VIRGO NTS96-36

Pulsar Amplitude modulation due to earth rotation

X.Grave B.Mours

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1 Introduction

The electro-magnetic waves emitted by a lonesome pulsar have a very well defined frequency. Which allow us to integrate a very long time. Models expect gravitational wave at the pulsar's frequency (ν) and at twice this frequency. As shown in [1] the metric tensor (in coordinate (x,y,z) with z along propagation axis, i the angle between the rotation axis and the direction of the earth) can be expressed as :

$$H_p = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

With i the angle between the rotation axis and the direction of the earth and γ the angle between the rotation axis of the pulsar and the deformation axis and $\Omega = 2\pi\nu$, we have :

$$\begin{aligned} h_+ &= h_0 \sin(\gamma) \left[\cos(\gamma) \frac{\sin(2i)}{4} \cos(\Omega t) - \sin(\gamma) \frac{1 + \cos^2(i)}{2} \cos(2\Omega t) \right] \\ h_\times &= h_0 \sin(\gamma) \left[\cos(\gamma) \frac{\sin(i)}{2} \sin(\Omega t) - \sin(\gamma) \cos(i) \sin(2\Omega t) \right] \end{aligned}$$

In this note we give the formula for the coordinate change from the pulsar frame to the detector frame, these is the new formula used in SIESTA (since version v2.10). Then we will compute the amplitude modulation due to the earth rotation and we will discus the consequences on the signal losses for pulsar search.

2 Coordinate change

Let's define α and δ the pulsar equatorial coordinates, ψ the polarization in the propagation frame. The position detector can be characterized in equatorial coordinates by the longitude $\alpha'(t)$ (which is the usual longitude + $\frac{2\pi}{86164}(t - t_0)$) and the latitude λ and by θ the angle between the west-east axis and the x-arm of the detector (For Virgo $\lambda = 43.67°$ and $\theta = 71°$). Let's call

H_d the metric tensor in the detector frame, H_p the metric tensor in the pulsar frame and $R(t)$ the rotation tensor which gives the changes from the propagation frame to the detector frame. So we have :

$$\begin{aligned} H_d(t) &= R(t) * H_p * R^{-1}(t) \\ &= R(t) * H_p * R^{\dagger}(t) \end{aligned}$$

To calculate the rotation matrix $R(t)$, we can break down $R(t)$ into five rotations matrix.

$$R(t) = R_5 R_4 R_3(t) R_2 R_1$$

Where :

$$\begin{aligned}
R_1 &= \begin{pmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
R_2 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\delta+\frac{\pi}{2}) & -\sin(\delta+\frac{\pi}{2}) \\ 0 & \sin(\delta+\frac{\pi}{2}) & \cos(\delta+\frac{\pi}{2}) \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\sin\delta & -\cos\delta \\ 0 & \cos\delta & -\sin\delta \end{pmatrix} \\
R_3(t) &= \begin{pmatrix} \cos(\alpha-\alpha'(t)) & -\sin(\alpha-\alpha'(t)) & 0 \\ \sin(\alpha-\alpha'(t)) & \cos(\alpha-\alpha'(t)) & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
R_4 &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\lambda-\frac{\pi}{2}) & \sin(\lambda-\frac{\pi}{2}) \\ 0 & -\sin(\lambda-\frac{\pi}{2}) & \cos(\lambda-\frac{\pi}{2}) \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin\lambda & -\cos\lambda \\ 0 & \cos\lambda & \sin\lambda \end{pmatrix} \\
R_5 &= \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{aligned}$$

In fact, the detector only 'see' the difference between two coefficients of H_d.

$$h(t) = H_{dxx}(t) - H_{dyy}(t)$$

Well we have :

$$\begin{aligned} H_{dxx} &= \left(R_{xx}^2 - R_{xy}^2\right) h_+ + 2 R_{xx} R_{xy} h_\times \\ H_{dyy} &= \left(R_{yx}^2 - R_{yy}^2\right) h_+ + 2 R_{yx} R_{yy} h_\times \end{aligned}$$

