

The Virgo detector

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GraSPA summer school

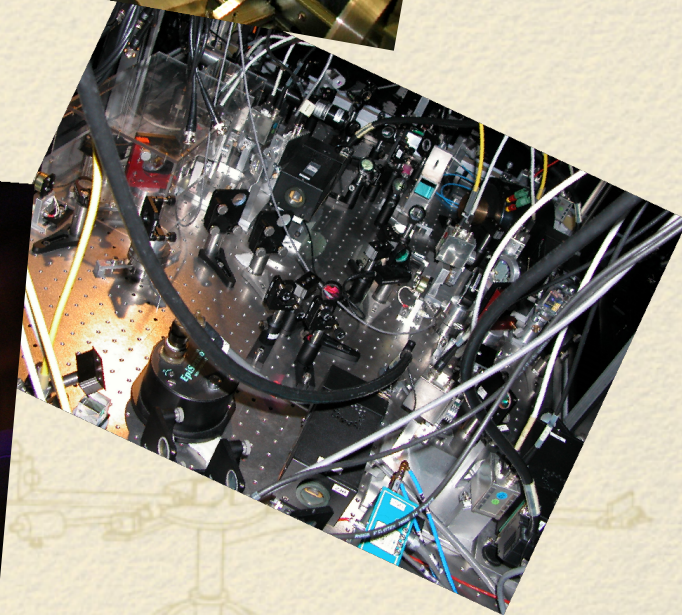
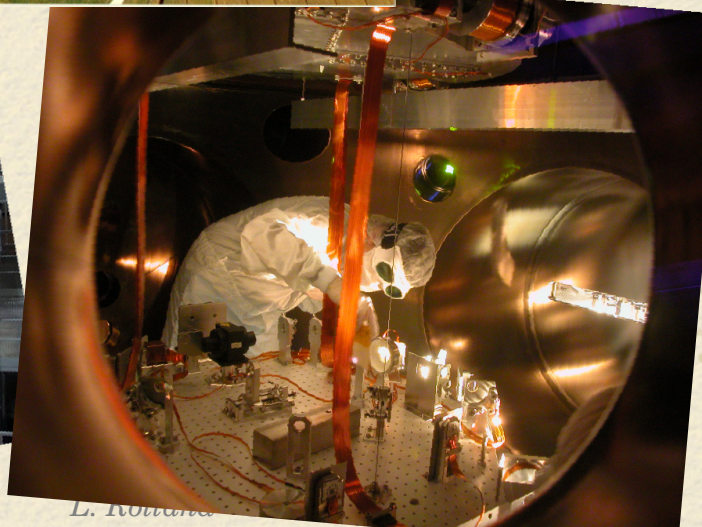


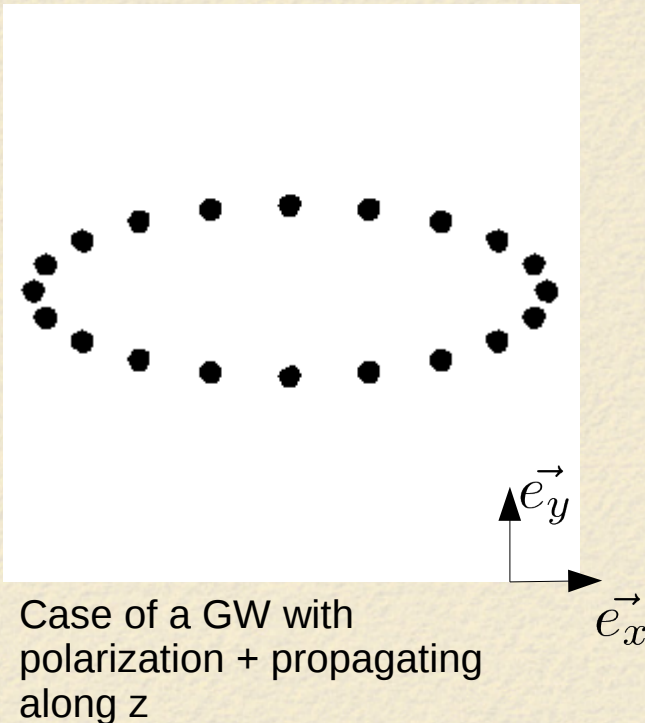
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- **How do we measure the GW strain, $h(t)$, from this detector ?**
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 - What is a noise ?
 - The fundamental noises: seismic, thermal, and shot noises
 - History of Virgo noise

Reminder: effect of a GW on free masses

A gravitational wave (GW) modifies the distance between free-fall masses

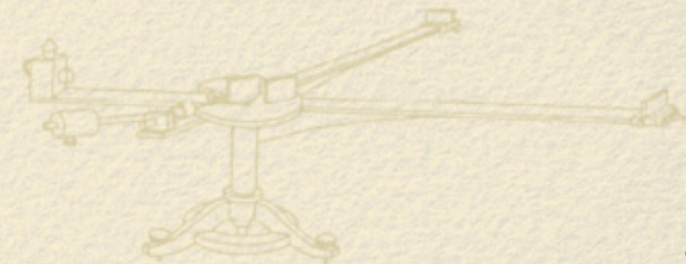
$$\delta x(t) = -\delta y(t) = \frac{1}{2} h(t) L_0$$



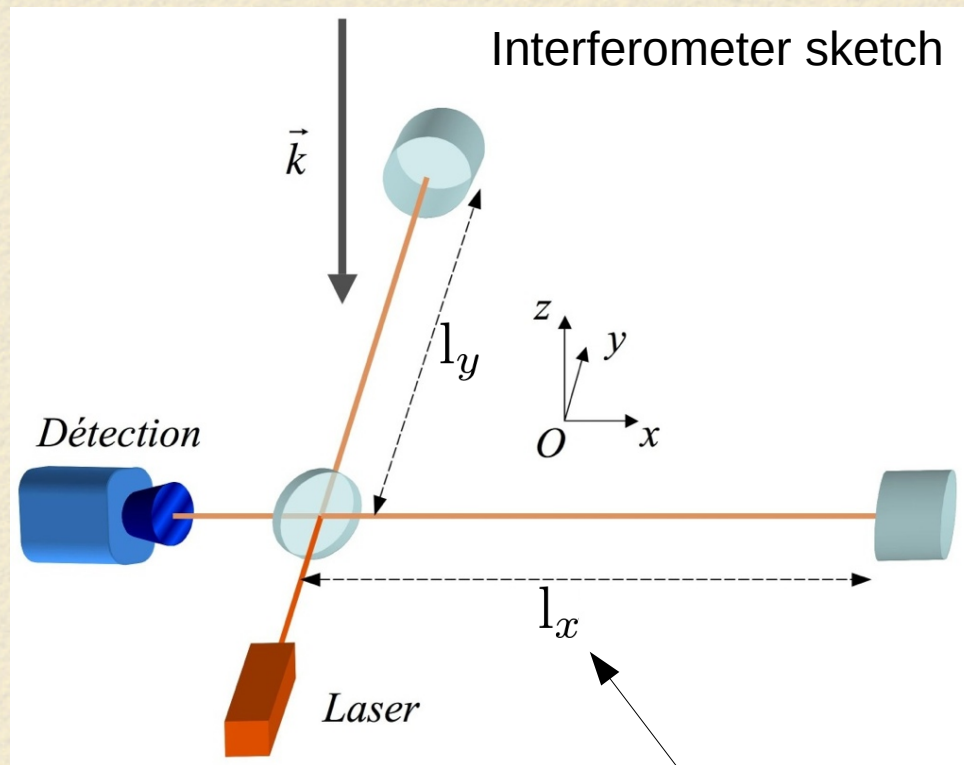
Typical amplitude of a GW crossing the Earth:

$$h \sim 10^{-23}$$

(h has no dimension/unit)

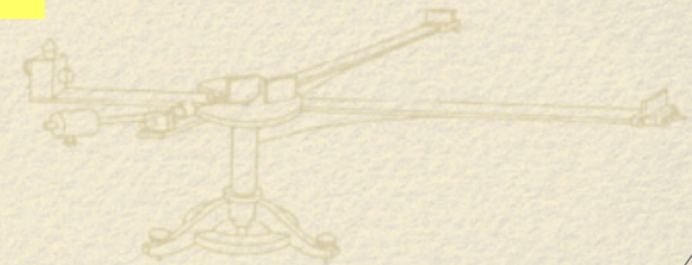


A general overview of the Virgo detector



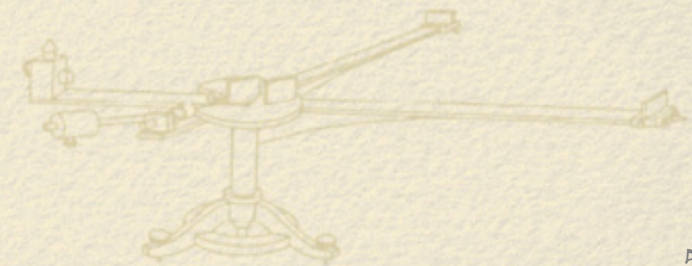
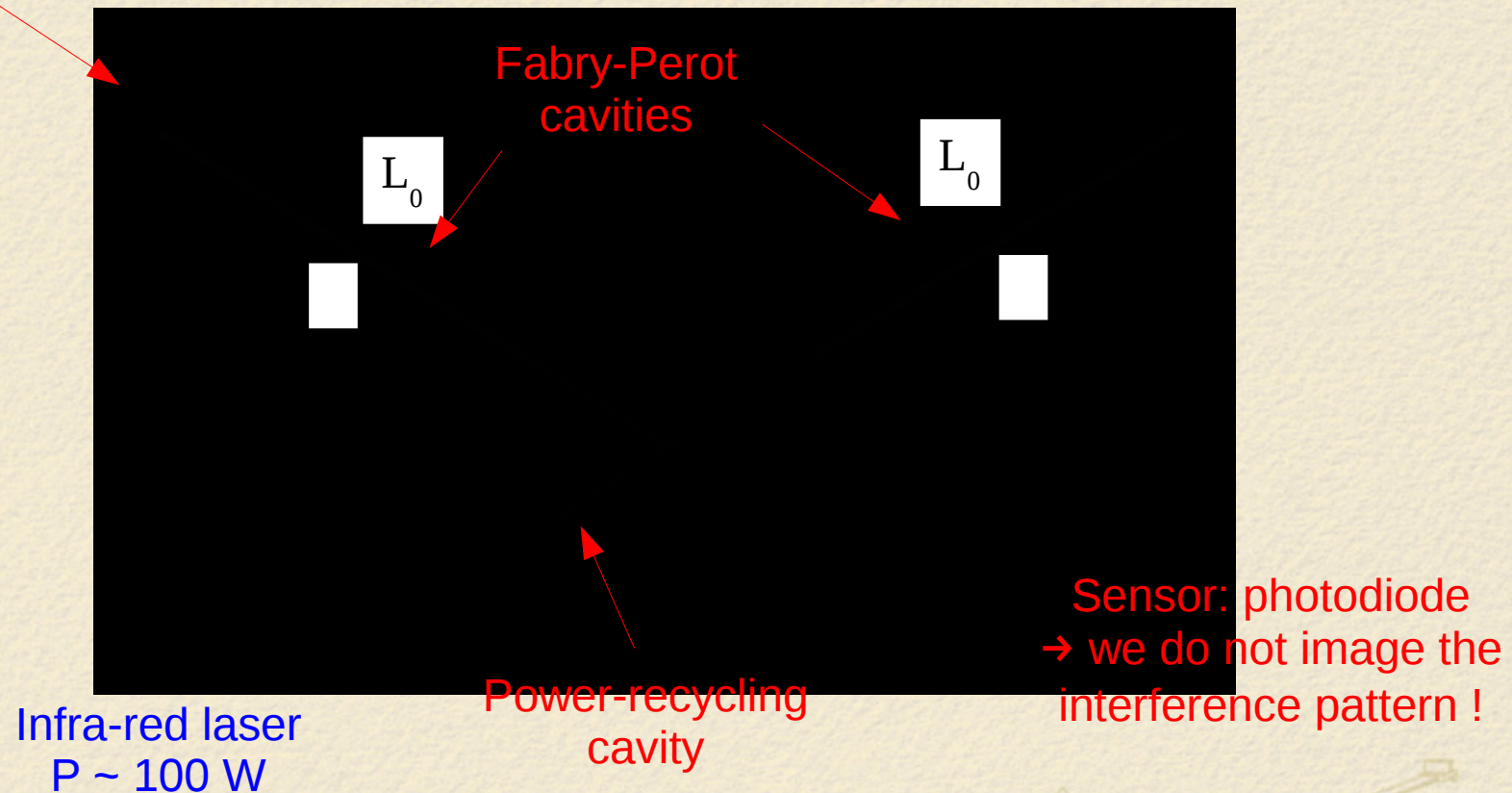
The interference pattern depends on ΔL :
 $\Delta L(t) = l_x(t) - l_y(t)$

Length of the arms: $L_0 = 3 \text{ km}$

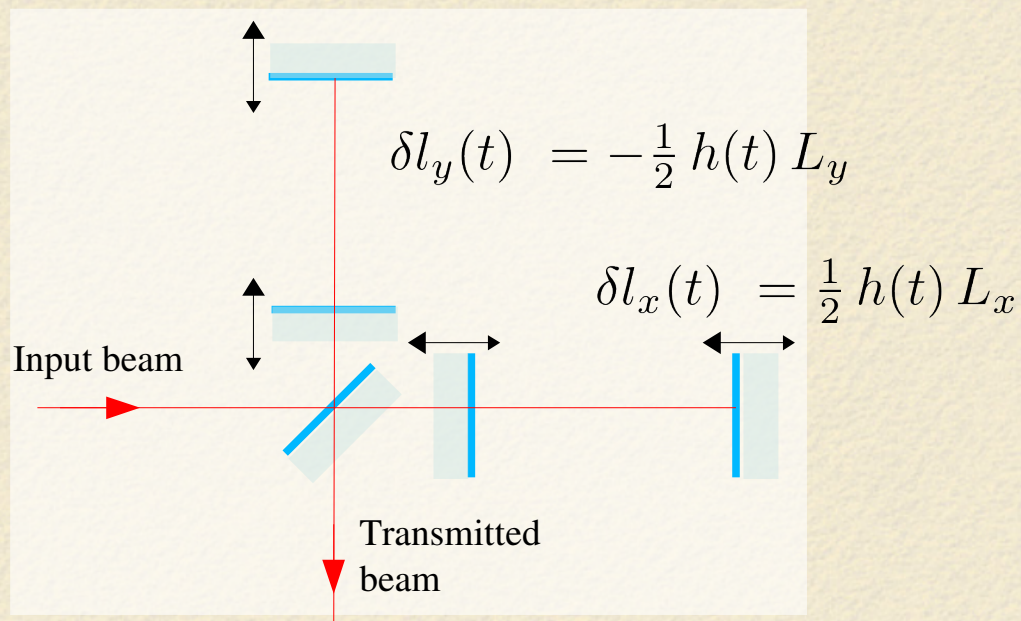


Virgo: a more complicated interferometer

Suspended mirrors → Mirrors can be considered as free for frequencies larger than ~ 10 Hz



Orders of magnitude



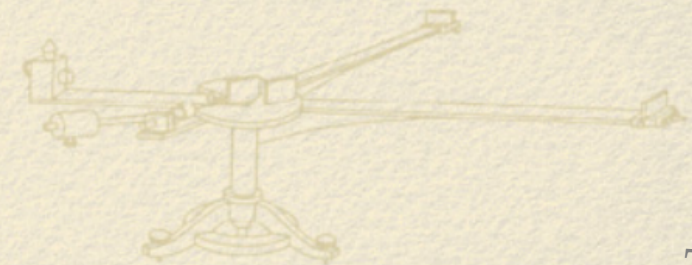
Typical amplitude of differential arm length variations when a GW crosses the Earth:

$$\delta \Delta L = \delta l_x(t) - \delta l_y(t) = h(t) L_0$$

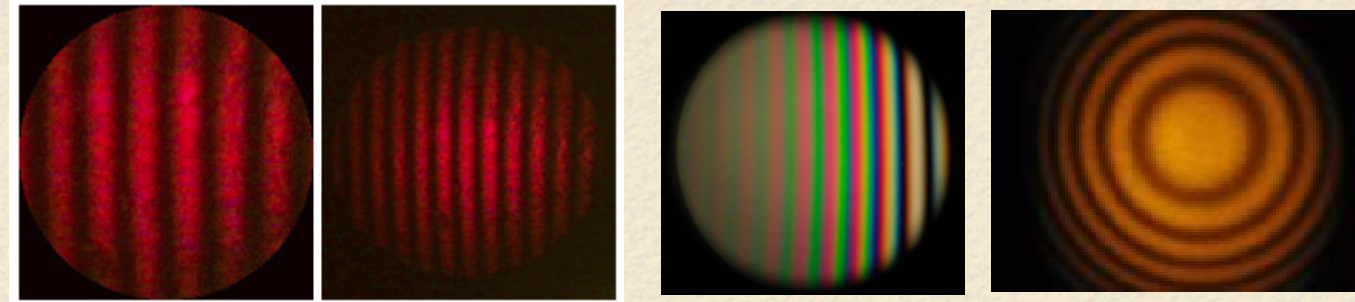
$$h \sim 10^{-23} \quad L_0 = 3 \text{ km}$$

$$\rightarrow \delta \Delta L \sim 3 \times 10^{-20} \text{ m}$$

$$\sim \frac{\text{size of a proton}}{100000}$$

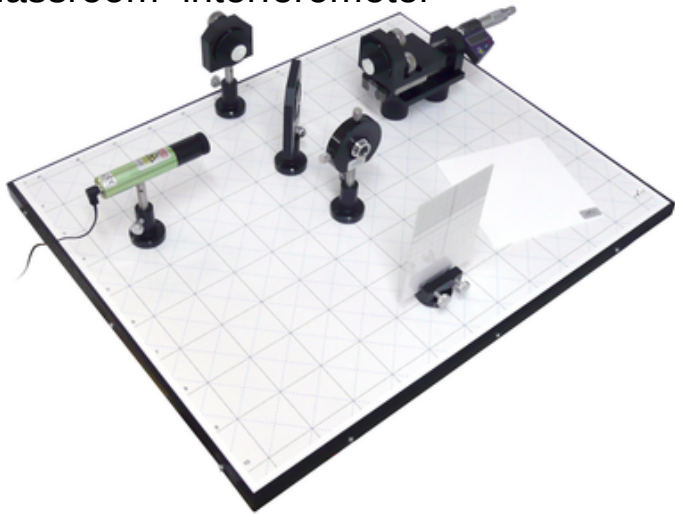


How and for what did you use interferometers ?



