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Pulsar signal losses due to their spinning down

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1 Introduction

Pulsars are often described as very stable clock. In fact, the period of their rotation and therefore the period of the electromagnetic and gravitational waves (which in the case of gravitational waves is half the usual period) drifts very slowly. This drift will limit the maximum integration time of a pulsar unless we correct for it. Of course if we search for an unknown pulsar (blind search) we will have to guess this correction and this will increase the already large combinatory problem of this kind of search [1].

In this note, we will use the 600 known pulsars [2] to evaluate the pulsar spin down consequences for a blind signal search.

2 Expected signal from the pulsar

The pulsar signal will be a periodic signal with the frequency slowly drifting. The period is well described by $P(t) = P(t_0) + \dot{P}t$. Figure 1 gives the P and $\dot{P}$ measured values for the known pulsars. As expected, the $\dot{P}$ values are very small.

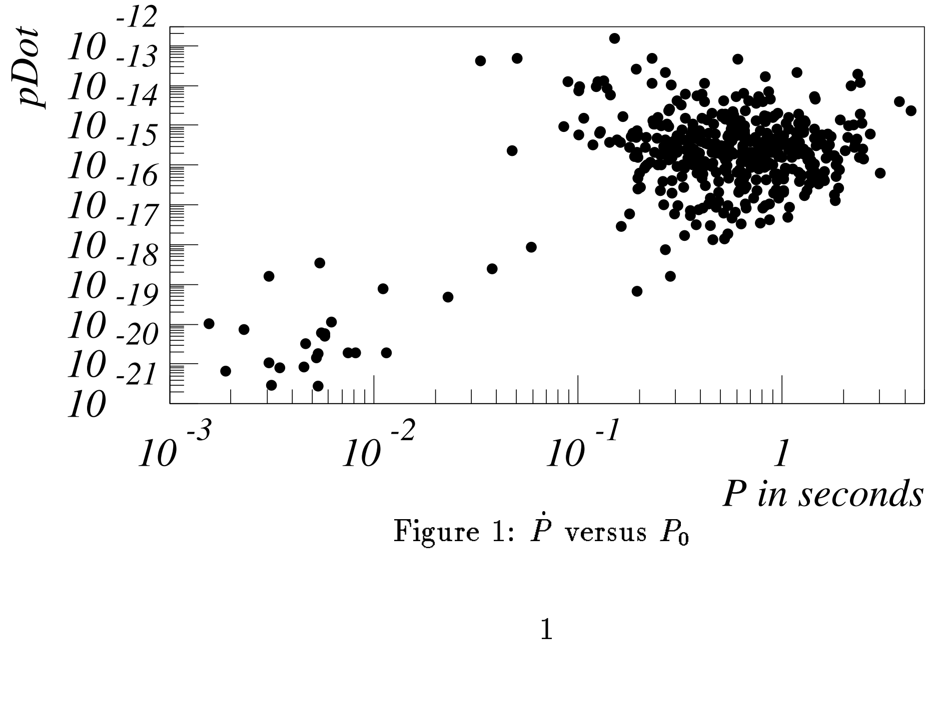

Figure 1: $\dot{P}$ versus P_0

The phase of the signal is:

$$\phi(t) = \int d\phi = \int \frac{2\pi}{P_0 + \dot{P}t} dt$$

As we can see from figure 1, $\dot{P}t \ll P_0$ for an integration time t up to several years ($10^8 sec$). Therefore we can do the following approximation :

$$\begin{aligned} \phi(t) &= \int \frac{2\pi}{P_0} dt - \int \frac{2\pi \dot{P}}{P_0^2} t\, dt \\ &= \frac{2\pi}{P_0} t - \frac{\pi \dot{P}}{P_0^2} t^2 dt \end{aligned}$$

The signal losses are coming from the quadratic term. Taking into account the factor two between gravitational and electromagnetic waves and since the mean value for $\dot{P}$ is $1.8 * 10^{-14}$ and for P_0 is 0.7, the quadratic term reach 2π with a time of integration about $2.15 * 10^6 sec$ (25 days).

3 Signal losses

We will now find a more accurate estimation of the signal losses. Some of the calculations here are similar to those in [1].