So in SIESTA we will only need to calculate four coefficients of the 3*3 matrix. Here is the list of all this coefficient :

$$
\begin{aligned}
R_{xx} &= R_{xx1}\cos(\alpha-\alpha'(t)) + R_{xx2}\sin(\alpha-\alpha'(t)) + R_{xx3} \\
R_{xy} &= R_{xy1}\cos(\alpha-\alpha'(t)) + R_{xy2}\sin(\alpha-\alpha'(t)) + R_{xy3} \\
R_{yx} &= R_{yx1}\cos(\alpha-\alpha'(t)) + R_{yx2}\sin(\alpha-\alpha'(t)) + R_{yx3} \\
R_{yy} &= R_{yy1}\cos(\alpha-\alpha'(t)) + R_{yy2}\sin(\alpha-\alpha'(t)) + R_{yy3}
\end{aligned}
$$

Where we have (computed with Maple):

$$
\begin{aligned}
R_{xx1} &= \cos\psi\cos\theta - \sin\psi\sin\delta\sin\theta\sin\lambda \\
R_{xx2} &= \cos\psi\sin\theta\sin\lambda + \sin\psi\sin\delta\sin\theta \\
R_{xx3} &= -\sin\psi\sin\theta\cos\lambda\cos\delta \\
R_{xy1} &= -\sin\psi\cos\theta - \cos\psi\sin\delta\sin\theta\sin\lambda \\
R_{xy2} &= -\sin\psi\sin\theta\sin\lambda + \cos\psi\sin\delta\cos\theta \\
R_{xy3} &= -\cos\psi\sin\theta\cos\lambda\cos\delta \\
R_{yx1} &= -\cos\psi\sin\theta - \sin\psi\sin\delta\cos\theta\sin\lambda \\
R_{yx2} &= \cos\psi\cos\theta\sin\lambda - \sin\psi\sin\delta\sin\theta \\
R_{yx3} &= -\sin\psi\cos\theta\cos\lambda\cos\delta \\
R_{yy1} &= \sin\psi\sin\theta - \cos\psi\sin\delta\cos\theta\sin\lambda \\
R_{yy2} &= -\sin\psi\cos\theta\sin\lambda - \cos\psi\sin\delta\sin\theta \\
R_{yy3} &= -\cos\psi\cos\theta\cos\lambda\cos\delta
\end{aligned}
$$

3 Amplitude modulation

The signal which is measured by VIRGO has the following form :

$$h(t) = H_{dxx} - H_{dyy}$$

Using the previous formula, we can write $h(t)$ in the pulsar case :

$$h(t) = A_1(t)\cos(\Omega t - \phi_1(t)) + A_2(t)\cos(2\Omega t - \phi_2(t))$$

With :

$$
\begin{aligned}
A_1(t) &= \sqrt{\left(R_{xx}^2 + R_{yy}^2 - R_{xy}^2 - R_{yx}^2\right)^2 h_{+1}^2 + 4\left(R_{xx}R_{xy} - R_{yy}R_{yx}\right)^2 h_{\times 1}^2} \\
A_2(t) &= \sqrt{\left(R_{xx}^2 + R_{yy}^2 - R_{xy}^2 - R_{yx}^2\right)^2 h_{+2}^2 + 4\left(R_{xx}R_{xy} - R_{yy}R_{yx}\right)^2 h_{\times 2}^2} \\
\cos(\phi_1(t)) &= \frac{(R_{xx}^2 + R_{yy}^2 - R_{xy}^2 - R_{yx}^2)h_{+1}}{A_1(t)}
\end{aligned}
$$

$$
\begin{aligned}
\sin(\phi_1(t)) &= \frac{2(R_{xx}R_{xy} - R_{yy}R_{yx})h_{\times 1}}{A_1(t)} \\
\cos(\phi_2(t)) &= \frac{(R_{xx}^2 + R_{yy}^2 - R_{xy}^2 - R_{yx}^2)h_{+2}}{A_2(t)} \\
\sin(\phi_2(t)) &= \frac{2(R_{xx}R_{xy} - R_{yy}R_{yx})h_{\times 2}}{A_2(t)}
\end{aligned}
$$

Where we have $h_+ = h_{+1}\cos(\Omega t) + h_{+2}\cos(2\Omega t)$ and $h_\times = h_{\times 1}\sin(\Omega t) + h_{\times 2}\sin(2\Omega t)$. A_1, A_2, ϕ_1 and ϕ_2 are only function of $\cos(\alpha - \alpha'(t))$ which means that their period is one day.