Wavelength of monochromatic source
Sodium doublet wavelength separation

Classroom interferometer



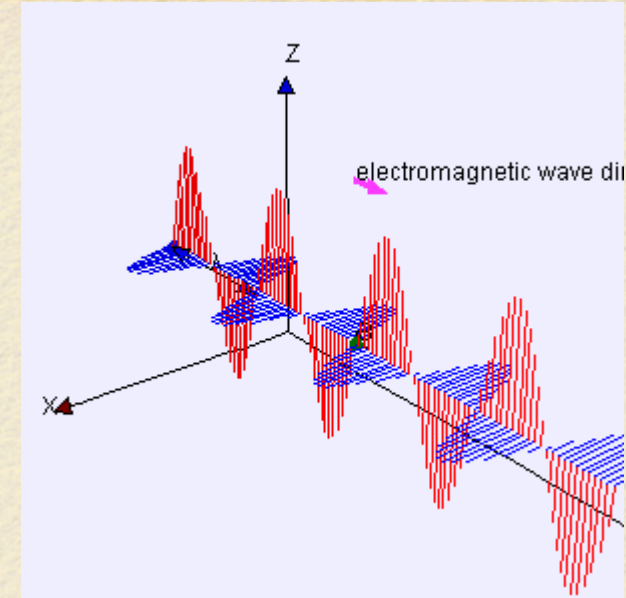
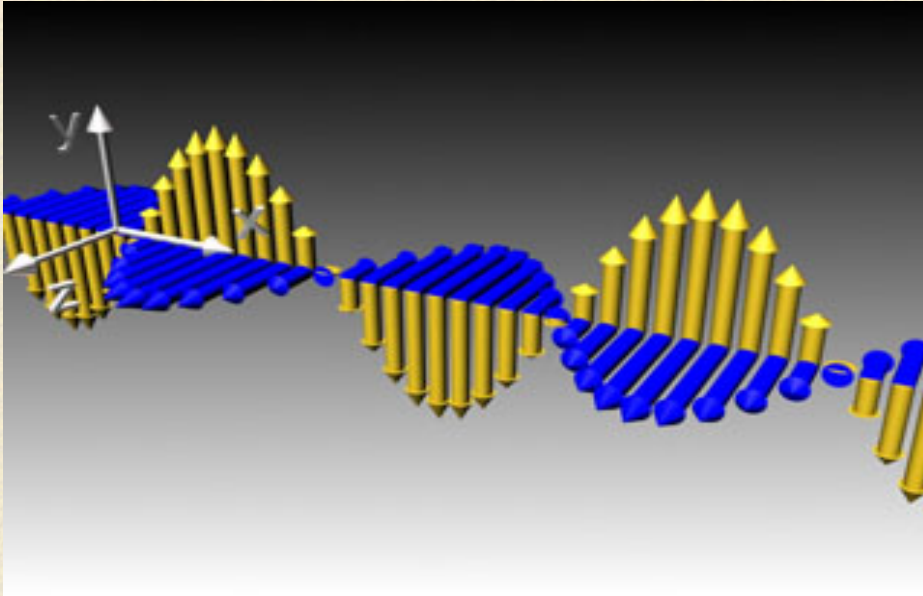
Virgo interferometer
Pisa, Italy



Part 2: Virgo optical configuration

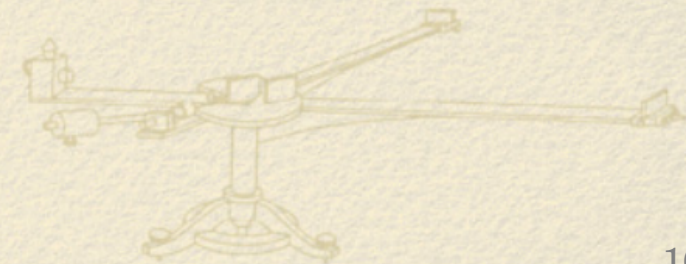
- Reminder about electromagnetic waves and plane waves
- How do we “observe” ΔL with a Michelson interferometer ?
 - Measurement of power variations
 - From power variations to ΔL (or to gravitational wave amplitude h)
- Improving the interferometer:
 - How do we increase the power on the beam-splitter mirror ?
 - How do we amplify the phase offset between the arms ?

Electromagnetic waves



- Propagation of a perturbation of electric and magnetic fields
 - Direction of propagation: along $\vec{k}$
 - E and B are in phase, and with perpendicular directions
 - E and B are perpendicular to the direction of propagation of the wave (transverse wave)
- Amplitude: amplitude of the E (or B) field,
- Two polarizations: defined by the direction of E (or B)

$$\vec{B} = \frac{\vec{k} \times \vec{E}}{c}$$



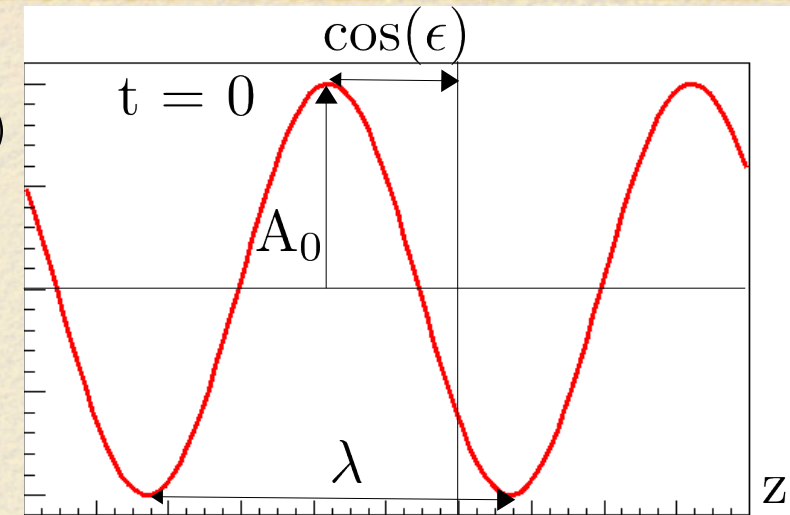
Description of plane waves

- Plane wave propagating along z , with speed c

$$A(z, t) = A_0 \cos(kz - \omega t + \epsilon) \quad (\text{since } \vec{k} \vec{r} = k z)$$

$$\begin{cases} A_0 & \text{amplitude} \\ \lambda & \text{wavelength (m)} \\ k = \frac{2\pi}{\lambda} & \text{wave number (rad/m)} \\ \omega = kc & \text{angular frequency (rad/s)} \end{cases}$$

- Average power: $P \propto A_0^2$

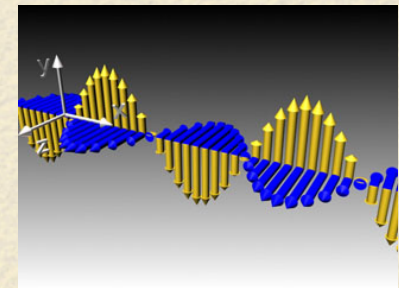


- Complex form

$$\begin{aligned} U(z, t) &= A_0 e^{j(kz - \omega t + \epsilon)} \\ &= \underline{A_0} e^{j(kz + \epsilon)} \quad \text{with} \quad \underline{A_0} = A_0 e^{-j\omega t} \end{aligned}$$

--> simpler algebraic calculations, for example $P \propto |U|^2 = UU^*$

--> real plane wave is the real part: $\Re(U(z, t)) = A(z, t)$

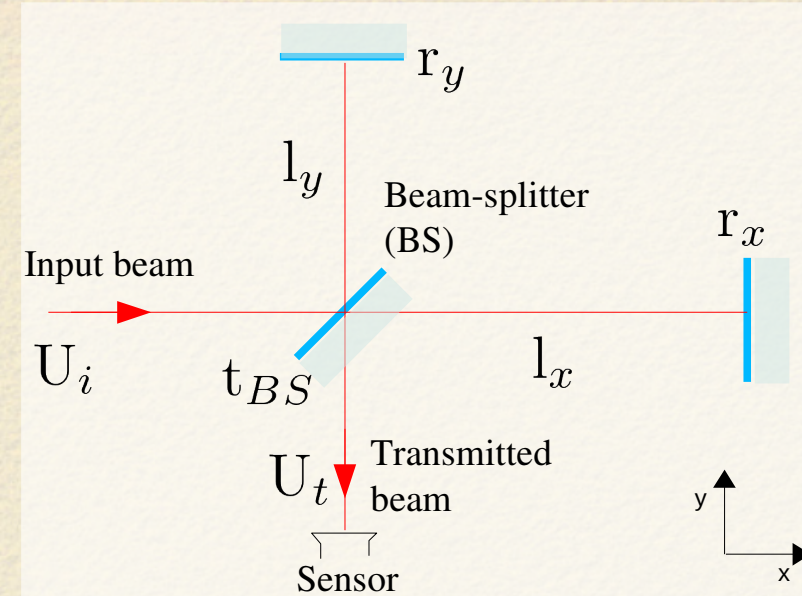


- Plane waves do not exist but they are a good approximation of many waves in localized region of space

How do we “observe” ΔL with a Michelson interferometer ?

- Input wave $U_i(x, t) = \underline{\mathcal{A}}_i e^{j k x}$
 $= \underline{\mathcal{A}}_i$ on BS
- BS located at (0,0)
- Sensor located at (0, - y_s)
- Amplitude reflection and transmission coefficients: r and t

→ We are interested in the beam transmitted by the interferometer: it is the sum of the two beams (fields) that have propagated along each arm.



Around the mirrors:

- Radius of curvature of the beam ~ 1400 m
- Size of the beam $\sim$ few cm

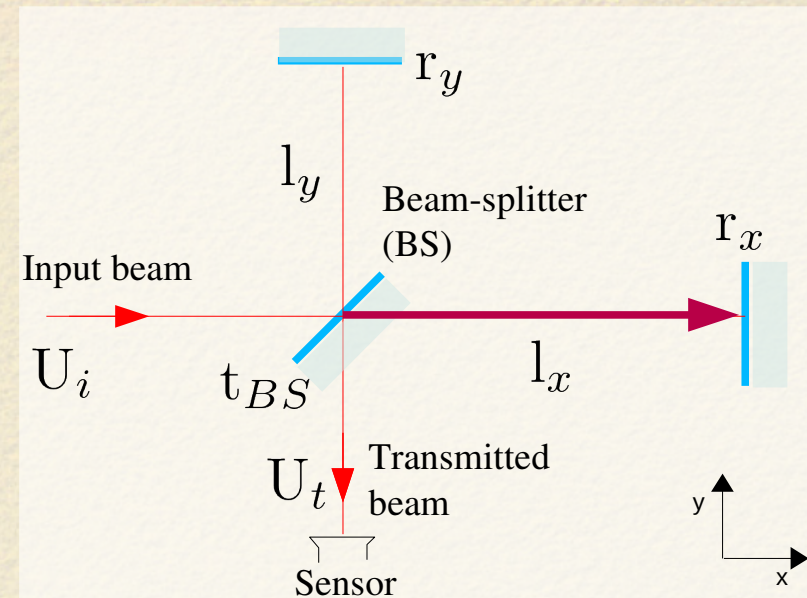
→ The beam can be approximated by plane waves

How do we “observe” ΔL with a Michelson interferometer ?

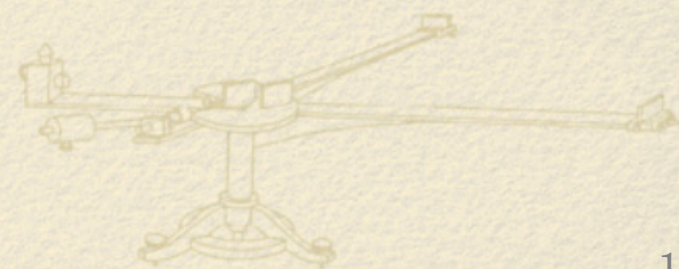
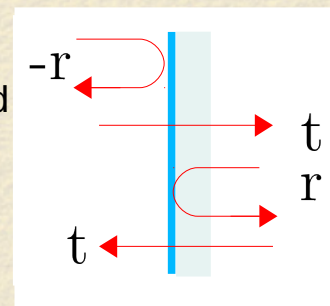
- Input wave $U_i(x, t) = \underline{\mathcal{A}}_i e^{j k x}$
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- Beam propagating along x-arm:

$$U_{tx} = \underline{\mathcal{A}}_i t_{BS} e^{j k l_x} \dots\dots$$



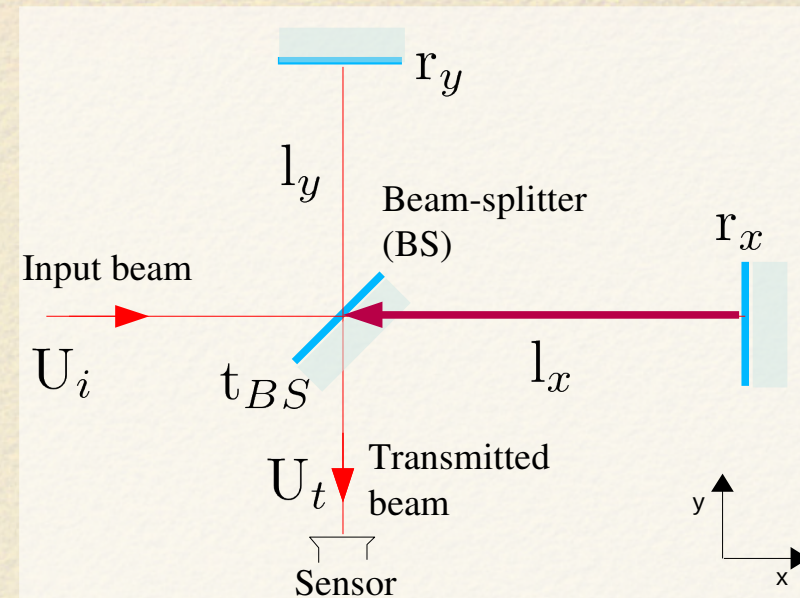
Sign convention for amplitude reflection and transmission coefficients



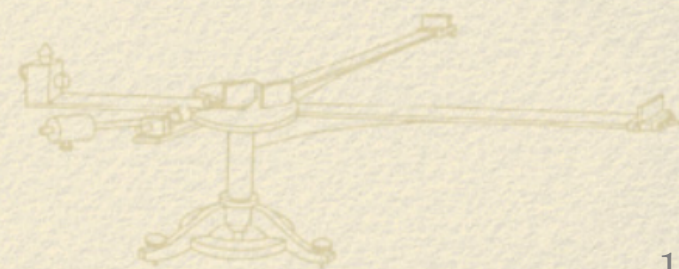
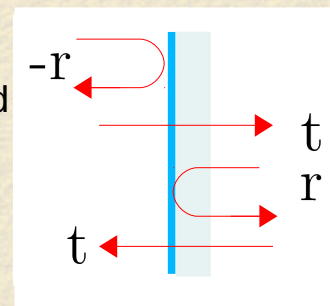
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$$U_{tx} = \underline{\mathcal{A}}_i t_{BS} e^{j k l_x} \quad (-r_x) e^{j k l_x} \dots\dots$$



Sign convention for amplitude reflection and transmission coefficients

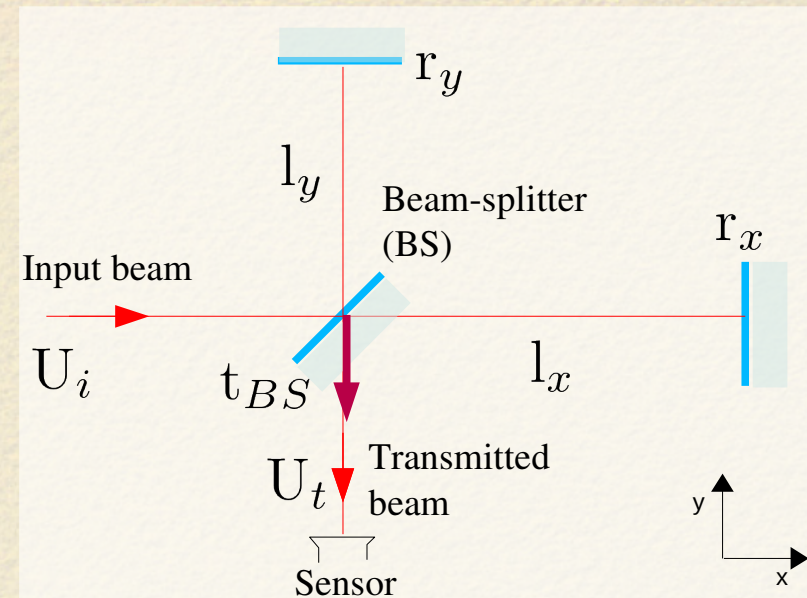


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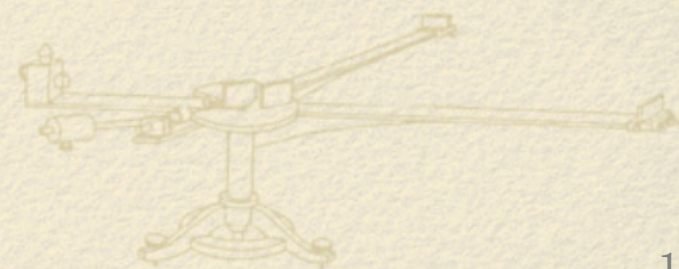
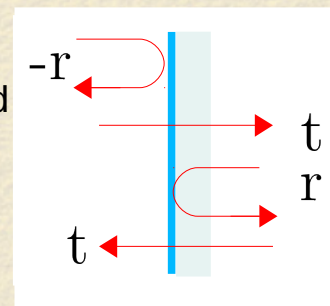
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Sign convention for amplitude reflection and transmission coefficients



