The plane-concave lens (the worst one for the aberration) produces more losses compared with the other cases.

It is interesting to notice that the results obtained for the mirror, using the first method (phase shift) that does not consider the reflection angle, are very closer with those calculated with the Kirchhoff method.

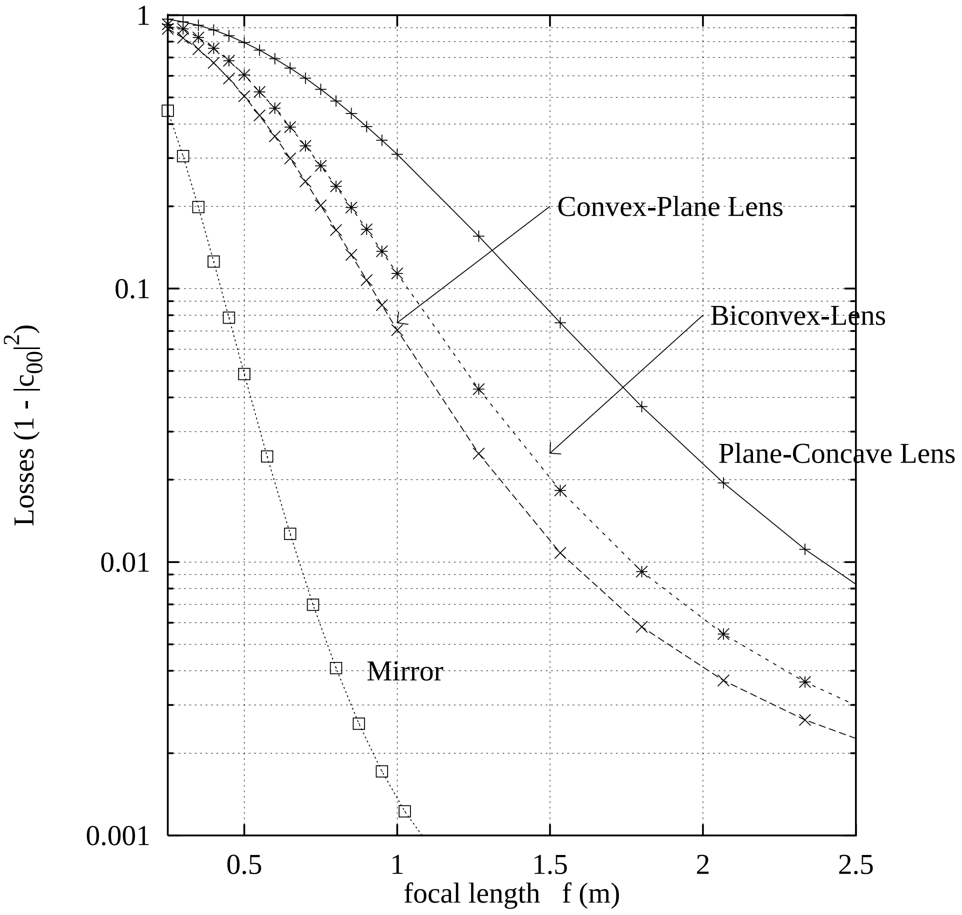

Figure 9: TEM_{00} losses computed using the Kirchhoff integral solution.

6 Conclusions

- For the evaluation of TEM_{00} losses due to the spherical aberrations of the lens case , it's necessary to use the Kirchhoff integral method.

- The phase-shift method is a good one for the spherical mirror case due to the fact that it gives results very near to the Kirchhoff method.

- The second method (OPD) gives results not in agreement with Kirchhoff method in the spherical mirror case and with the parameters used ,and it over-estimates the losses.

- The specifications of the detection bench telescope imposes losses for the TEM_{00} of $\sim .1\%$. This means that there are some restriction for the first optical element which belongs to the telescope which are

    - convex-plane lens with focal length $f \geq 2\ m$,

    - mirrors with focal length $f \geq 1\ m$.

  Of course, no particular restriction has to be applied for the subsequents optical elements, if the beam dimensions remain less than that of the telescope input.

References

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