4 Effect in the case of the Crab Pulsar

The figure 1 shows $\frac{A_1(t)}{h_0}$ and $\frac{A_2(t)}{h_0}$ for the VIRGO detector and the Crab Pulsar ($\alpha = 5h34min$, $\delta = 22°01'$ and deformation angle $\gamma = 15°$ and $\psi = 0rad$, $i = 45°$). These results are exactly the same as in [1], except for a time translation and a scale factor because we didn't do any assumption on the distortion origin and in [1] there is the following assumption : the distortion is due to the magnetic field.

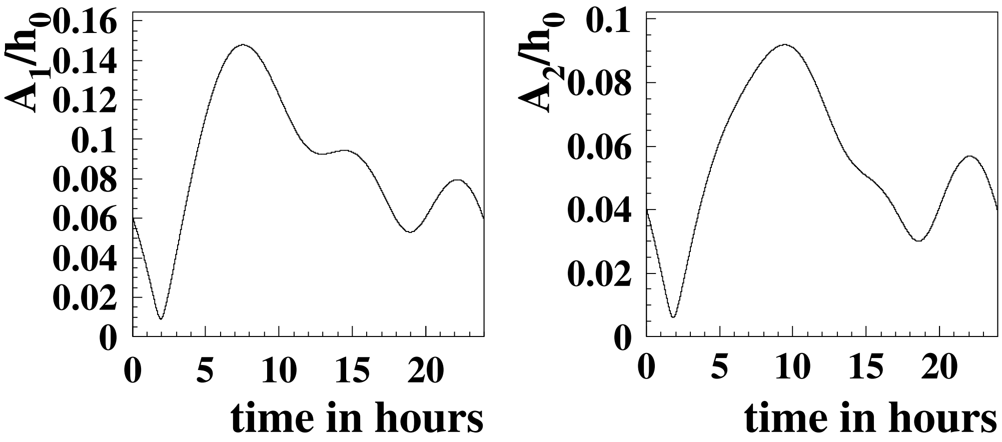

Figure 1: Coefficients of the amplitude modulation

The figure 2 shows the FFT of $\frac{A_1}{h_0}$ and $\frac{A_2}{h_0}$. After the continuous level, we can observe a first peak an order of magnitude below. This peak is at $1.1610^{-5}Hz$ which correspond at a period of one day. It will create side bands on the signal. The second peak after as twice the first peak frequency and a comparable level, it will create side bands too. All this will generate signal losses.

The figure 3 shows ϕ_1 and ϕ_2 also for the case of the Crab pulsar. The periodic phase variation as two components. First it increase or decrease by 2π every day because of the daily rotation of the detector around the direction of the pulsar. This will produce a small permanent frequency shift. Then, when we subtract this effect has shown by figure 4, we will have phase fluctuation which will induce some signal losses.