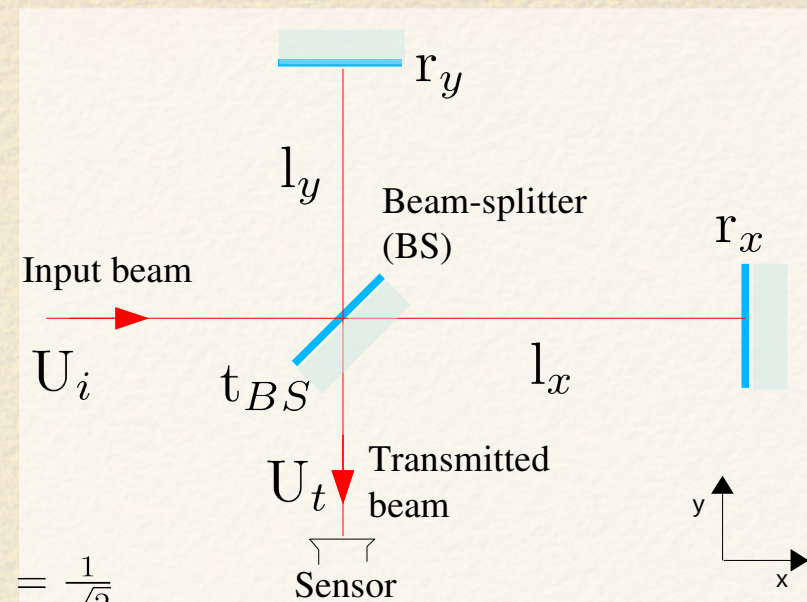
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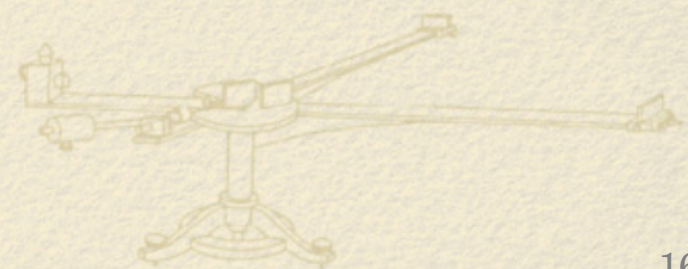
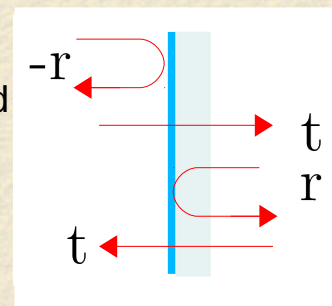
- Beam propagating along x-arm:

$$\begin{aligned} U_{tx} &= \underline{A}_i t_{BS} e^{j k l_x} \quad (-r_x) e^{j k l_x} \quad r_{BS} e^{j k y_s} \\ &= \underline{A}_i t_{BS} r_{BS} (-r_x) e^{2 j k l_x} e^{j k y_s} \\ &= \frac{\underline{A}_i}{2} \times \underbrace{(-r_x e^{2 j k l_x})}_{\text{Complex reflection of the x-arm}} e^{j k y_s} \quad \text{with } t_{BS} = r_{BS} = \frac{1}{\sqrt{2}} \end{aligned}$$

Complex reflection of the x-arm



Sign convention for amplitude reflection and transmission coefficients



How do we “observe” ΔL with a Michelson interferometer ?

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Complex reflection of the x-arm

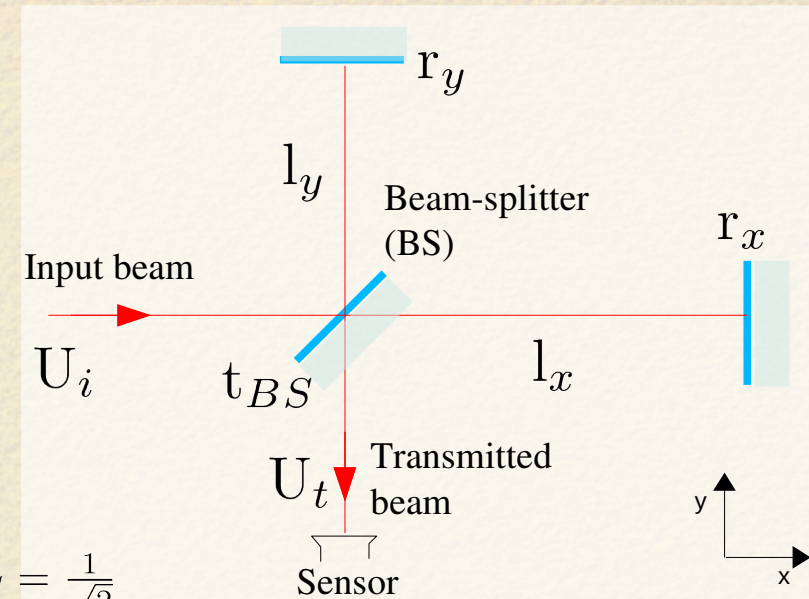
- Beam propagating along y-arm:

$$U_{ty} = -\frac{\underline{A_i}}{2} \times \underbrace{(-r_y e^{2j k l_y})}_{\text{Complex reflection of the y-arm}} e^{j k y_s}$$

Complex reflection of the y-arm

- Transmitted field:

$$\begin{aligned} U_t &= U_{tx} + U_{ty} \\ &= \frac{\underline{A_i}}{2} e^{j k y_s} (r_y e^{2j k l_y} - r_x e^{2j k l_x}) \end{aligned}$$



Power transmitted by a simple Michelson

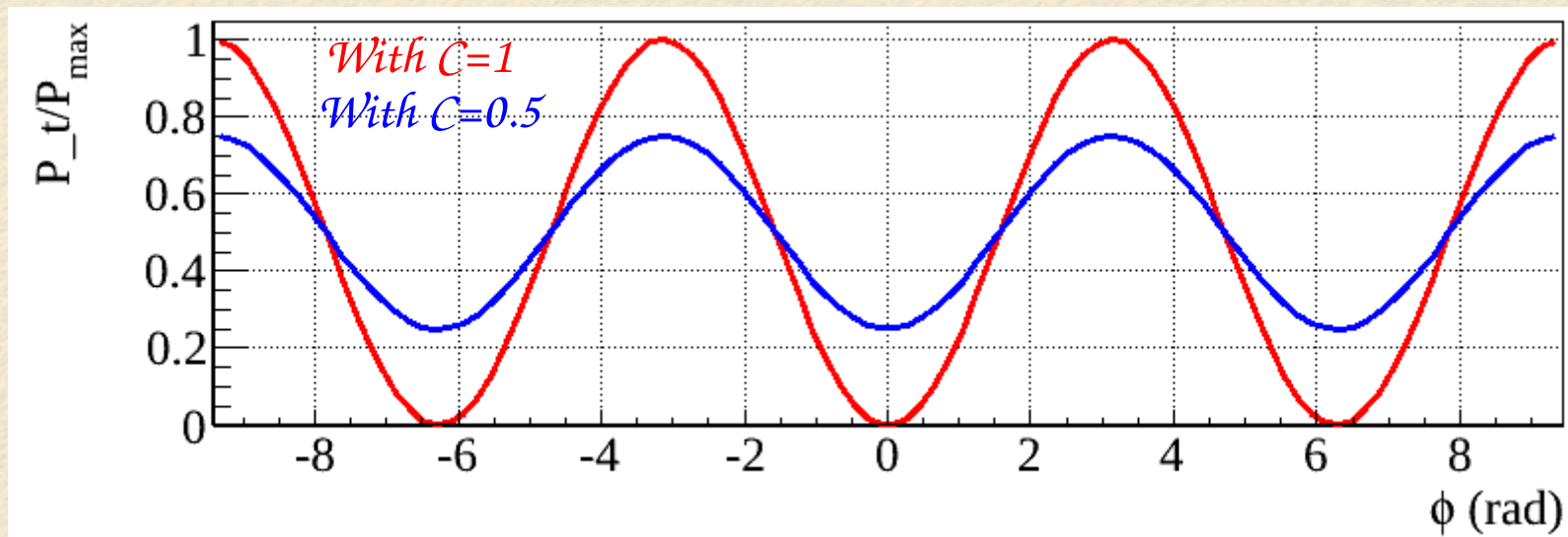
- Transmitted field:
$$U_t = \frac{A_i}{2} e^{jky_s} (r_y e^{2jkl_y} - r_x e^{2jkl_x})$$

- Calculation of the transmitted power:

$$P_t \propto |U_t|^2 = \frac{P_{max}}{2} (1 - C \cos(\phi)) \quad \text{where } \phi = 2k(l_y - l_x)$$

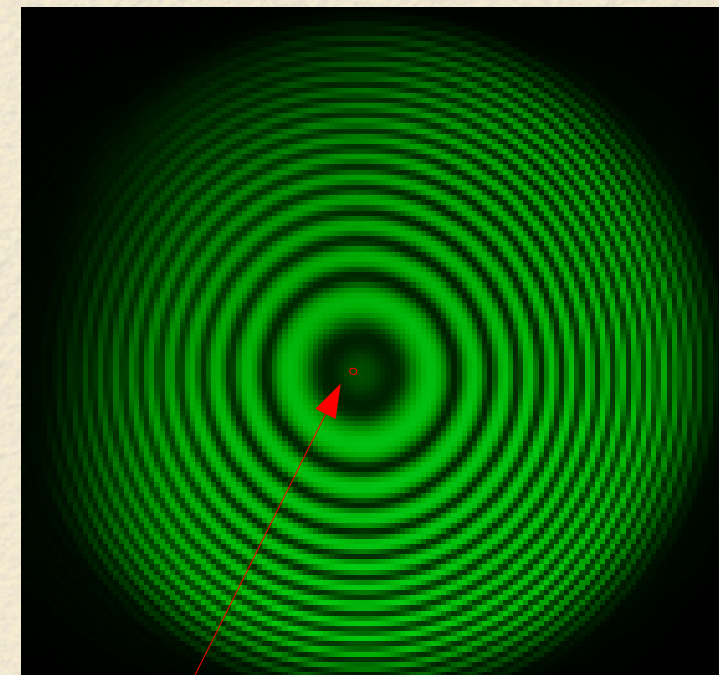
$$C = 2 \frac{r_x r_y}{r_x^2 + r_y^2}$$

$$P_{max} = \frac{P_i}{2} (r_x^2 + r_y^2)$$

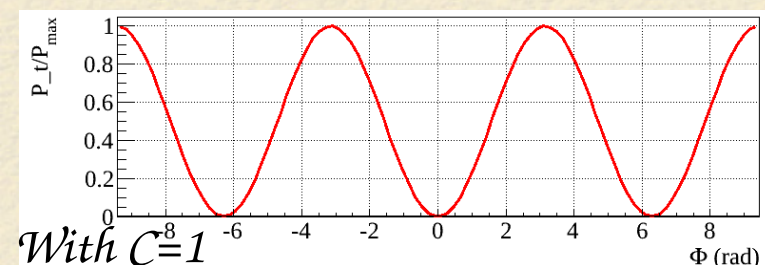


What power does Virgo measure ?

- In general, the beam is not a plane wave but a spherical wave
 - interference pattern
(and the complementary pattern in reflection)
- Virgo interference pattern much larger than the beam size:
size: ~ 1 m between 2 consecutive fringes
 - we do not study the fringes in nice images !

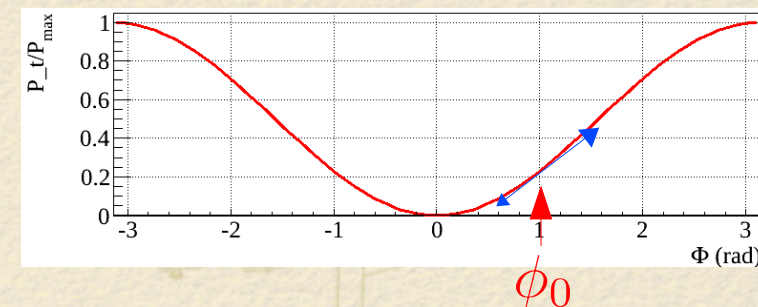


Equivalent size of Virgo beam



Freely swinging mirrors

Setting a working point



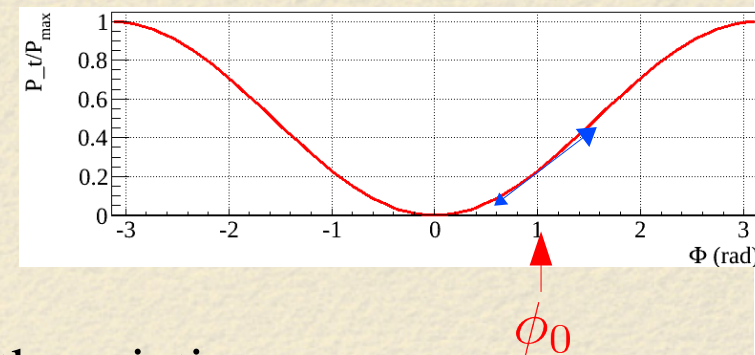
Controlled mirror positions

From the power to the gravitational wave

$$P_t = \frac{P_i}{2} (1 - C \cos(\phi)) \quad \text{where } \phi = 2\frac{2\pi}{\lambda}(l_y - l_x)$$

- Around the working point:

$$\left. \frac{dP_t}{d\phi} \right|_{\phi_0} = \frac{P_i}{2} C \sin(\phi_0) \quad \text{where } \phi_0 = \frac{4\pi}{\lambda} \Delta L_0$$



- Power variations as function of small differential length variations:

$$\delta P_t = \frac{P_i}{2} C \sin(\phi_0) \delta \phi$$

$$\delta P_t = P_i C \frac{4\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda} \Delta L_0\right) \delta \Delta L$$

$$\delta P_t \propto \delta \Delta L = h L_0 \quad \text{around the working point !}$$

From the power to the gravitational wave

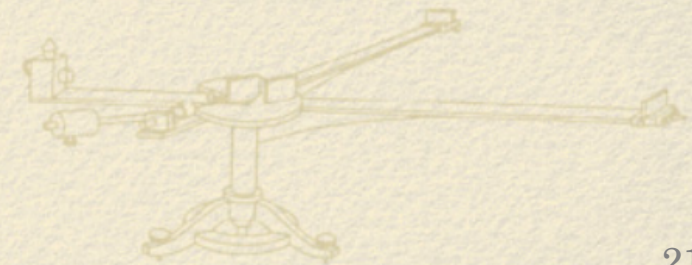
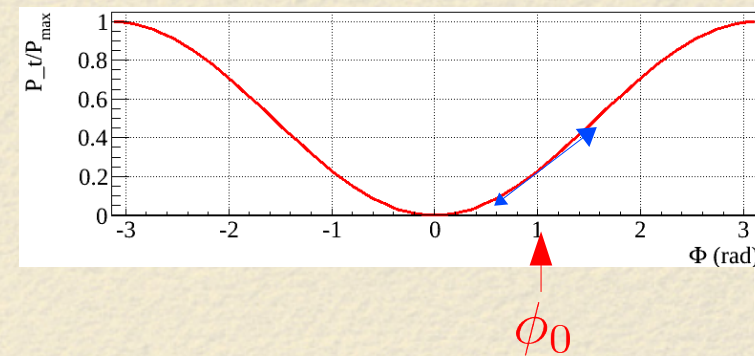
- Around the working point:

$$\delta P_t = P_i C \frac{4\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda} \Delta L_0\right) \delta \Delta L$$

$$\delta P_t = \underbrace{\left(\text{Interferometer response} \right)}_{\text{(W/m)}} \times \delta \Delta L$$

Measurable
physical quantity

Physical effect to be detected



Improving the interferometer sensitivity

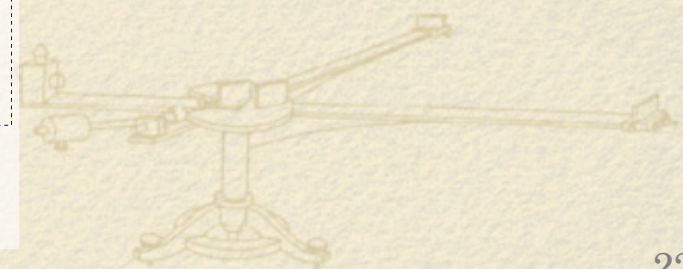
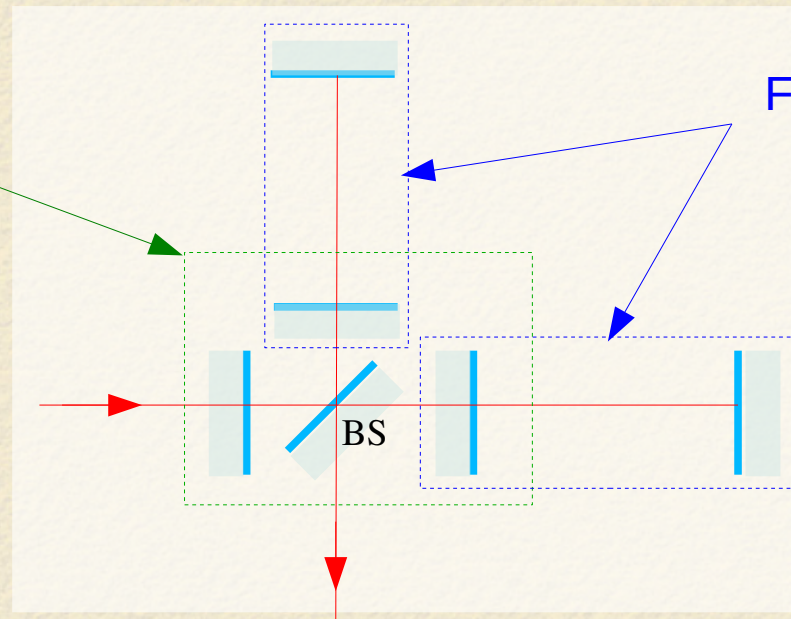
$$\delta P_t = P_i C \sin\left(\frac{4\pi}{\lambda} \Delta L_0\right) (2k \delta \Delta L)$$