Let $\hat{f}_e(P)$ be the value of the Fourier Transform with the period P computed with the quadratic term and $\hat{f}(P)$ without the quadratic term. When the frequency of the signal changes, the maximum value in the Fourier Transform is observed at the mean frequency value. Therefore we will compute the Fourier Transform for $P = P_0$ and we will integrate the signal for a total time $T = 2nP$ with the time origin centred at the middle of the total integration time.

$$\begin{aligned} \hat{f}_e(P_0) &= \frac{2h_0}{T} \int_{-T/2}^{T/2} e^{i\frac{2\pi}{P_0}t} \sin(\phi) dt \\ &= \frac{2h_0}{T} \int_{-T/2}^{T/2} e^{i\frac{2\pi}{P_0}t} \sin(\frac{2\pi}{P_0}t - \frac{\pi \dot{P}}{P_0^2} t^2) dt \\ &= \frac{2h_0}{T} \sum_{j=-n}^{n-1} \int_{jP_0}^{(j+1)P_0} e^{i\frac{2\pi}{P_0}t} \sin(\frac{2\pi}{P_0}t - \frac{\pi \dot{P}}{P_0^2} t^2) dt \\ &= \frac{2h_0}{T} \sum_{j=-n}^{n-1} \int_{jP_0}^{(j+1)P_0} e^{i\frac{2\pi}{P_0}t} \sin(\frac{2\pi}{P_0}t) \cos(\frac{\pi \dot{P}}{P_0^2} t^2) dt - \int_{jP_0}^{(j+1)P_0} e^{i\frac{2\pi}{P_0}t} \cos(\frac{2\pi}{P_0}t) \sin(\frac{\pi \dot{P}}{P_0^2} t^2) dt \end{aligned}$$

Since $\dot{P}$ is very small, the term $\frac{\pi \dot{P}}{P_0^2} t^2$ is quasi constant during one period and we have :

$$\begin{aligned} \hat{f}_e(P_0) &= \frac{2h_0}{T} \sum_{j=-n}^{n-1} \left[\cos(\frac{\pi \dot{P}}{P_0^2}(jP_0)^2) \int_{jP_0}^{(j+1)P_0} e^{i\frac{2\pi}{P_0}t} \sin(\frac{2\pi}{P_0}t) dt \right. \\ &\quad \left. - \sin(\frac{\pi \dot{P}}{P_0^2}(jP_0)^2) \int_{jP_0}^{(j+1)P_0} e^{i\frac{2\pi}{P_0}t} \cos(\frac{2\pi}{P_0}t) dt \right] \\ &= \frac{2h_0}{T} \sum_{j=-n}^{n-1} \cos(\pi \dot{P} j^2) \frac{-P_0}{2i} - \sin(\pi \dot{P} j^2) \frac{P_0}{2} \end{aligned}$$

$$= \frac{-h_0}{2n} \sum_{j=-n}^{n-1} \sin(\pi \dot{P} j^2) - i\cos(\pi \dot{P} j^2)$$

By comparing the above expression to the perfect case with $\dot{P} = 0$, the fraction of signal losses L is:

$$L = 1 - \frac{1}{2n}\sqrt{\left[\sum_{i=-n}^{n-1} \cos(\pi \dot{P} i^2)\right]^2 + \left[\sum_{i=-n}^{n-1} \sin(\pi \dot{P} i^2)\right]^2}$$

If we define $\alpha = \pi \dot{P} n^2 = \pi \dot{P} (\frac{T}{2P_0})^2$ we have a formula which depend on only one parameter : α. Then the previous formula becomes :

$$L = 1 - \frac{1}{2n}\sqrt{\left[\sum_{i=-n}^{n-1} \cos(\alpha \frac{i^2}{n^2})\right]^2 + \left[\sum_{i=-n}^{n-1} \sin(\alpha \frac{i^2}{n^2})\right]^2}$$

In the case of small α, this formula could be approximate by :

$$L = \frac{2}{45}\alpha^2$$

The figure 2 shows the losses as a function of α. If we do not want losses larger than 5% we need an $\alpha \leq 1.068 rad$.

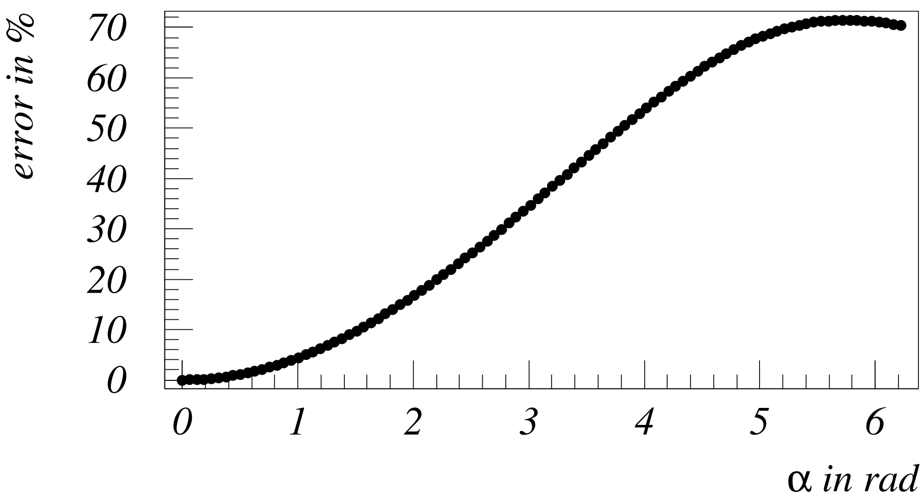

Figure 2: Evaluation of α

We can define a characteristic time of each pulsar defined as the integration time for a pulsar to reach 5% of losses.

$$\begin{aligned} \alpha &= 1.068 \\ \pi \dot{P} (\frac{T_{car}}{2P_0})^2 &= 1.068 \\ T_{car} &= 2P_0 \sqrt{\frac{1.068}{\pi \dot{P}}} \end{aligned}$$

The figure 3 shows the distribution of T_{car} for all the known pulsars. The mean value of this distribution is 234.3 days.