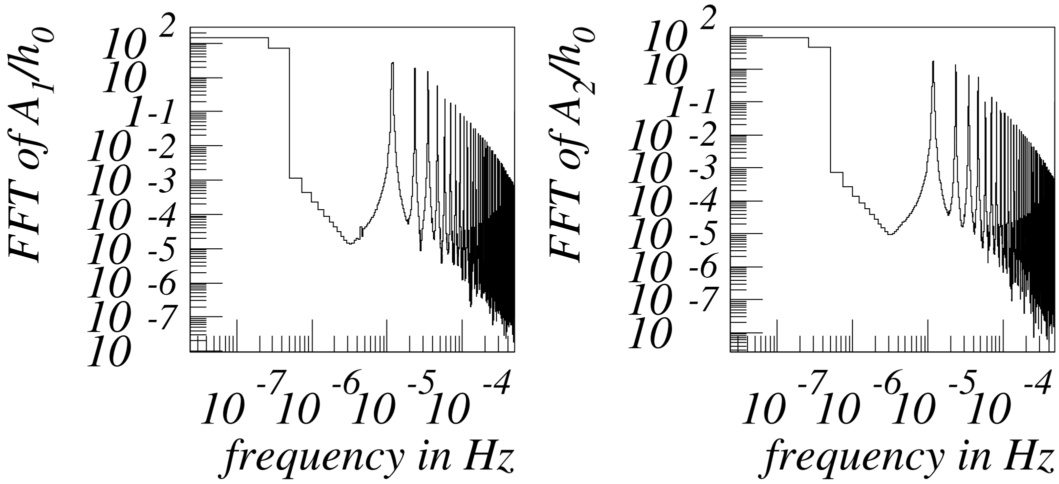

Figure 2: FFT of $\frac{A_1}{h_0}$ and $\frac{A_2}{h_0}$

5 Conclusion

In this note we derive the formula for the amplitude and phase modulation due to earth rotation. These formula have been implemented in SIESTA for a pulsar and for a binary coalescence. This effect will allow us to get the signature of an extraterrestrial signal if it isn't corrected but will reduce the signal available.

References

[1] astro-ph/9602107
S. Bonazzola and E. Gourgoulhon
Département d'Astrophysique Relativiste et de Cosmologie
Observatoire de Paris
Section de Meudon

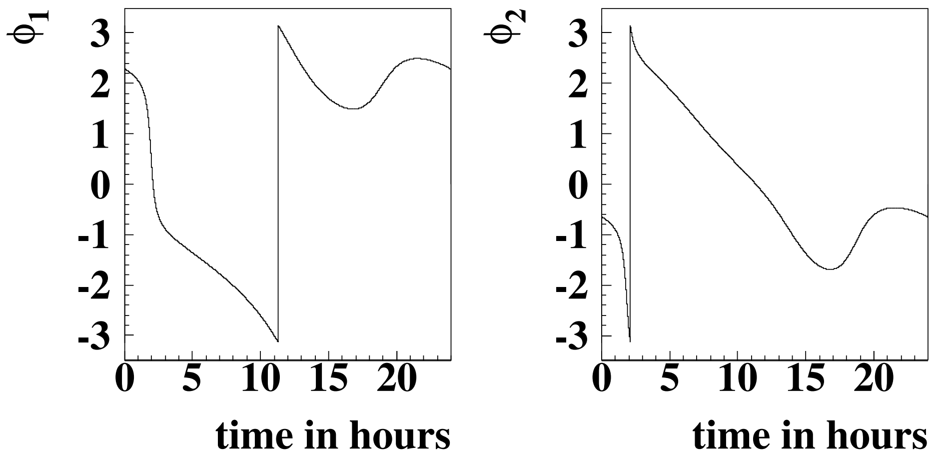

Figure 3: ϕ_1 and ϕ_2

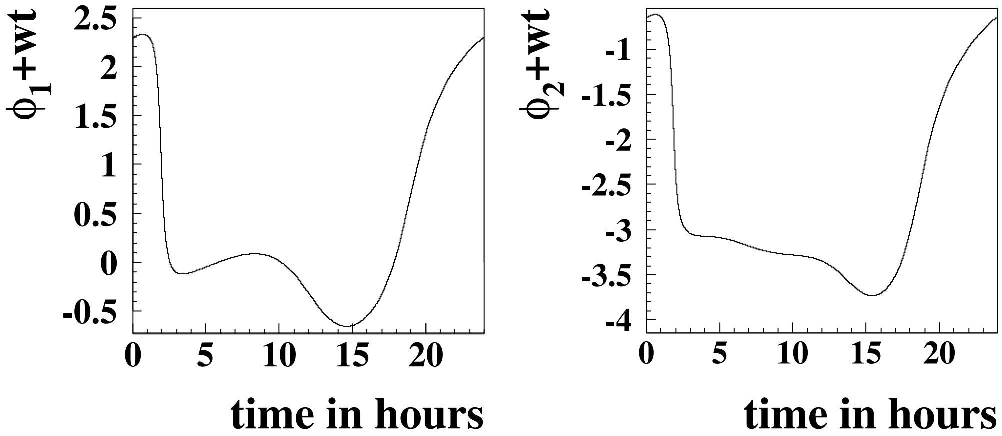

Figure 4: $\phi_1 + \frac{2\pi}{86164}t$ and $\phi_2 + \frac{2\pi}{86164}t$