Increase the input power
on BS

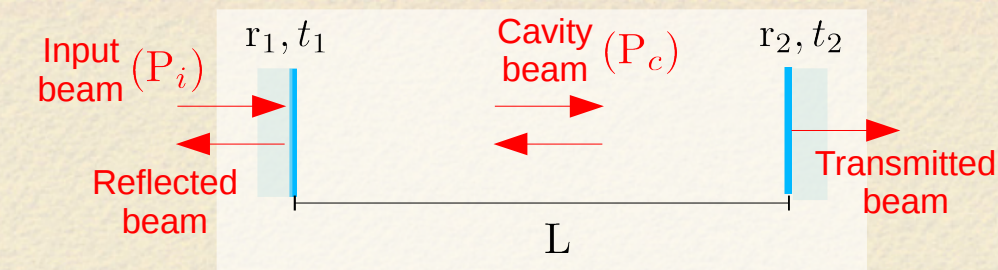
Increase the phase difference
between the arms for a given
differential arm length variation

Recycling cavity

Fabry-Perot cavities in the arms



In Virgo, the beam is resonant inside the cavities

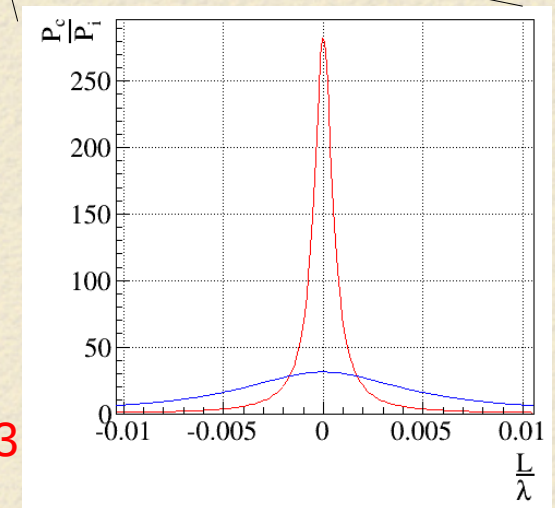
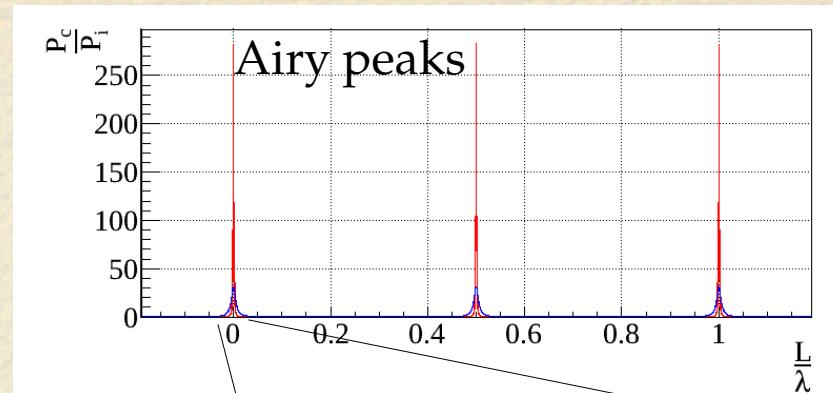


$$P_c = P_i \frac{t_1^2}{(1 - r_1 r_2)^2} \frac{1}{1 + \left(\frac{2\mathcal{F}}{\pi}\right)^2 \sin^2(kL)}$$

$$\text{Finesse } \mathcal{F} = \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2}$$

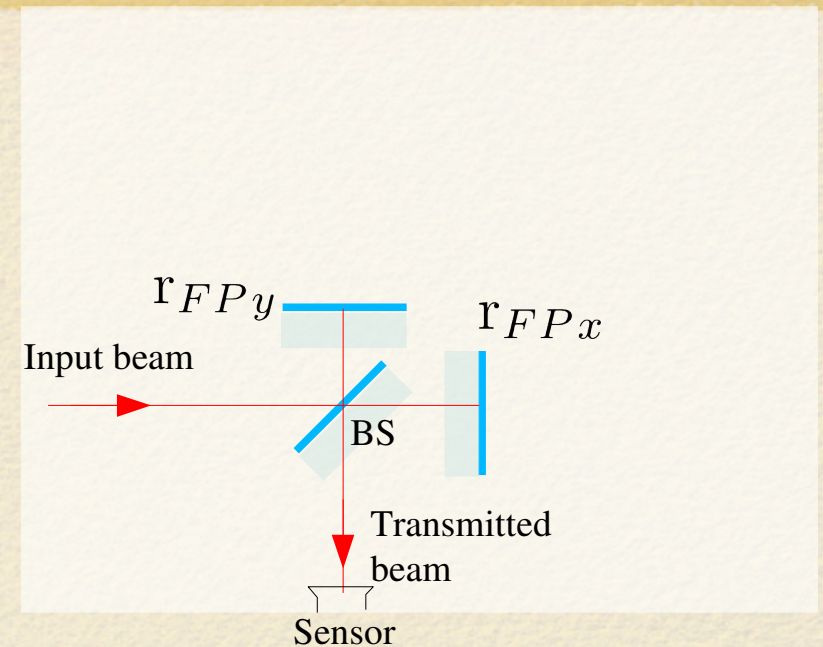
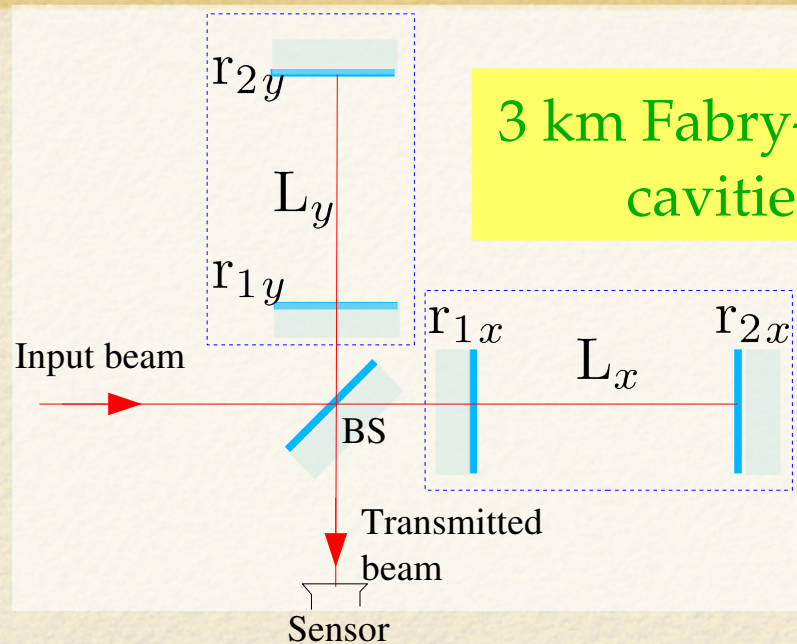
Virgo cavity at resonance: $L = n \frac{\lambda}{2} \quad (n \in \mathbb{N})$

Virgo $\mathcal{F} = 50$
AdVirgo $\mathcal{F} = 443$



Average number of light round-trips in the cavity: $N = \frac{2\mathcal{F}}{\pi}$

How do we amplify the phase offset ?



$$r_{FPx} = -1 \times e^{j \frac{2\mathcal{F}}{\pi} 2k \delta L_x}$$

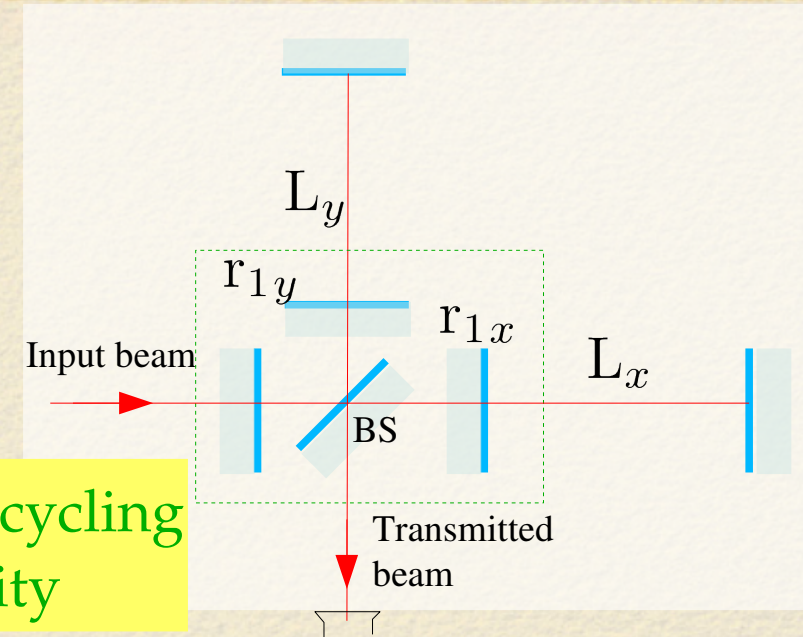
~number of round-trips in the arm
~300 for AdVirgo

(instead of $r_{armx} = -1 \times e^{j2k(L_x + \delta L_x)}$

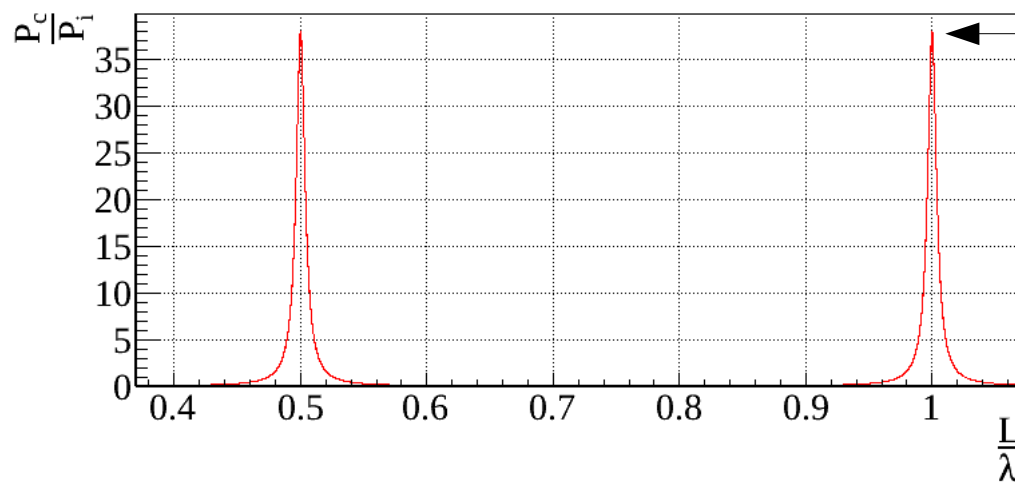
in the arm of a simple Michelson)

How do we increase the power on BS ?

Detector working point close to a dark fringe
→ most of power go back towards the laser



Resonant power recycling cavity



$$G_{PR} = 38 \quad (r_{PR}^2 = 0.95)$$

→ input power on BS
increased by a factor 38 !

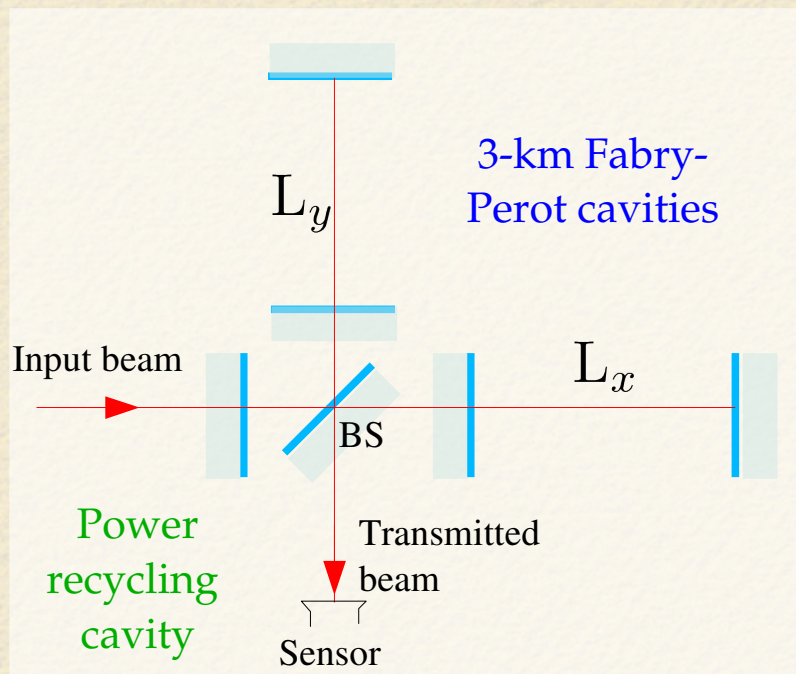
The improved interferometer response

- Response of simple Michelson:**

$$\delta P_t = P_i C \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda} \Delta L_0\right) \delta \Delta L$$

$$\delta P_t = \text{(Michelson response)} \times \delta \Delta L$$

(W/m)



- Response of recycled Michelson with Fabry-Perot cavities:**

$$\delta P_t = G_{PR} P_i C \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda} \Delta L_0\right) \frac{2\mathcal{F}}{\pi} \delta \Delta L$$

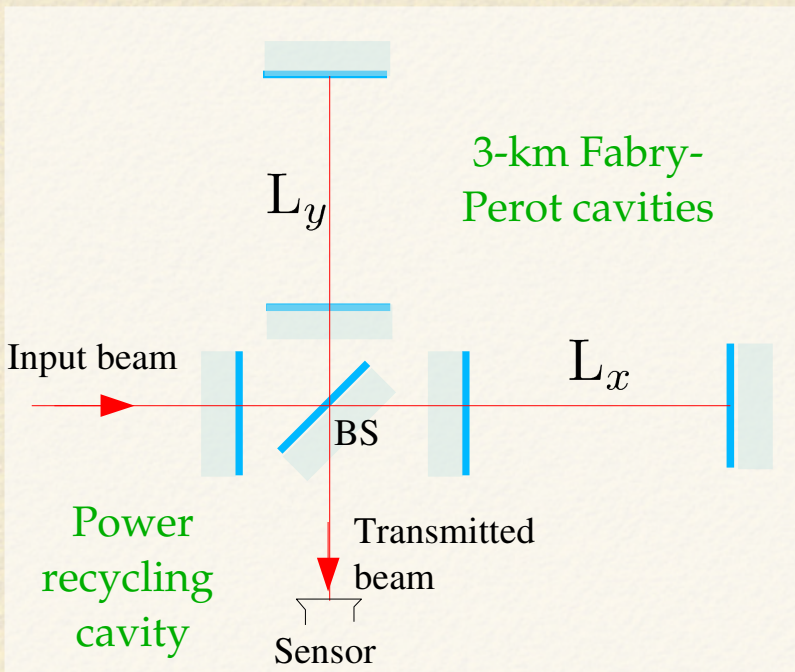
~ 38 ~ 300

For the same $\delta \Delta L$, δP_t has been increased by a factor ~ 12000 .