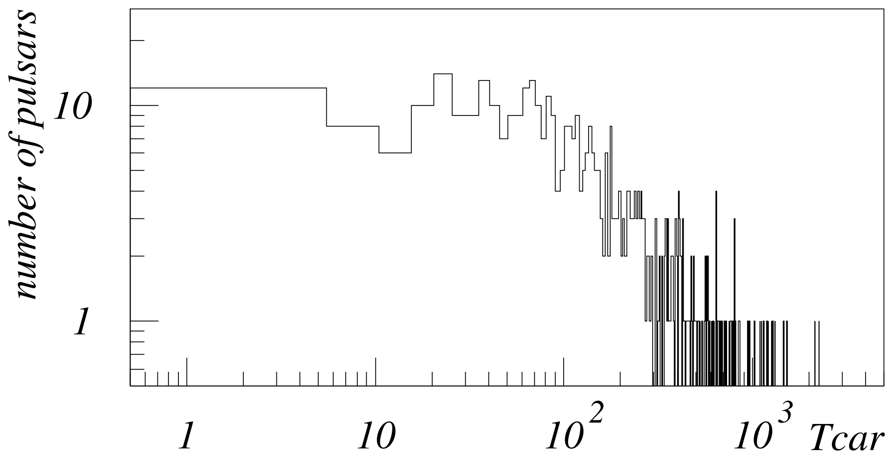

Figure 3: Distribution of T_{car} for known pulsars

The figure 4 shows the proportions of pulsars which have a T_{car} under given integration time. We can integrate up to a few weeks without many losses for most pulsars (only 10 % of the pulsars have reached the 5 % losses limit).

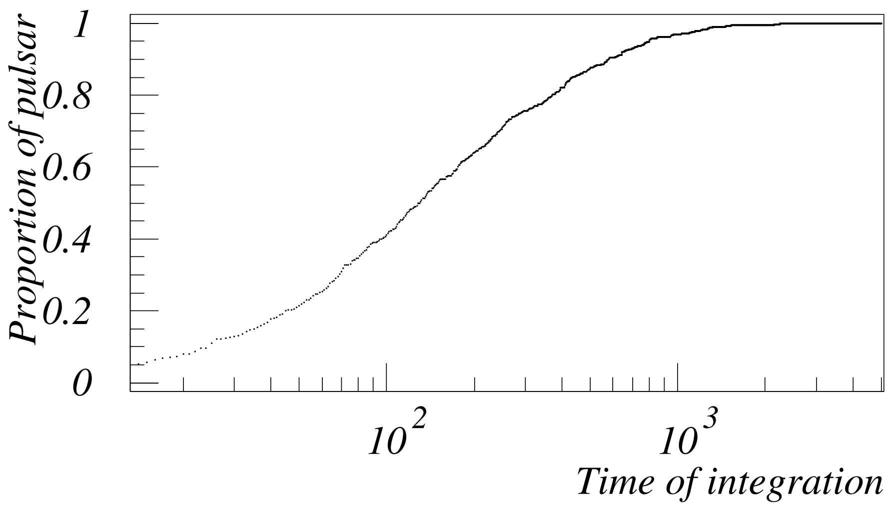

Figure 4: Proportion of pulsars affected by the spin down

It should be noticed that this result is not very sensitive to the level of losses since T_{car} varies as the square root of α and α varies as the square root of the losses (for small losses).

4 Conclusion

This study shows us that we can neglect the correction of the spin down of pulsars, as long as we integrate less than a few weeks. This is enough for the first step of the pulsar search, as explain in [1]. But when we have to validate the signal by integrating more than a few weeks we will have to correct the data for this effect.

In this case, it is possible to try to measure $\dot{P}$ by comparing the measured frequency for the beginning of the integration time to the value measured at the end of the integration time (if we split the total integration time T_i in two pieces, the frequency resolution is $\frac{2}{T_i}$ then the resolution on $\dot{P}$ is $4\frac{P^2}{T_i^2}$ and the corresponding T_{car} is of the order the integration time).

References

[1] *NTS95-40*
X.Grave, B.Mours, LAPP Annecy

[2] *Pulsar Catalogue*
J.H.Taylor, Physics Department, Princeton University
R.N.Manchester, Australia Telescope National Facility
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