A hint of Advanced Virgo sensitivity

- Response of recycled Michelson with Fabry-Perot cavities:

$$\delta P_t = G_{PR} P_i C \frac{2\pi}{\lambda} \sin\left(\frac{4\pi}{\lambda} \Delta L_0\right) \frac{2\mathcal{F}}{\pi} \delta \Delta L$$



Laser wavelength: $\lambda = 1.064 \mu\text{m}$

Input power: $P_i \sim 100 \text{ W}$

Interferometer contrast: $C \sim 1$

Input mirror reflection: $r_1 = \sqrt{0.986}$

Working point: $\Delta L_0 \sim 10^{-11} \text{ m}$

Power recycling gain: $G_{PR} \sim 38$

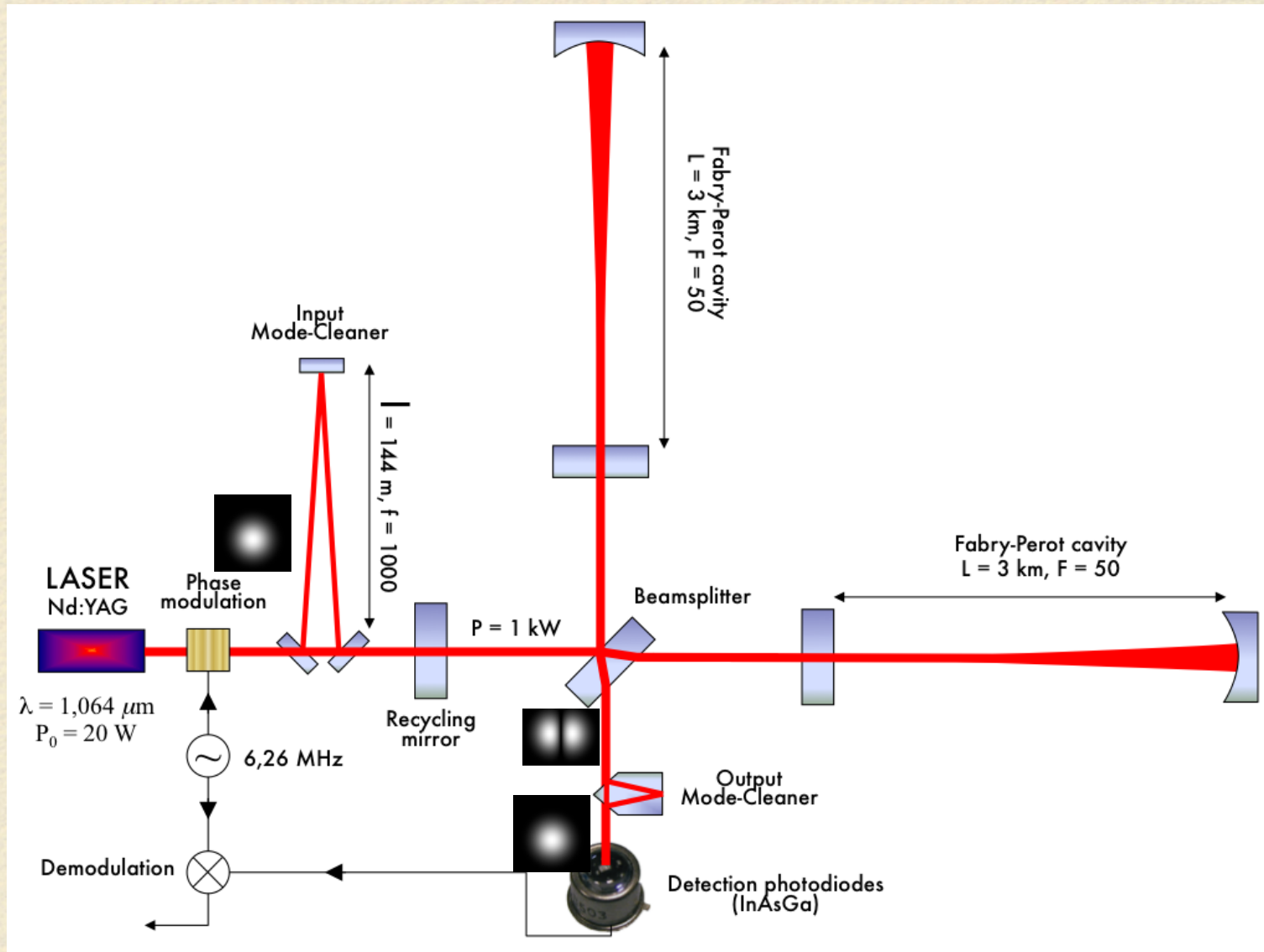
Power noise: $\delta P_{t,min} \sim 0.1 \text{ nW}$ $\longrightarrow$ $\delta \Delta L_{min} \sim 5 \times 10^{-20} \text{ m}$

$$\longrightarrow h_{min} = \frac{\delta \Delta L_{min}}{L} \sim 10^{-23}$$



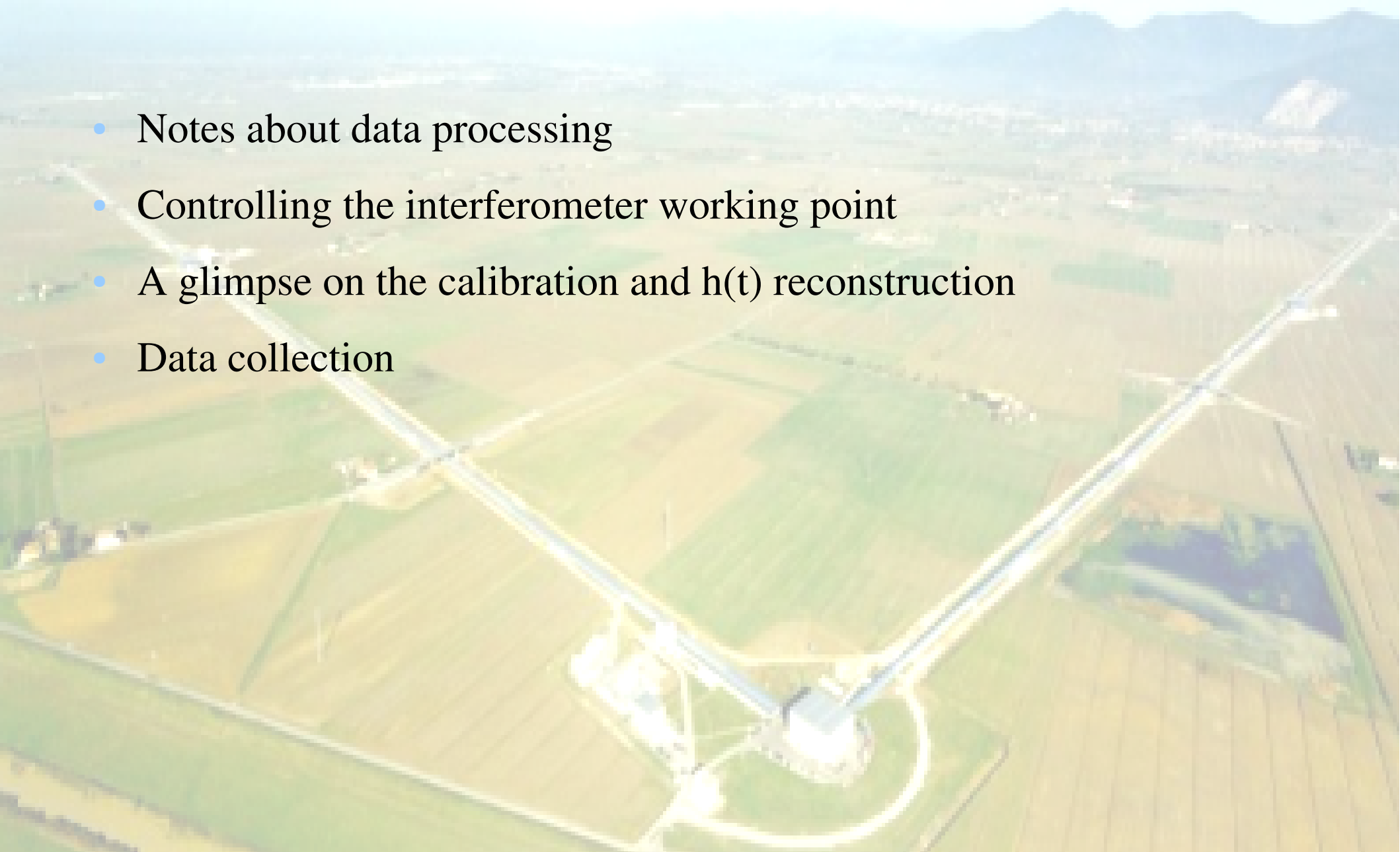
In reality, the detector response depends on frequency...

Optical layout of Virgo



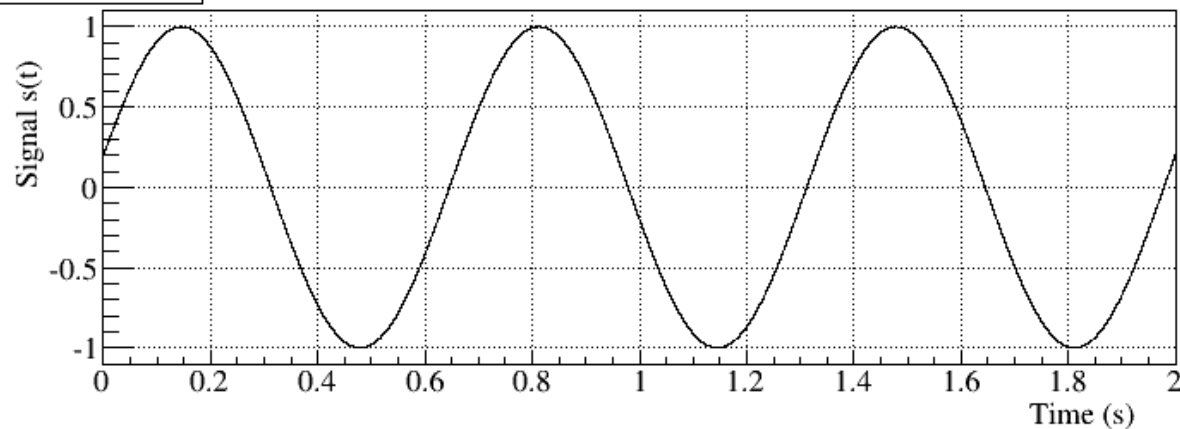
Part 3: How do we measure the GW strain, $h(t)$, from this detector ?

- Notes about data processing
- Controlling the interferometer working point
- A glimpse on the calibration and $h(t)$ reconstruction
- Data collection

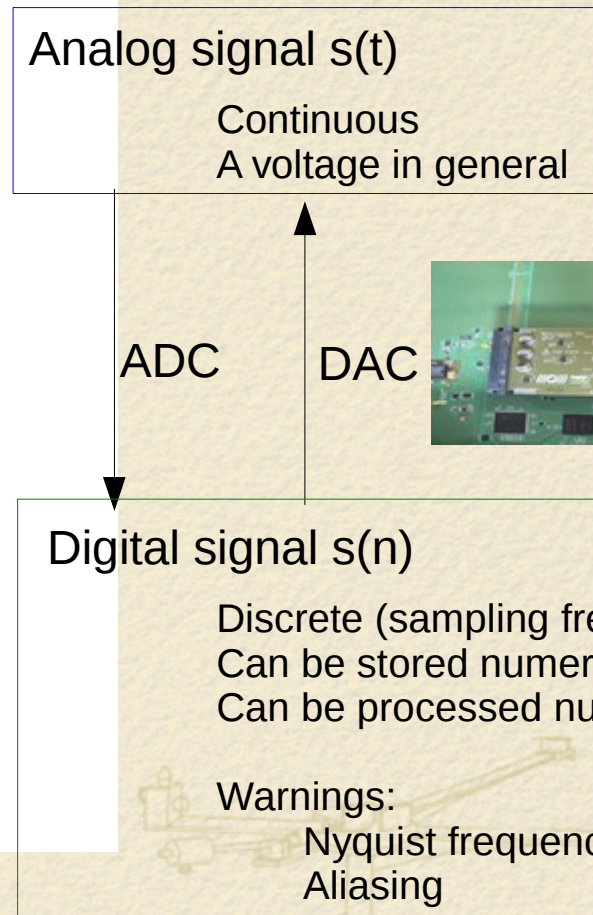
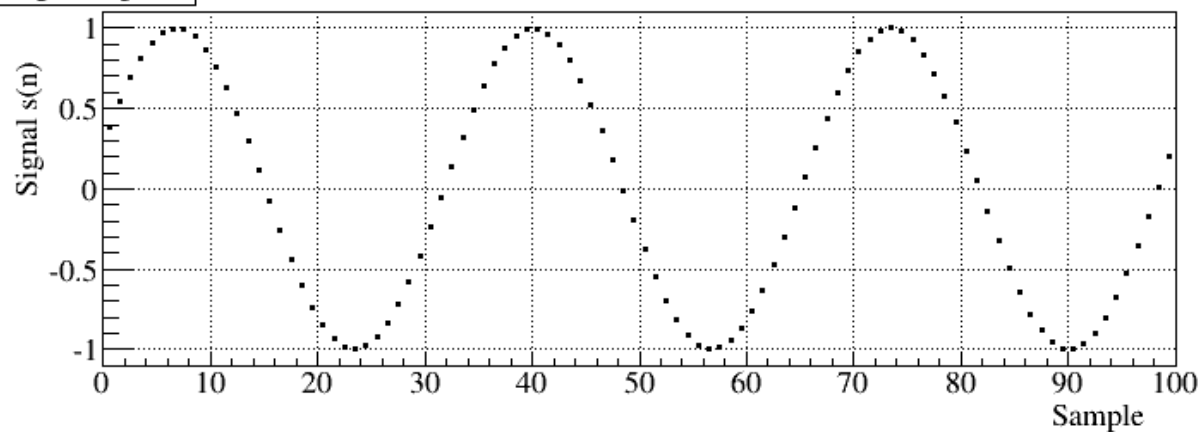


Notes about data processing: digitization

Analog signal



Digital signal



Notes about data processing: spectral analysis

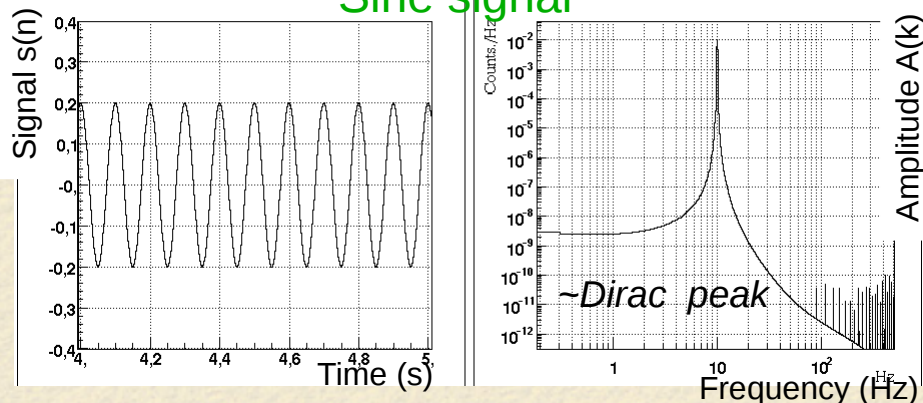
A signal can be decomposed in different frequency components.

$s(n)$ $\xrightarrow{\text{(Discrete) Fourier transform}}$ $A(k)$ and $\Phi(k)$
 $\xleftarrow{\text{Inverse Fourier transform}}$

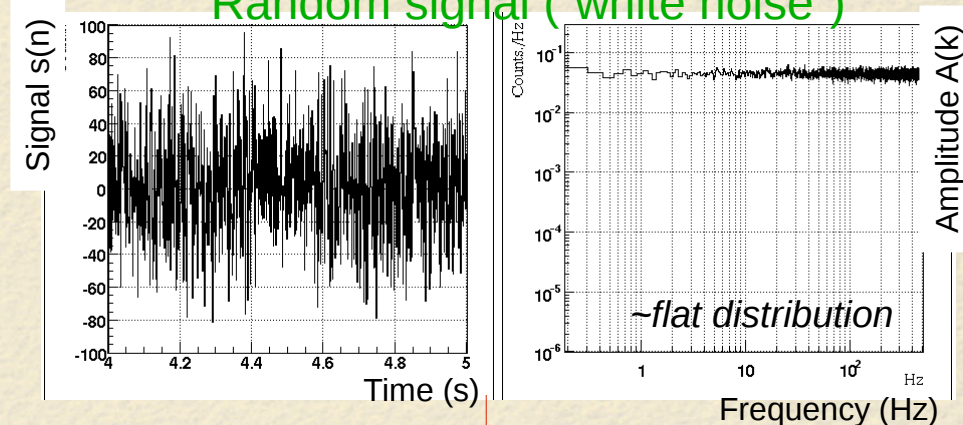
$$S(k) = \sum_{n=1}^N s(n) e^{-j2\pi k \frac{n}{N}}$$

$$= A(k) e^{j\Phi(k)}$$

Sine signal

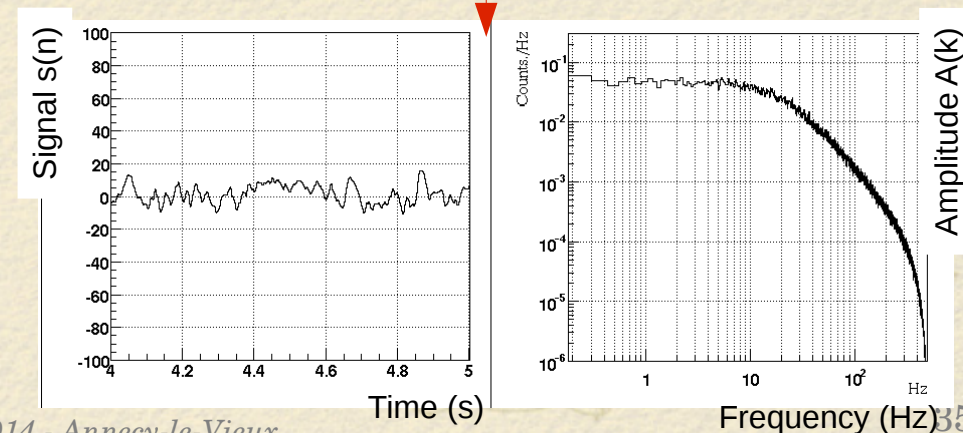


Random signal ("white noise")

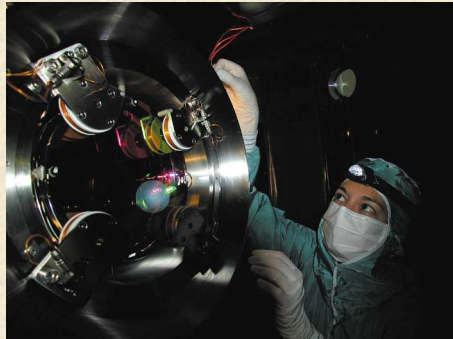


Apply a low-pass filter on the signal

Filtering the data
 = modifying the frequency components



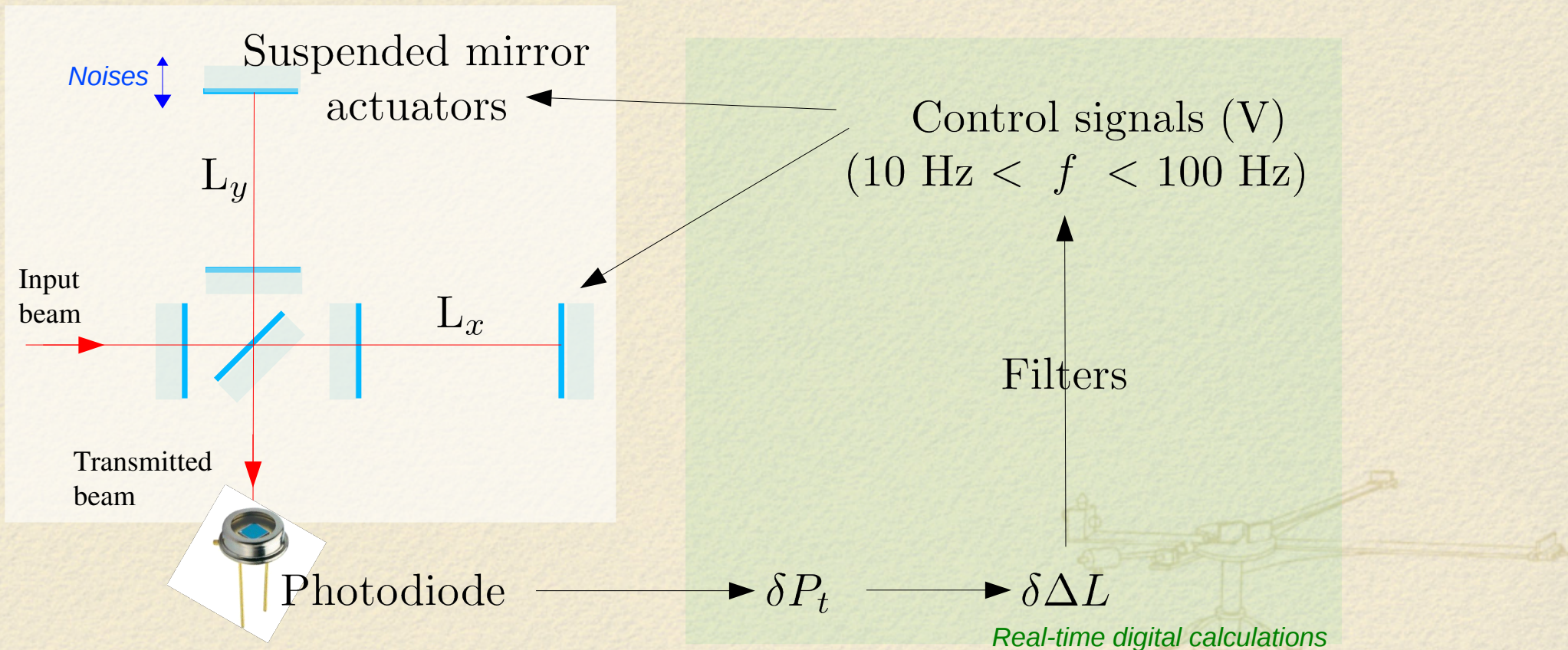
How do we control the working point ?



We want $\Delta L_0 = n \frac{\lambda}{2} + 10^{-11} \text{ m}$ to be (almost) fixed !

Control loop done for noises with f between $\sim 10 \text{ Hz}$ and $\sim 100 \text{ Hz}$

Precision of the control $\sim 10^{-16} \text{ m}$



From the detector data to the GW strain $h(t)$

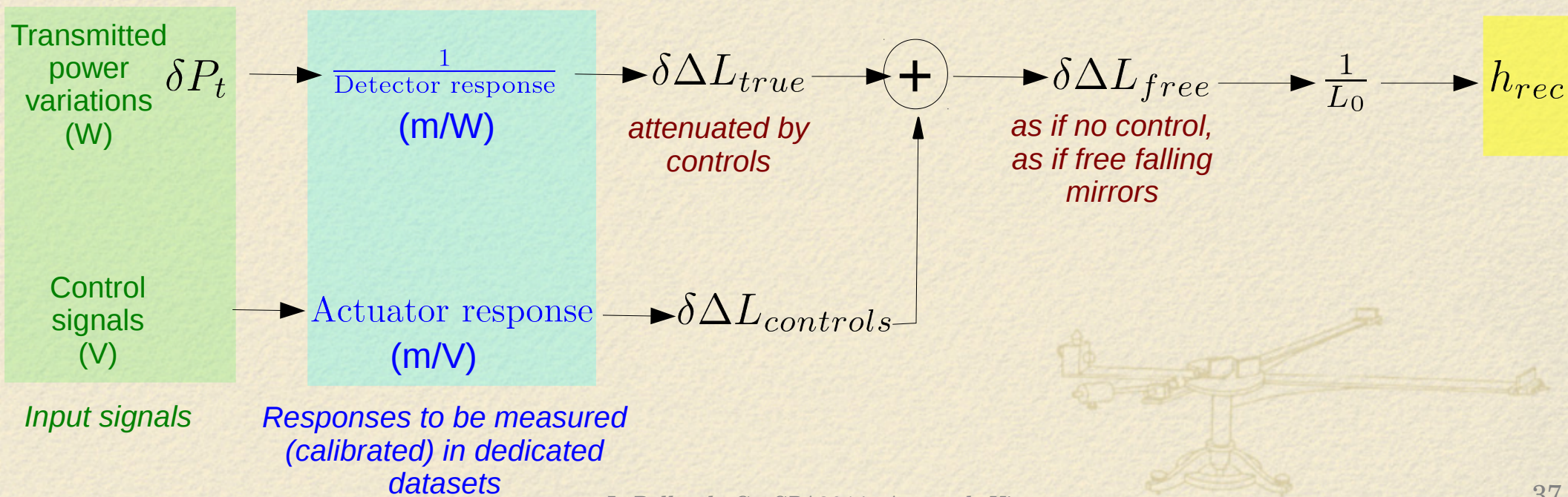
For free falling masses, $h(t) = \frac{\delta \Delta L(t)}{L_0}$



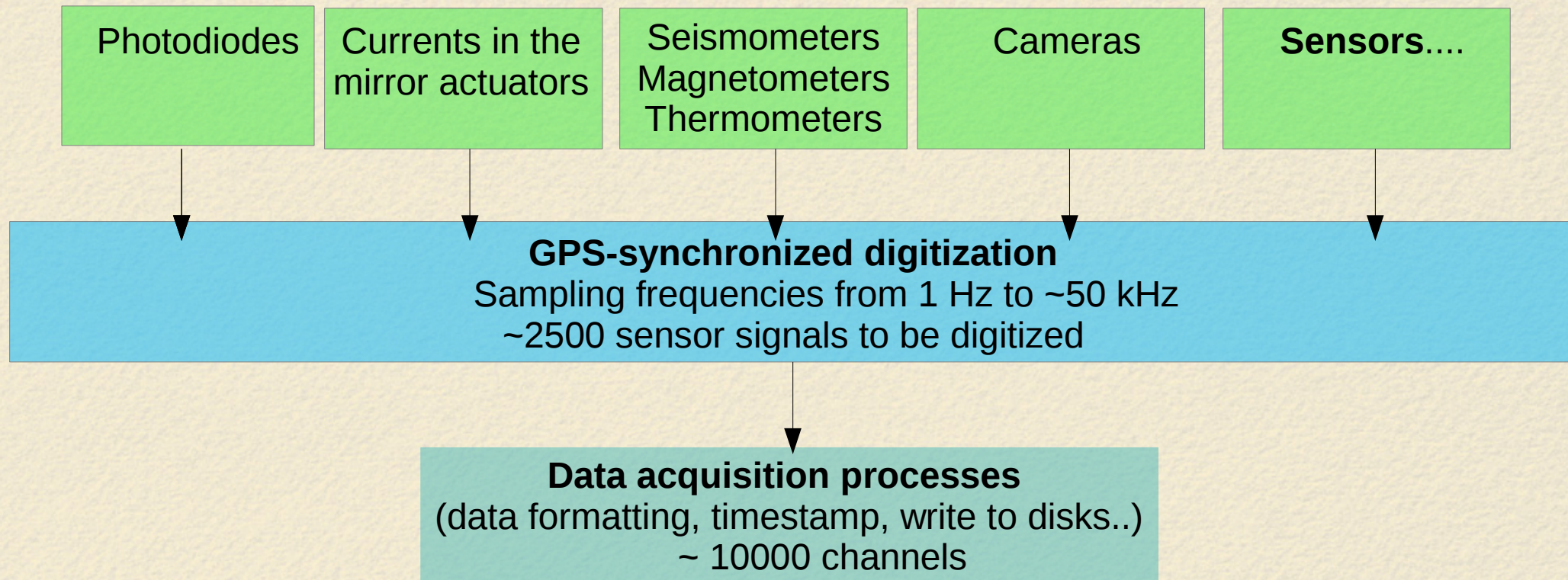
→ this condition is valid for the suspended mirrors above ~ 100 Hz
(no control signal)

At lower frequencies, the controls attenuate the noise...
but also the gravitational wave signal !

→ the control signals contain information on $h(t)$



AdVirgo data acquisition summary



→ { Continuous flow of ~2 TBytes/day (20 to 40 MBytes/s)
Disk space on Virgo site: ~400 TB for 6 months of data
Longer storage: data sent via Ethernet to computing centers (Lyon, Bologna)

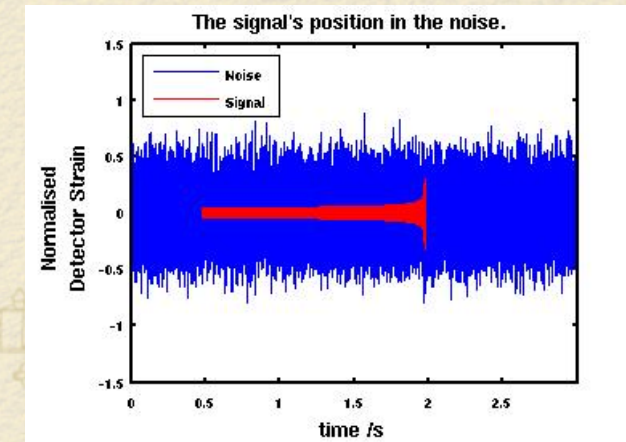
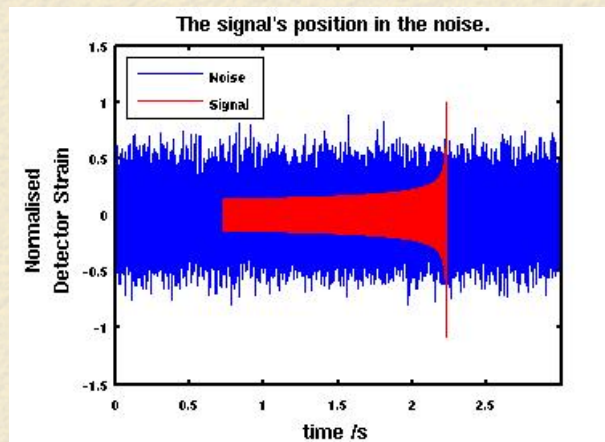
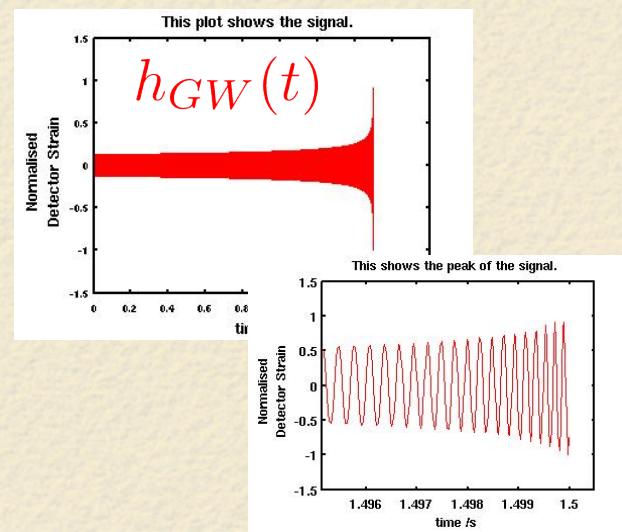
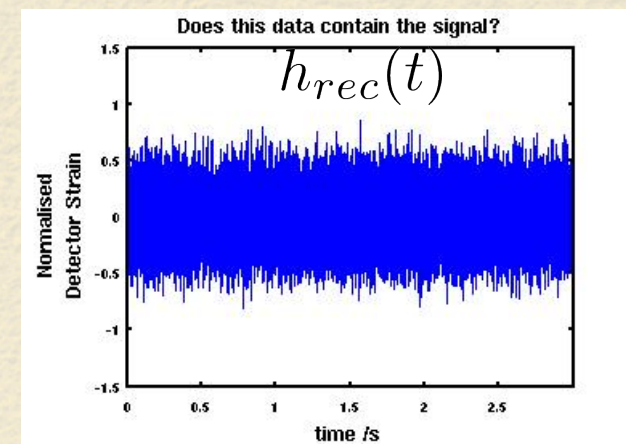
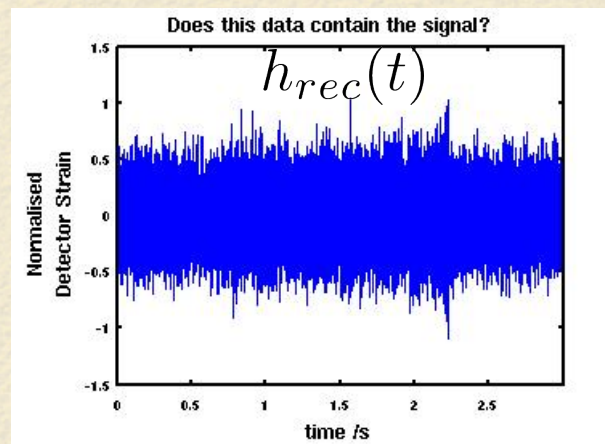
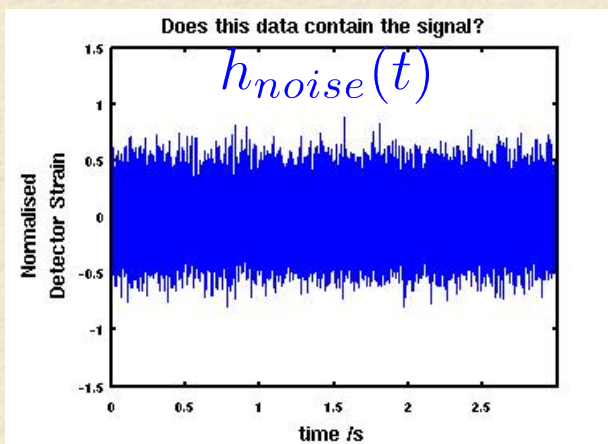
Part 4: Virgo noises



What is a noise in Virgo ?

- Stochastic (random) signal that contributes to the signal $h_{rec}(t)$ but does not contain information on the gravitational wave strain $h_{GW}(t)$

$$h_{rec}(t) = h_{noise}(t) + h_{GW}(t)$$



Extracted from Black Hole Hunter: <http://www.blackholehunter.org/>

How do we characterize a noise ?

- Hypothesis:
- we are looking for a constant signal S_0 ($=0.5$) in the data
 - data are noisy (Gaussian noise)

$$x(t) = S_0 + \text{Noise}(t)$$

Data points

Distribution of the data

Gaussian distribution:

$$N e^{-\frac{1}{2} \frac{(x - \langle x \rangle)^2}{\sigma_x^2}}$$

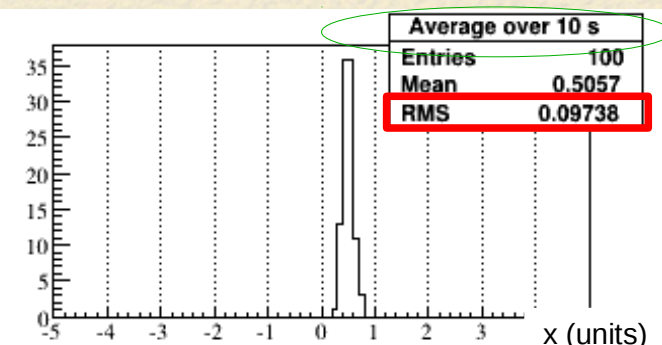
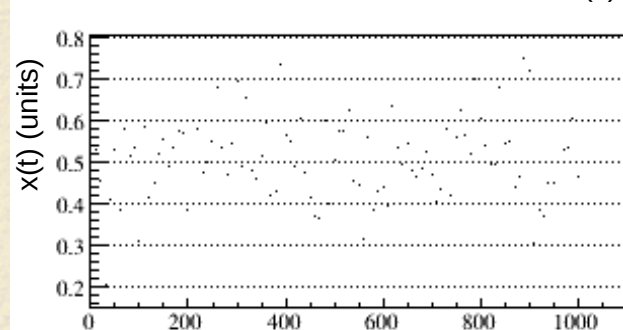
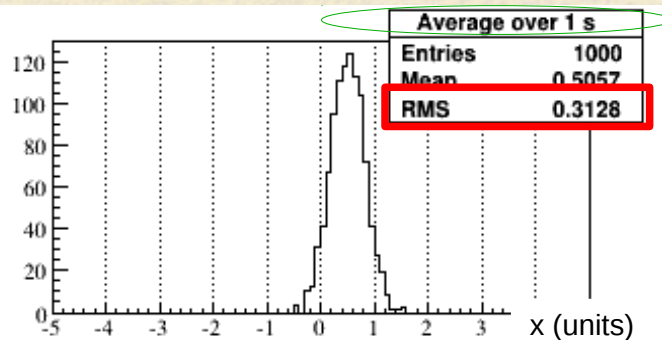
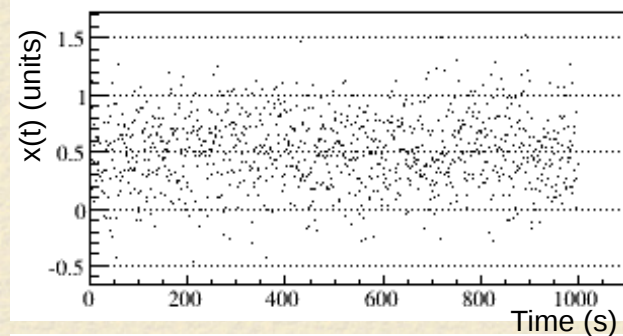
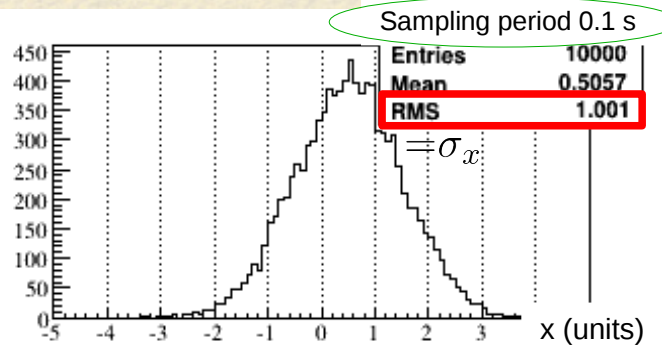
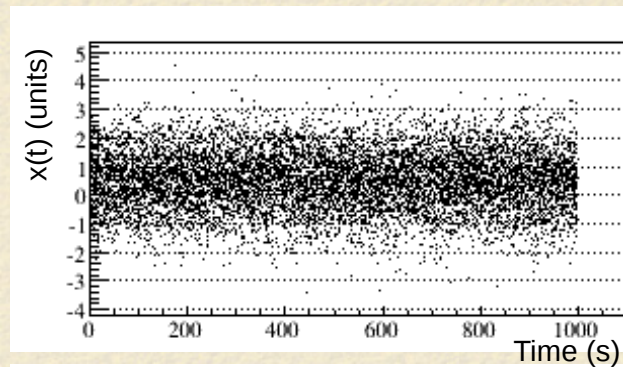
The mean value of the noise stays around 0

The mean value of the signal stays around S_0 .

The variations of the noise decrease when the data are averaged over longer time

$$\sigma_x \propto \frac{1}{\sqrt{\text{average duration}}}$$

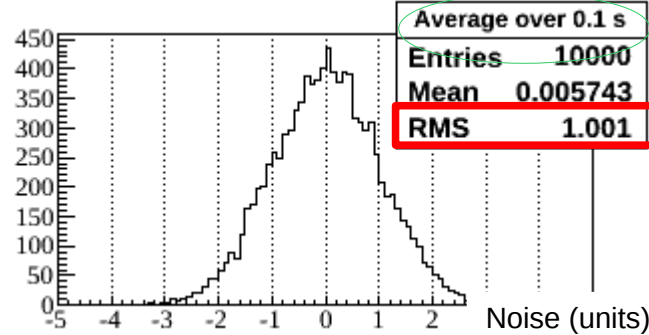
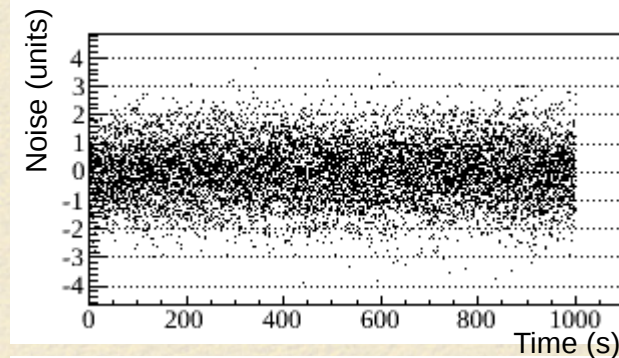
→ What is important to characterize a noise is its dispersion σ_x !



How do we characterize a noise ?

Data points of noise only

Projection of noise data

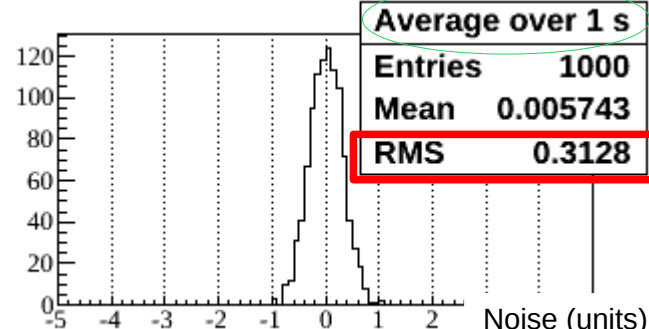
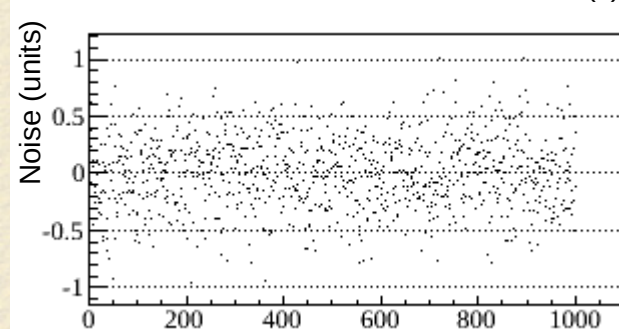


The variations of the noise decrease when the data are averaged over longer time

$$\sigma_x \propto \frac{1}{\sqrt{\text{average duration}}}$$



The noise can be characterized by the coefficient of proportionality D



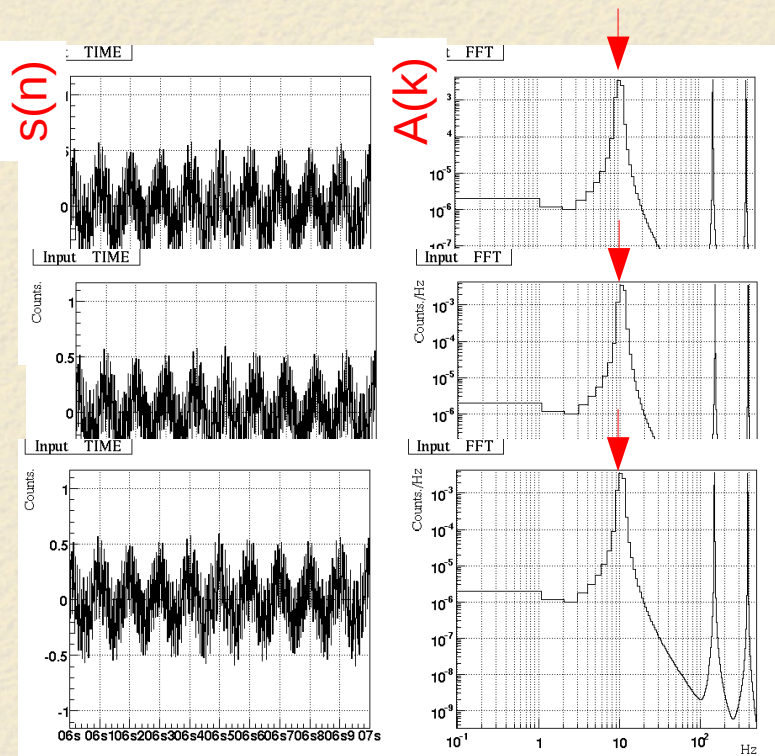
$$\sigma_x = \frac{D}{\sqrt{\text{average duration}}}$$

D is in (Data units $\times \sqrt{s}$) or $\frac{\text{Data units}}{\sqrt{\text{Hz}}}$

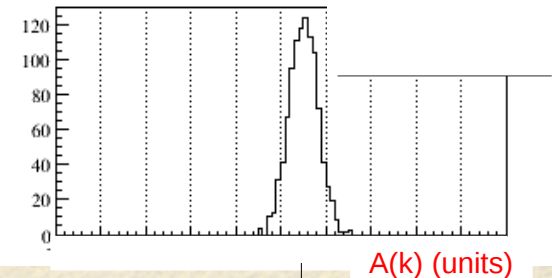
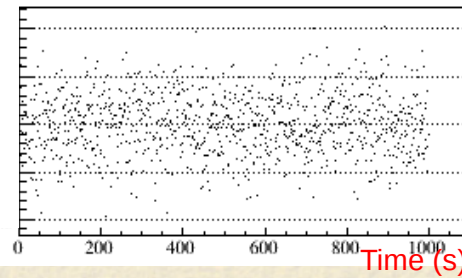
its absolute value is equal to the dispersion of the noise when it is averaged over 1 s.

How do we characterize a noise ...in frequency-domain ?

$s(n)$ $\xrightarrow{\text{Discrete Fourier transform}}$ $A(k)$ (and $\Phi(k)$)



$[A(k)](t)$ (units)



$\sigma_{A(k)}$

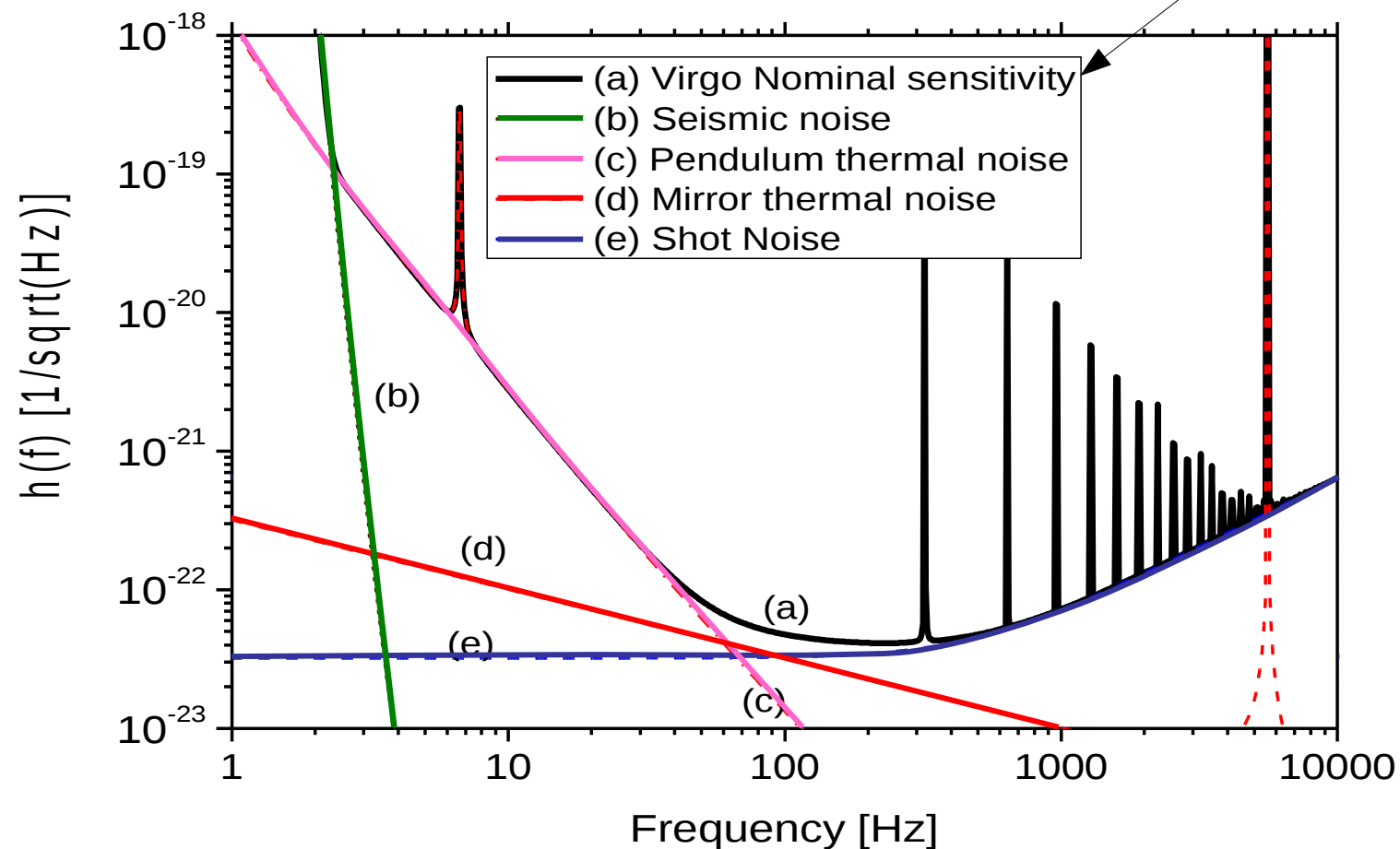
$D(k)$

also in $\frac{\text{Data units}}{\sqrt{\text{Hz}}}$

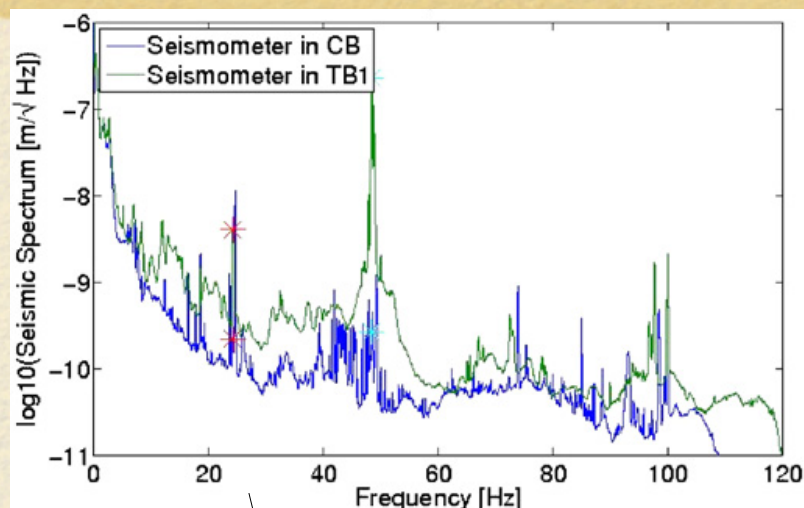
$D(k)$ (amplitude spectral density) is
a curve that characterizes the noise dispersion as function of frequency

What is the noise level of Virgo ?

Noise level of $h_{rec}(t)$, shown as function of frequency

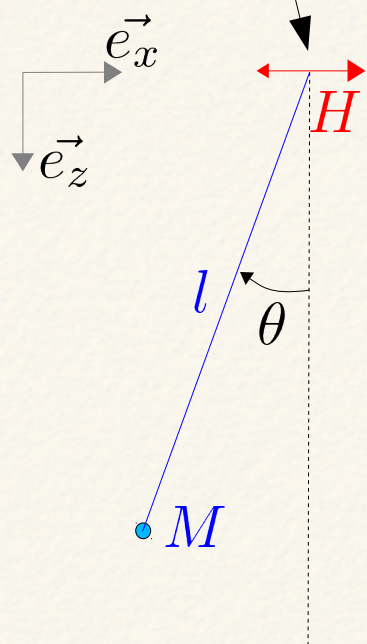


Seismic noise and suspended mirrors



Ground vibrations up to $\sim 1 \mu\text{m}/\sqrt{\text{Hz}}$ at low frequency decreasing down to $\sim 10 \text{ pm}/\sqrt{\text{Hz}}$ at 100 Hz

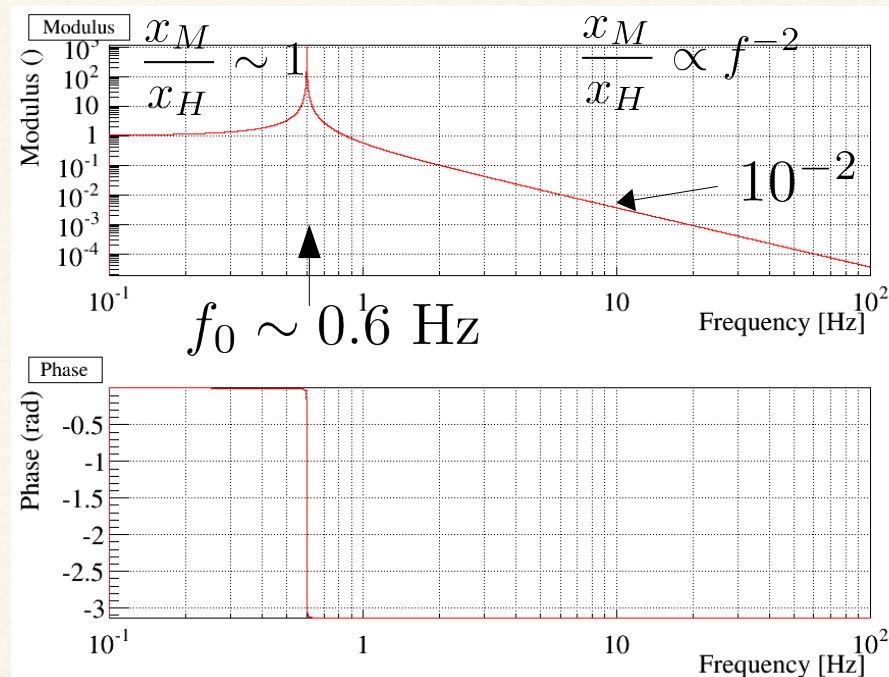
$\gg 10^{-19} \text{ m}/\sqrt{\text{Hz}}$ needed to detect GW !!



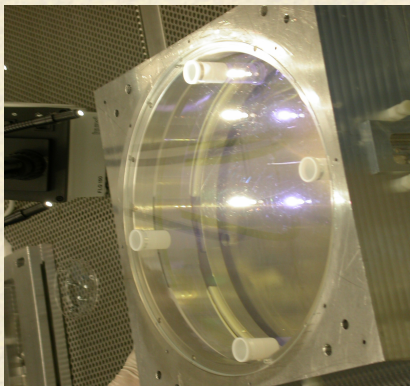
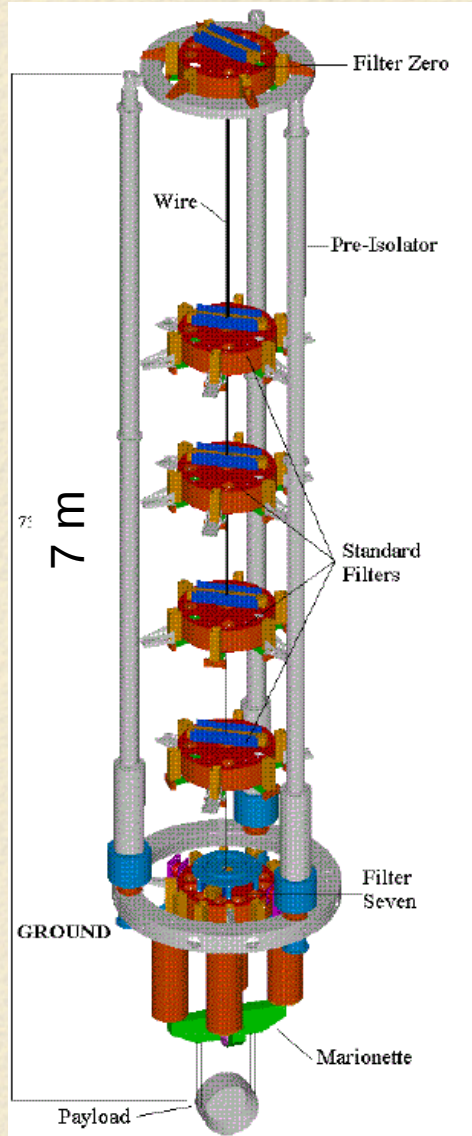
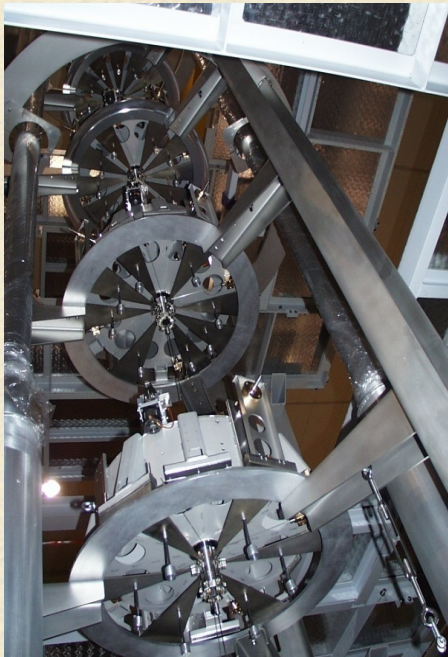
Assuming δx_H small and sinusoidal and θ small:

$$\underline{x_M} = \underline{\mathcal{H}} \times \underline{x_H}$$

Transfer function



Seismic noise and the Virgo suspension



- **Passive attenuation:** 7 pendulum in cascade

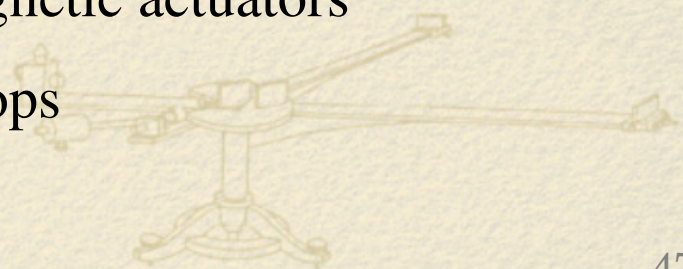
$$\text{At 10 Hz: } \frac{x_{\text{mirror}}}{x_{\text{ground}}} \sim (10^{-2})^7 = 10^{-14}$$

$$x_{\text{ground}} \sim 10^{-9} \text{ m}/\sqrt{\text{Hz}}$$

$$\rightarrow x_{\text{mirror}} \sim 10^{-23} \text{ m}/\sqrt{\text{Hz}}$$

This noise directly modifies the positions of the mirror surfaces, and thus $\delta\Delta L$ and $h_{\text{rec}}(t)$!

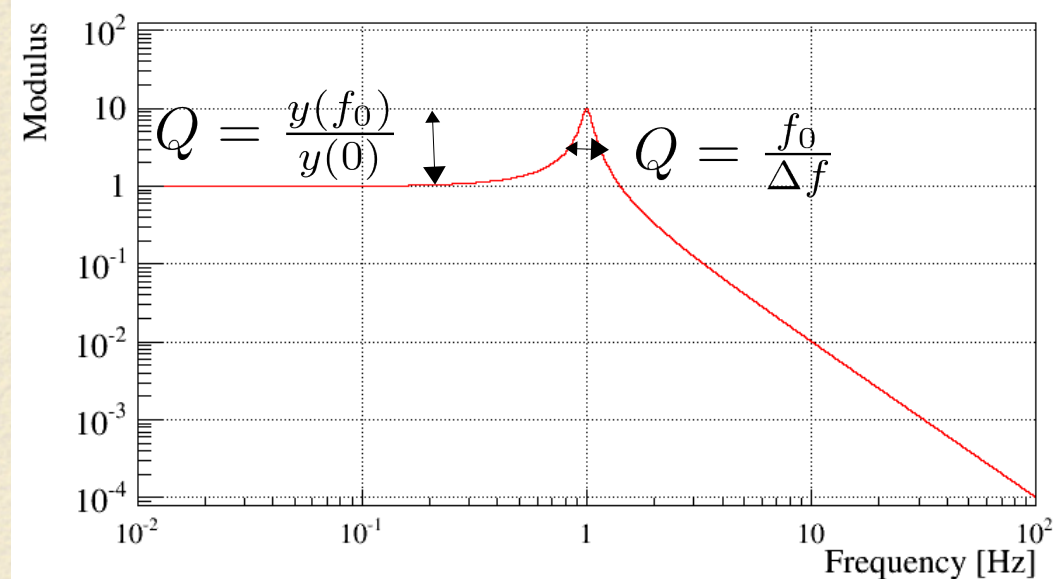
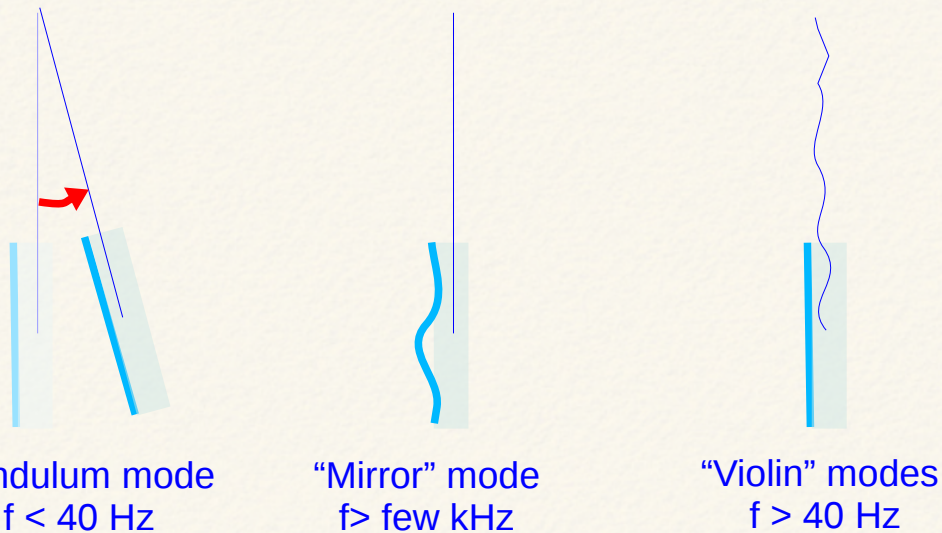
- **Active controls** at low frequency
 - Accelerometers or interferometer data
 - Electromagnetic actuators
 - Control loops



Some noises: thermal noise

- Microscopic thermal fluctuations

--> dissipation of energy through excitation of the macroscopic modes of the mirror



This noise directly modifies the positions of the mirror surfaces, and thus $\delta\Delta L$ and $h_{rec}(t)$!

- We want high quality factors Q to concentrate all the noise in a small frequency band

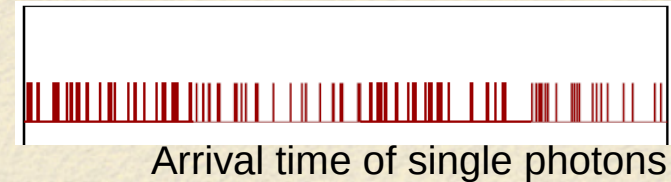
What is the shot noise ?



- Fluctuations of arrival times of photons (quantum noise)

Power received by the photodiode: P_t

→ $N = \frac{P_t}{h\nu}$ photons/s on average.



Standard deviation on this number: $\sigma_N = \sqrt{N}$

→ $\sigma_{P_t} = \sigma_N \times h\nu = \sqrt{\frac{P_t}{h\nu}} h\nu = \sqrt{P_t h\nu}$

Virgo laser: $\lambda = 1.064 \mu\text{m} \rightarrow \nu = \frac{c}{\lambda} \sim 2.8 \times 10^{14} \text{ Hz}$

Working point: $P_t \sim 80 \text{ mW} \rightarrow \sigma_{P_t} = 0.1 \text{ nW}/\sqrt{\text{Hz}}$

→ a variation of power is interpreted as a variation of distance $\delta\Delta L$

$$\delta P_t = (\text{Virgo response}) \times L_0 \times h$$

(in W/m)

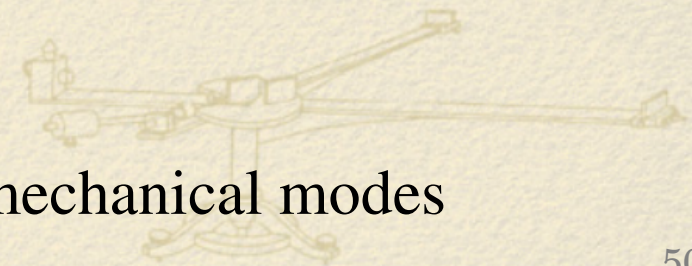
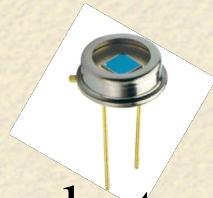
$$h_{\text{equivalent}} = \frac{1}{L_0} \frac{\sigma_{P_t}}{(\text{Virgo response})}$$

Some other noises

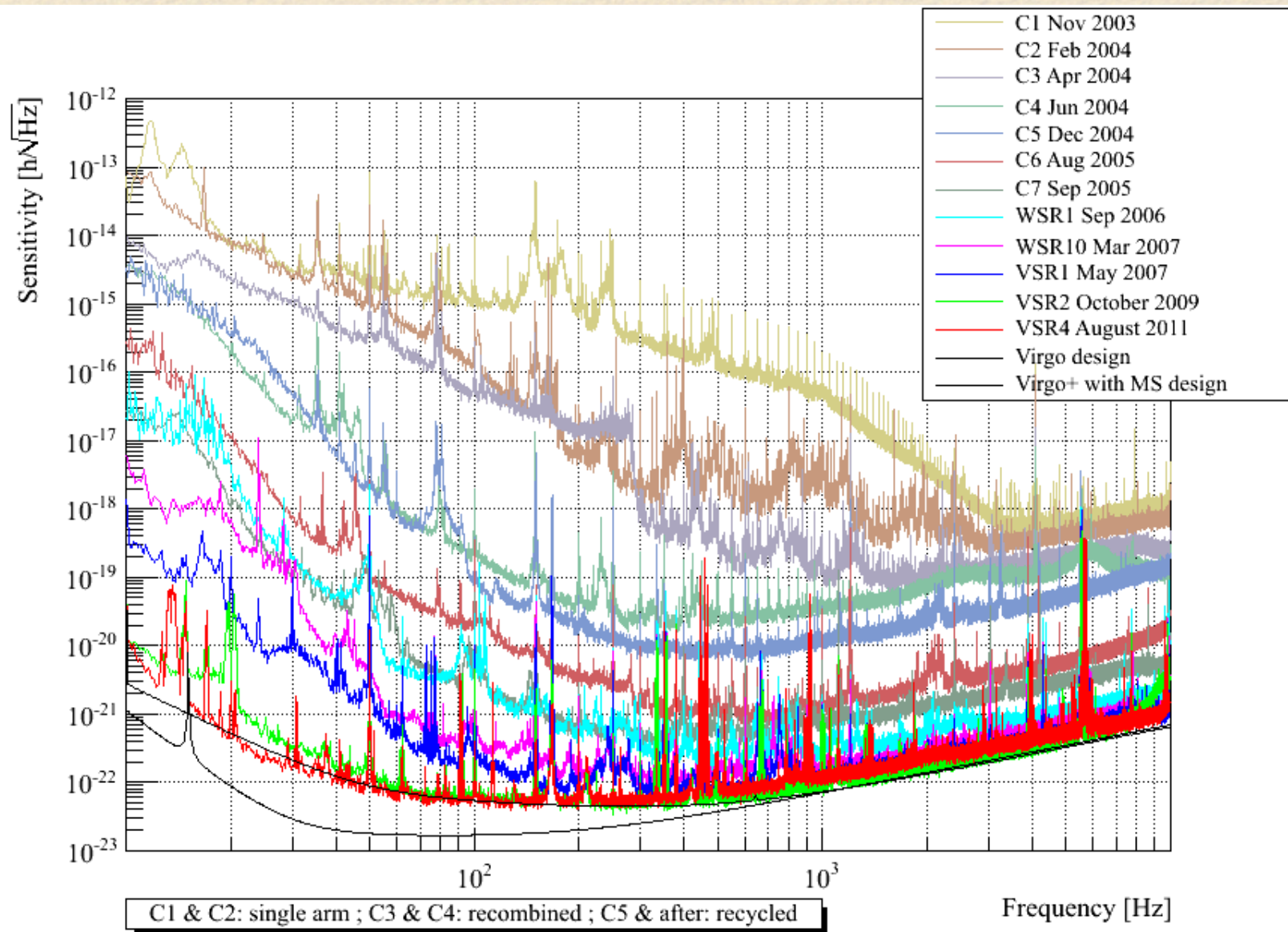
- Acoustic vibrations and refraction index fluctuations
 - Main elements installed in vacuum
- Laser: amplitude, frequency, jitter noise
 - Lots of control loops to reduce these noises



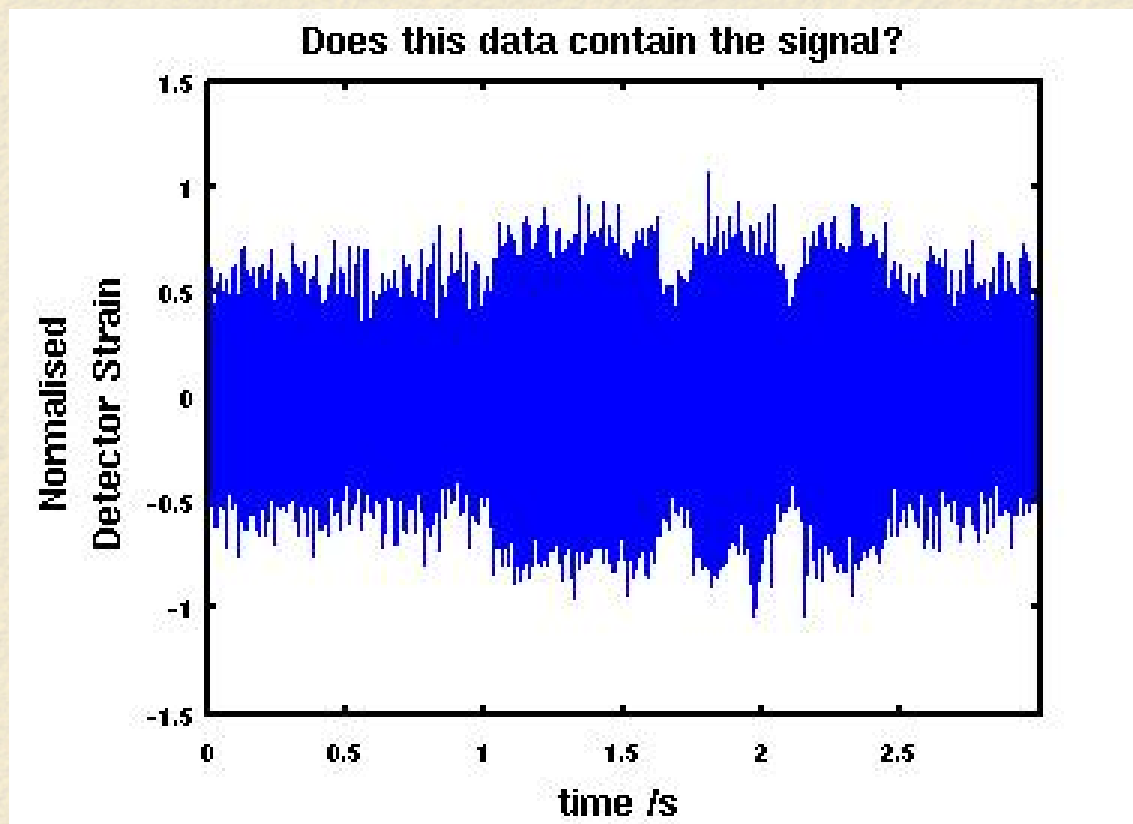
- Electronics noise
 - Challenge for the electronics to measure down to $0.1 \text{ nW}/\sqrt{\text{Hz}}$
- Non-linear noise from diffuse light
 - Need dedicated optical elements with specific mechanical modes



History of Virgo noise curve



Noises are not always stationary...



“Glitches” are impulses of noise. They might look like a transient GW signal...

→ Now it is time to play with the data analysis !